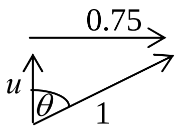
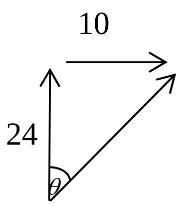
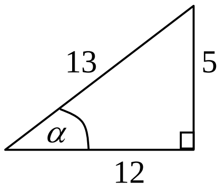
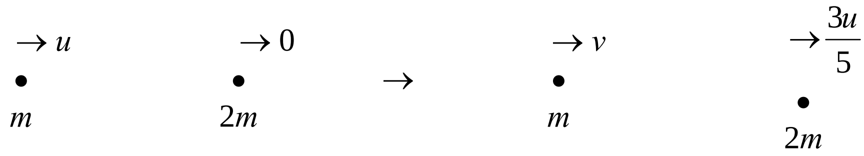
Question Number	Scheme	Marks
1.	Let boy's velocity be ↑ u, → 0.75	M1
	Speed = 1 ⇒ $1^2 = u^2 + \frac{9}{16}$, $\therefore\ u^2 = \frac{7}{16}$ or $u = \frac{\sqrt{7}}{4}$ or 0.661…	M1 A1
	Time = $\frac{100}{\sqrt{7}/4}$ = 151.18… = 151s	A1
	 $\sin\theta = \frac{0.75}{1}$ $\Rightarrow \theta = 48.6$	M1
	∴ Bearing is 049° or 048.6°	A1 (6)
		(6 marks)
2.	Let wind be ↑ W_y, → W_x	M1
	Relative to A: ↑ W_y, → $W_x - 10$ From South, ⇒ $W_x = 10$	M1, A1
	Relative to B: ↑ $W_y - 14$, → W_x From SW, ⇒ $W_y - 14 = W_x$ $\therefore\ W_y = 24$	M1, A1
	∴ Magnitude of $W = \sqrt{10^2 + 24^2}$ = 26 km h^{-1}	A1
	 $\tan\alpha = \frac{10}{24} \Rightarrow \alpha = 22.6$	
	∴ Bearing 023° or 022.6°	A1
		(7 marks)

Question Number	Scheme	Marks
3.	(↓) $mg - mkv^2 = ma$	M1 A1
	$g - kv^2 = v\dfrac{\mathrm{d}v}{\mathrm{d}x}$ $\quad v\dfrac{\mathrm{d}v}{\mathrm{d}x}$	M1
	$x = \displaystyle\int \frac{v}{g - kv^2}\,\mathrm{d}v$	M1
	$x = -\dfrac{1}{2k}\ \ln\left\lvert g - kv^2\right\rvert + c$	M1 A1
	$x = 0,\ v = 0 \Rightarrow 0 = -\dfrac{1}{2k} + c$	M1
	$x = \dfrac{1}{2k}\ \ln\left\lvert\dfrac{g}{g - kv^2}\right\rvert$	
	$\mathrm{e}^{2kx} = \dfrac{g}{g - kv^2}$	A1
	$kv^2 = g(1 - \mathrm{e}^{-2kx})$	M1
	$v = \sqrt{\dfrac{g}{k}\left(1 - e^{-2kD}\right)}$ $\quad$ must use D	A1
		(11 marks)

Question Number	Scheme	Marks
4. (*a*)	P.E.of rod = $mg \times 2a \sin 2\theta$	B1
	$AC = a \cot \theta$	B1
	EPE in String $= \frac{1}{2} \times \frac{3}{4} \times \frac{mg}{a}(a \cot \theta - a)^2$	M1 A1
	Total P.E $V = mg.2a \sin 2\theta + \frac{3}{8}\frac{mg}{a}(a \cot \theta - a)^2$	M1
	$= \frac{mga}{8}[16 \sin 2\theta + 3 \cot^2 \theta - 6 \cot \theta + 3]$	M1
	i.e. $V = \frac{mga}{8}[16 \sin 2\theta + 3\cot^2 \theta - 6 \cot \theta] + \text{const}$ (*)	A1 cso (7)
(*b*)	$\frac{dv}{d\theta} = \frac{mga}{8}[32 \cos 2\theta - 6 \cot \theta \operatorname{cosec}^2 \theta + 6 \operatorname{cosec}^2 \theta]$	M1 A2, 1, 0
	$\left.\frac{dv}{d\theta}\right\vert_{\theta=0.535} = \frac{mga}{8}(-0.5^{0.1....}_{.....})$	M1
	$\left.\frac{dv}{d\theta}\right\vert_{\theta=0.545} = \frac{mga}{8}(0.2^{99....}_{........})$	A1
	Change of sign $\therefore \frac{dv}{d\theta} = 0$ in range, so $\exists$ find a position of equilibrium	A1 (6)
	(c) $\left.\frac{dv}{d\theta}\right\vert_{0.535} < 0,\ \left.\frac{dv}{d\theta}\right\vert_{\theta=0.545} > 0$	M1
	So turning point is *minimum*, $\therefore$ equilibrium is *stable*	A1, A1 (3)
		(16 marks)

Question Number	Scheme	Marks
5. (*a*)	Auxiliary Equation.: $m^2 + 2m + 2 = 0, \quad \Rightarrow m = -1 \pm \mathrm{i}$	M1, A1
	$\therefore$ Complementary. Function is: $x = \mathrm{e}^{-t}\,(\mathrm{A}\cos t + \mathrm{B}\sin t)$	M1 ft
	Let $x = p\cos 2t + q\sin 2t,\ \dot{x} = -2p\sin 2t + 2q\cos 2t,\ \ddot{x} = -4x$	M1
	Sub. in D.E.	
	$-2p\cos 2t - 2q\sin 2t - 4p\sin 2t + 4q\cos 2t = 12\cos 2t - 6\sin 2t$	M1
	$-2p + 4q = 12,\ -4p - 2q = -6$	A1
	$-10p = 0 \quad \Rightarrow p = 0,\ q = 3$	M1
	$\therefore \quad x = 3\sin 2t + \mathrm{e}^{-t}\,(\mathrm{A}\cos t + B\sin t)$	A1
	$t = 0, x = 0 \quad \Rightarrow \quad 0 = A$	B1
	$\dot{x} = 6\cos 2t - \mathrm{e}^{-t} B\sin t + \mathrm{e}^{-t} B\cos t$	M1
	$t = 0,\ x = 0 \ \Rightarrow \quad 0 = 6 + B \qquad \therefore B = -6$	
	$\therefore \quad x = 3\sin 2t - 6\,\mathrm{e}^{-t}\sin t$	A1 (11)
(*b*)	$\dot{x} = 6[\cos 2t + \mathrm{e}^{-t}\sin t - \mathrm{e}^{-t}\cos t]$	
	Sub $t = \frac{\pi}{4}$ $\quad \dot{x} = 6[\cos 2t + \mathrm{e}^{-t} - 6\,\mathrm{e}^{-t}\cos t]$	
	$\dot{x} = 6\left[0 + \mathrm{e}^{-\frac{x}{4}} \times \frac{1}{\sqrt{2}} - \mathrm{e}^{-\frac{x}{4}} \times \frac{1}{\sqrt{2}}\right] = 0$	M1
	$\therefore$ *P* comes to instantaneous rest when $t = \frac{\pi}{4}$	A1 (2)
(*c*)	sub $t = \frac{\pi}{4}$ in $x \quad = 3\sin\frac{\pi}{2} - 6e^{-\frac{\pi}{4}}\frac{1}{\sqrt{2}}$, $= 1.07$	M1, A1 (2)
(*d*)	$t \to \infty \qquad x \approx 3\sin 2t$, approximate period is π	M1, A1 (2)
		(17 marks)

Question Number	Scheme	Marks
6. (*a*)	 *P* before: $\rightarrow \frac{13u}{12}\cos\alpha = u$, $\uparrow \frac{13u}{12}\sin\alpha = \frac{5u}{12}$	B1, B1
		
	PCLM (→) $mu = mv + 2m\frac{3u}{5}, \Rightarrow v = \frac{-u}{5}$, i.e. $\frac{u}{5}$ // CB	M1 A1 (4)
(*b*)	NLI → $eu = v_2 - v_1 \Rightarrow eu = \frac{3u}{5} - -\frac{u}{5}$, i.e. $e = \frac{4}{5}$	M1, A1 (2)
(*c*)	$Q \rightarrow C$ $t_1 = \frac{d_1}{3u/5} = \frac{5d_1}{3u}$	B1
	P travels $\frac{u}{5} \times \frac{5d_1}{3u} = \frac{d_1}{3}$ in direction *CB*	M1
	$\therefore$ *P* is $d_1 + \frac{d_1}{3} = \frac{4d_1}{3}$ from *w* (*)	A1 c.s.o (3)
(*d*)	After hitting *w*, *Q* has speed $\frac{3u}{10}$ in direction *CB*	B1
	Velocity of *Q* relative to *P* in direction *CB* is $\frac{u}{10}$	M1
	Time for *Q* to travel $\frac{4}{3}d_1$ is: $\frac{4d_1}{3u} \times 10 = \frac{40d_1}{3u}$	A1
	Total time between collisions is: $\frac{40d_1}{3u} + \frac{5d_1}{3u} = \frac{15d_1}{u}$ (*)	A1 c.s.o (4)
(*e*)	For collision to occur *P* must travel $\uparrow d_2$ and $\downarrow d_2$ in time $\frac{15d_1}{u}$	
	$d_2\uparrow$ $t_2 = \frac{d_2}{5u/12} = \frac{12d_2}{5u}$	B1
	$\downarrow d_2$ velocity $\downarrow$ is $\frac{5u}{24}$, $\therefore$ $t_3 = \frac{d_2}{5u/24} = \frac{24d_2}{5u}$	B1, B1
	Total time is $\frac{36d_2}{5u} = \frac{15d_1}{u}$,	M1
	$\therefore 12d_2 = 25d_1$, i.e. $d_1{:}d_2 = 12{:}25$	A1 (5)
		(18 marks)