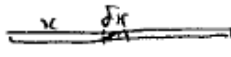
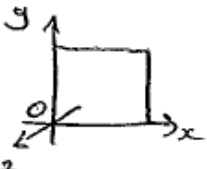
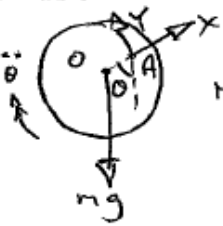
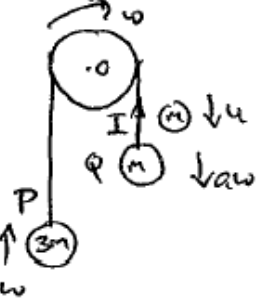


June 2006
6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
1. (a)	 $I = \int_0^{2a} \frac{m}{2a} x^2 dx$ $= \frac{m}{2a} [x^3]_0^{2a}$ $= \frac{4}{3} ma^2$	M1 A1 A1 (3)
(b)	 $I_x = I_y = \frac{4}{3} ma^2 \text{ (stitching rule)}$ $I_z = I_x + I_y = \frac{8}{3} ma^2 \text{ (per axes)}$	M1 M1A1 (3) (6)
2.	$\underline{d} = \begin{pmatrix} 4 \\ 5 \\ -5 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ $\underline{E} \cdot (2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = \frac{1}{2} \times \frac{1}{2} \times 12^2 = 36$ <p>but $\underline{E} = \lambda(2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ (particle starts at rest)</p> $\Rightarrow \lambda(2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \cdot (2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = 36$ $\Rightarrow \lambda^2(4 + 4 + 1) = 36$ $\Rightarrow \lambda^2 = 36$ $\Rightarrow \lambda = 4$ $\underline{F}_2 = 4 \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \underline{7\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}}$	B1 M1A2 M1 M1 A1 M1A1 (9)
3. (a)	$m^2 - 2m = m(m-2) = 0$ $\Rightarrow m = 0 \text{ or } m = 2$ $\Rightarrow \underline{r} = \underline{A} + \underline{B}e^{2t}$ $t=0, \underline{r} = 3\mathbf{i} \Rightarrow \underline{A} + \underline{B} = 3\mathbf{i}$ $\underline{\dot{r}} = 2\underline{B}e^{2t}$ $t=0, \underline{\dot{r}} = \mathbf{j} \Rightarrow \underline{B} = \frac{1}{2}\mathbf{j}$ $\Rightarrow \underline{r} = (3\mathbf{i} - \frac{1}{2}\mathbf{j}) + \frac{1}{2}\mathbf{j}e^{2t} = 3\mathbf{i} + \frac{1}{2}\mathbf{j}(e^{2t} - 1)$	M1 A1 A1 M1A1 M1 A1 A1 (8)
(b)	<p>Particle moves in a straight line</p> <p>Equation of line is $x = 3$</p>	B1 B1 (2) (10)

Question Number	Scheme	Marks
4. (a)	$\underline{R} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} = \underline{4\underline{i} + 2\underline{k}}$	M1 A1 (2)
(b)	$\begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} 0 \\ -6 \\ -6 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$ $= \begin{pmatrix} 3 \\ 3 \\ -6 \end{pmatrix}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -6 \end{pmatrix}$ $\begin{pmatrix} 2y - 2xz \\ 4z - 2xy \\ -4xy \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -6 \end{pmatrix}$ <p>e.g. $x = -3/2, y = 3/2, z = 0$</p> $\underline{r} = \begin{pmatrix} -3/2 \\ 3/2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$	M1 A1 A1 A1 A1 M1 A1 ft. B1 M1 A1 (10) (12)
5. (a)	$m\underline{v} \equiv (m + \delta m)(\underline{v} + \delta \underline{v}) + (-\delta m)(\underline{k} + \underline{v} + \delta \underline{v})$ $m\underline{v} \equiv m\underline{v} + m\delta \underline{v} + \underline{v}\delta m - \underline{k}\delta m - \underline{v}\delta m$ $\underline{k}\delta m \approx m\delta \underline{v}$ <p>In the limit, as $\delta t \rightarrow 0$,</p> $\frac{dm}{dv} = \frac{m}{K} *$	M1 A3 M1 A1 (6)
(b)	$\int_M^{m_1} \frac{dm}{m} = \int_U^V \frac{dv}{K}$ $\ln m_1 - \ln M = \frac{1}{K} (V - U)$ $\ln \frac{m_1}{M} = \frac{1}{K} (V - U)$ $m_1 = M e^{\frac{V-U}{K}}$ $\text{Amount of fuel} = M - m_1 = M \left(1 - e^{\frac{V-U}{K}} \right)$	M1 A1 M1 A1 M1 A1 (6) (12)

Question Number	Scheme	Marks
6(c)	 $I_A = \frac{1}{2}ma^2 + m\left(\frac{1}{2}a\right)^2 = \frac{3}{4}ma^2$ $N(A), -mg\frac{a}{2}\sin\theta = \frac{3}{4}ma^2\ddot{\theta}$ $-\frac{2g}{3a}\sin\theta = \ddot{\theta}$	M1 A1 M1 A2 (5)
(b)	$F\sin\theta = 0, \quad -\frac{2g}{3a}\theta = \ddot{\theta}$ $T = 2\pi\sqrt{\frac{3a}{2g}}$	M1 A1 (2)
(c)	$R(F), \quad Y - mg\sin\theta = m\frac{a}{2}\ddot{\theta}$ $\Rightarrow Y = mg\sin\theta + \frac{ma}{2}\left(-\frac{2g}{3a}\sin\theta\right)$ $= \frac{2mg\sin\theta}{3}$	M1 A2 M1 A1 (5) (12)
7.(a)	 $u = \sqrt{2ag}$ <p style="text-align: center;"><u>CAM about O:</u></p> $m\sqrt{2ag}a = 2ma^2\omega + 3ma^2\omega + \frac{1}{2}2ma^2\omega$ $\frac{\sqrt{2ag}}{6a} = \omega$ $\frac{1}{3}\sqrt{\frac{g}{2a}} = \omega \quad *$	B1 M1 A2 A1 (5)
(b)	<p><u>For Q:</u></p> $-I = 2maw - ma^2\alpha$ $\Rightarrow I = 6maw - 2maw = 4maw$ $= \frac{4ma}{3}\sqrt{\frac{g}{2a}} = \frac{m}{3}\sqrt{8ag}$	M1 A1 A1 (5)
(c)	$\text{PE Gain of P} = \text{KE loss of P} + \text{KE loss of Q} + \text{KE loss of pulley} + \text{PE loss of Q}$ $3mgd = \frac{1}{2}3ma^2\omega^2 + \frac{1}{2}2ma^2\omega^2 + \frac{1}{2}ma^2\omega^2 + 2mgd$ $\cancel{gd} = 3ma^2\omega^2$ $gd = 3a^2 \cdot \frac{1}{9} \frac{g}{2a} = \frac{a}{6}$	M1 A3 M1 A1 (6) (14)

