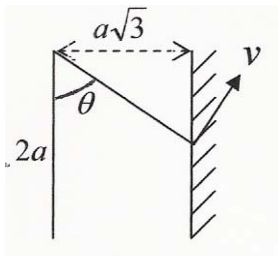
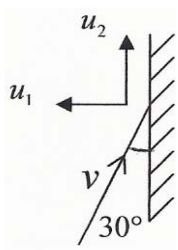
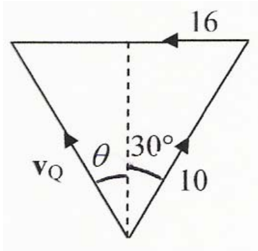
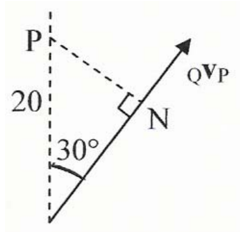
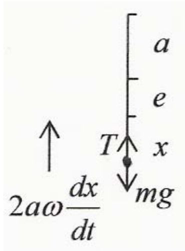
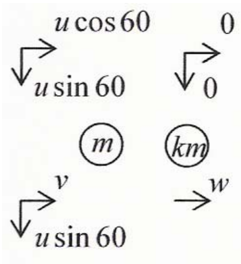
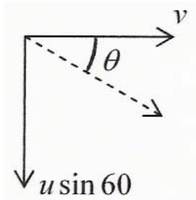
Question Number	Scheme	Marks
1.	(a) $\frac{1}{2}\frac{dv}{dt} = \frac{1}{2}g - 2v$	M1
	$\Rightarrow 5\frac{dv}{dt} = 49 - 20v$ (*)	A1 (2)
	(b) $\int \frac{5dv}{49-20v} = \int dt$ (separate variables)	M1
	$\frac{-5}{20}\ln(49-20v) = t$ (+c)	A1
	$t = 0, v = 0 \Rightarrow c = -\frac{1}{4}\ln 49$ (attempt to get c)	M1
	$t = \frac{1}{4}\ln\left(\frac{49}{49-20v}\right)$	
	$t = 1 : 1 = \frac{1}{4}\ln(\frac{49}{49-20v})$ (correct use of logs/exp)	M1
	$\rightarrow v \approx 2.41ms^{-1}$ or $2.4ms^{-1}$	A1 (5)
		Total 7 marks
2.	(a) Energy: $\frac{1}{2}m\left(\frac{37ga}{5} - v^2\right) = mg.2a(1-\cos\theta)$ 	M1 A1
	Using $\theta = \frac{\pi}{3}$ & solve: $\rightarrow v = \sqrt{\frac{27ga}{5}}$ (*)	M1 A1 (4)
	(b) Impact: $u_1 = ev\sin 30$ 	M1 A1
	KE loss $= \frac{1}{2}m\left(v^2 \sin^2 30 - e^2v^2\sin^2 30\right)$	
	$\left[+\frac{1}{2}mv^2\cos^2 30 - \frac{1}{2}mu_2^2\right] = \frac{3mga}{5}$	M1 A1
	[Using $u_2 = v\cos 30$ if necessary &] reducing to equation in (m, g, a) e alone	
	$\frac{3mga}{5} = \frac{1}{2}m.\frac{27ga}{5}.\frac{1}{4}(1-e^2)$	A1
	Solve for e: $\rightarrow e = \frac{1}{3}$	M1 A1 (7)
		Total 11 marks

Question Number	Scheme	Marks
3.	(a)	
	(i) $\mathbf{v}_Q = {}_Q\mathbf{v}_P + \mathbf{v}_P$	
	$\lvert\mathbf{v}_Q\rvert^2 = (10\cos 30)^2 + (16 - 10\sin 30)^2$	M1 A1
	$= 75 + 121$	
	$\Rightarrow \underline{\lvert\mathbf{v}_Q\rvert = 14 ms^{-1}}$	A1
		
	(ii) $\tan\theta = \dfrac{16 - \sin 30}{10\cos 30}$ (o.e.)	M1
	$\theta \approx 51.8°$, $\Rightarrow$ bearing $\underline{308°}$ (nearest degree)	A1, A1
		(6)
	(b) At nearest approach: $PN = 20\sin 30$	M1 A1
	$= \underline{10\ km}$	A1
		(3)
	(c) $Time = \dfrac{20\cos 30}{10} \approx 1.732\ hrs$	M1 A1
	$\Rightarrow \underline{Time \approx 4.44\ pm}$ (AWRT)	A1
		(3)
		Total 12 marks
	<u>Alternatives</u>	
	(a) Use of cosine rule in velocity vector triangle.	
	(b) & (c) Use of scalar product of relative velocity and relative position or differentiating magnitude of relative position vector squared to find the minimum distance and time at which it occurs.	

Question Number	Scheme	Marks
4.	(a) R(↓) $m\frac{d^2x}{dt^2} = mg - T - 2m\omega\frac{dx}{dt}$ (4 terms)	M1 A1 ↓
	$m\frac{d^2x}{dt^2} = mg - \frac{2m\omega^2 a}{a}(e+x) - 2m\omega\frac{dx}{dt}$	M1 ↓
	$\rightarrow \frac{d^2x}{dx^2} + 2\omega\frac{dx}{dt} + 2\omega^2 x = 0$ (*)	M1 A1
		(5)
	(b) $x = e^{-\omega t}(A\cos\omega t + b\sin\omega t)$	
	$t = 0, x = 0 \Rightarrow A = 0$	B1
	$\frac{dx}{dt} = -\omega e^{-\omega t}.B\sin\omega t + e^{-\omega t}.B\omega\cos\omega t$ (use of product rule)	M1
	$t - 0, \frac{dx}{dt} = U : U = B\omega \Rightarrow B = \frac{U}{\omega}$	M1 A1
		(4)
	(c) $\frac{dx}{dt} = -Ue^{-\omega t}\sin\omega t + Ue^{-\omega t}\cos\omega t = 0$	M1
	$\Rightarrow \tan\omega t = 1$ (solve for $\tan\omega t$)	M1
	$\Rightarrow t = \frac{\pi}{4\omega}$	A1
		(3)
		Total 12 marks

Question Number	Scheme	Marks
5.	(a) $CLM(\leftrightarrow): mu\cos 60 = mv + kmw$	M1 A1
	$NLI: \frac{1}{2}u\cos 60 = w - v$	M1 A1
	 	
	Solve for w: $(1+k)w = \frac{1}{2}u\left(1+\frac{1}{2}\right)$	M1
	$\Rightarrow w = \frac{3u}{4(k+1)}$ (*)	A1 (6)
	(b) Solve for v $\rightarrow v = \frac{u(2-k)}{4(k+1)}$	M1 A1
	$\tan\theta = 2\sqrt{3} = \frac{u\sin 60}{v}$	M1 A1
	$= \frac{u\sqrt{3}}{2}\cdot\frac{4(k+1)}{u(2-k)}$	
	 	
	Solve k: $\rightarrow k = \frac{1}{2}$	M1 A1 (6)
	(c) $k = \frac{1}{2} \Rightarrow v = \frac{u}{4}, w = \frac{u}{2}$	B1
	KE loss $= \frac{1}{2}mu^2 - \left(\frac{1}{2}m.\frac{u^2}{16} + \frac{1}{2}m.\frac{3u^2}{4} + \frac{1}{2}.\frac{1}{2}m.\frac{u^2}{4}\right)$	M1 A1
	$= \frac{1}{2}mu^2\left(1 - \frac{1}{16} - \frac{3}{4} - \frac{1}{8}\right)$	
	$= \frac{1}{32}mu^2$	A1 (4)
		Total 16 marks

Question Number	Scheme	Marks
6.	(a) PE of R $= -\sqrt{2}mga\cos 2\theta$ (+c) (1)	B1
	PE of LH mass $= -\frac{3}{2}mg(2a - 2a\sin(45+\theta))$ (+c) (2)	M1 A1
	PE of RH mass $= -\frac{3}{2}mg(2a - 2a\sin(45-\theta))$ (+c) (3)	A1
	V = (1) + (2) + (3) (in terms of θ etc.)	M1
	$= -\sqrt{2}mga\cos 2\theta - \frac{3}{2}mg\left[4a - a\sqrt{2}(\cos\theta + \sin\theta + \cos\theta - \sin\theta)\right]$	M1
	$= -\sqrt{2}mga\cos 2\theta - \frac{3}{2}mga\left(-2\sqrt{2}\cos\theta + 4\right)$	
	$= \sqrt{2}mga(3\cos\theta - \cos 2\theta)$ + constant (*)	A1 (7)
	(b) $\frac{dV}{d\theta} = \sqrt{2}mga(-3\sin\theta + 2\sin 2\theta)$	M1 A1
	$\frac{dV}{d\theta} = 0 \Rightarrow 2\sin 2\theta - 3\sin\theta = 0$	M1
	$\Rightarrow \sin\theta(4\cos\theta - 3) = 0$	M1
	$\Rightarrow \theta = 0$, or $\theta = \pm\arccos\frac{3}{4} (= \pm 0.723)$	A1, A1 (6)
	(c) $\frac{d^2V}{d\theta^2} = \sqrt{2}mga(-3\cos\theta + 4\cos 2\theta)$	M1 A1
	$\cos\theta = \frac{3}{4}: \frac{d^2V}{d\theta^2} = \sqrt{2}mga\left(-3.\frac{3}{4} + 4\left(2.\frac{9}{16} - 1\right)\right)$	M1
	$= \sqrt{2}mga\left(-\frac{9}{4} + \frac{1}{2}\right)$	
	<0 $\therefore$ Unstable	A1 (4)
		Total 17 marks