

Question Number	Scheme	Marks
1	$x^2 - 2x + 17 = (x - 1)^2 + 16$ $I = \int_1^4 \frac{1}{\sqrt{(x-1)^2 + 16}} dx = \left[\operatorname{arsinh} \frac{(x-1)}{4} \right] \text{ or equiv. } = \operatorname{arsinh} \frac{3}{4}$ <p>[M1 does not require limits; A1 f.t. on completing square, providing arsinh]</p> <p>Into ln form $\left[\ln \left[\frac{3}{4} + \sqrt{\frac{9}{16} + 1} \right] \right]$; = ln2</p> <p>[If straight to ln form : B1, $\ln \left[(x-1) + \sqrt{(x-1)^2 + 16} \right]$ M1</p> <p>Using limits correctly M1A1√, ln2 A1]</p>	<p>B1</p> <p>M1 A1√</p> <p>M1A1</p> <p>[5]</p>
2	<p>(a) Using $b^2 = a^2 (e^2 - 1)$; $[4 = 16 (e^2 - 1)]$ $e = \frac{\sqrt{5}}{2}$ or equiv. (1.12)</p> <p>(b) Distance between foci = $2ae$ $[2 \times 4 \times \frac{\sqrt{5}}{2}]$; = $4\sqrt{5}$</p> <p>[A1√ dependent on both Ms]</p> <p>(c)</p> <p style="padding-left: 40px;">Ellipse, centred on origin</p> <p style="padding-left: 40px;">Hyperbola, both branches</p> <p style="padding-left: 40px;">Totally correct, touching, with correct intercepts</p>	<p>M1A1 (2)</p> <p>M1A1√ (2)</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p> <p>[7]</p>

<p>3</p>	<p>$\frac{dx}{dt} = 1 + \cos t, \quad \frac{dy}{dt} = \sin t, \quad (\text{both})$</p> <p>$s = \int \sqrt{(1 + \cos t)^2 + (\sin t)^2} dt \quad ; \quad = \int \sqrt{2 + 2 \cos t} dt$</p> <p>Use of "half-angle formula" $[\int \sqrt{4 \cos^2 t} dt]$; $s = \left[4 \sin \frac{t}{2} \right]_{(0)}^{\left(\frac{\pi}{2}\right)}$</p> <p>Using limits correctly and surd form; $= 2\sqrt{2}$ (allow $\frac{4}{\sqrt{2}}$)</p>	<p>B1</p> <p>M1A1</p> <p>M1A1√</p> <p>M1A1 [7]</p>
<p>4</p>	<p>Using $\cosh x = \frac{e^x + e^{-x}}{2}$ and attempt to progress</p> <p>Correct intermediate step as far as $4 \left(\frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8} \right) - \left[3 \left(\frac{e^x + e^{-x}}{2} \right) \right]$</p> <p>$= \frac{e^{3x} + e^{-3x}}{2} = \cosh 3x$</p> <p>(b) Using part (a) to reduce to $\cosh^2 x = [2]$</p> <p>Correct method to form $\ln x$ or find e^x or e^{2x}</p> <p>$x = \ln(\sqrt{2} + 1), \quad \ln(\sqrt{2} - 1)$ or equivalent</p> <p>or $\frac{1}{2} \ln(3 + 2\sqrt{2}), \quad \frac{1}{2} \ln(3 - 2\sqrt{2}),$ (after finding $e^{2x} = \dots$)</p>	<p>M1</p> <p>A1</p> <p>A1 (3)</p> <p>M1</p> <p>M1</p> <p>A1 A1√</p> <p>(4) [7]</p>

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5	$\frac{dy}{dx} = \frac{1}{\sec x} \cdot \sec x \tan x = \tan x ; \quad \psi = x$ $s = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int \sqrt{1 + \tan^2 x} dx$ $= \int \sec x dx ; \quad = \ln(\sec x + \tan x) (+c) \text{ or equivalent}$ <p>Method to find “c”: $s = 0, x = \frac{\pi}{3} \Rightarrow 0 = \ln(2 + \sqrt{3}) + c$</p> $c = -\ln(2 + \sqrt{3}) \text{ or } -\ln \tan \frac{5\pi}{12} \text{ or } -1.32$ $\Rightarrow s = \ln(\sec \psi + \tan \psi) - \ln(2 + \sqrt{3}) \text{ or equivalent}$	<p>B1B1</p> <p>M1</p> <p>A1A1</p> <p>M1</p> <p>A1</p> <p>A1√</p> <p>[8]</p>
6	<p>(a) $\frac{dy}{dx} = 3 \cosh^2 x \sinh x$</p> $\frac{d^2y}{dx^2} = 3 \cosh^3 x + 6 \cosh x \sinh^2 x \text{ or equivalent}$ <p>Application of formula for radius of curvature $[\rho = \frac{(1 + 9 \cosh^4 x \sinh^2 x)^{\frac{3}{2}}}{3 \cosh^3 x + 6 \cosh x \sinh^2 x}]$</p> <p>Use of $\sinh^2 x = \cosh^2 x - 1$</p> $\rho = \frac{(1 + 9c^4(c^2 - 1))^{\frac{3}{2}}}{3c\{c^2 + 2(c^2 - 1)\}} = \frac{(9c^6 - 9c^4 + 1)^{\frac{3}{2}}}{3c\{3c^2 - 2\}} \quad \text{AG}$ <p>(b) $\cosh x = \frac{5}{4}$</p> <p>Using found value of $\cosh x$ in formula for ρ; $\rho = 4.8$</p>	<p>B1</p> <p>M1A1</p> <p>M1</p> <p>M1</p> <p>A1* (6)</p> <p>B1</p> <p>M1A1 (3)</p> <p>[9]</p>

7	<p>(a) $I_n = -\frac{2}{3} \left[x^n (4-x)^{\frac{3}{2}} \right]_0^4 + \frac{2}{3} n \int_0^4 x^{n-1} (4-x)^{\frac{3}{2}} dx$</p> $= \frac{2}{3} n \int_0^4 x^{n-1} (4-x)^{\frac{3}{2}} dx$ $= \frac{2}{3} n \int_0^4 4x^{n-1} (4-x)^{\frac{1}{2}} dx - \frac{2}{3} n \int_0^4 x^n (4-x)^{\frac{1}{2}} dx$ $\Rightarrow I_n = \frac{8}{3} n I_{n-1} - \frac{2}{3} n I_n$ $[(2n+3)I_n = 8nI_{n-1}] \quad \Rightarrow I_n = \frac{8n}{2n+3} I_{n-1} \quad \text{AG}$ <p>(b) Relating I_2 to I_0 using result from (a)</p> $I_2 = \frac{16}{7} \cdot \frac{8}{5} I_0 = \frac{2048}{105} \left(19 \frac{53}{105} \right)$	<p>M1A1</p> <p>A1√</p> <p>M1A1</p> <p>A1* (6)</p> <p>M1</p> <p>A1A1 (3)</p> <p>[9]</p>
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8	<p>(a) $\operatorname{ar\,tanh} \frac{1}{\sqrt{2}} = \frac{1}{2} \ln \left(\frac{1 + \frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}} \right)$ or equivalent</p> <p style="text-align: center;">$= \frac{1}{2} \ln \left[\frac{\sqrt{2} + 1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} \right]$ or equivalent</p> <p style="text-align: center;">$= \frac{1}{2} \ln(\sqrt{2} + 1)^2 = \ln(\sqrt{2} + 1)$ AG</p> <p><i>Alternative Approach</i></p> <p>If using $y = \operatorname{ar\,tanh}(\sin x) \Rightarrow \tanh y = \left(\frac{1}{\sqrt{2}} \right)$ [or $\cosh^2 y = 2$] and then use exponentials:</p> <p>Progression as far as $e^y = \dots\dots$ or $e^{2y} = \dots\dots$ M1 Converting to ln form M1 Answer as given A1*</p> <p>Note: $\frac{1}{2} \ln(3 + 2\sqrt{2})$ can earn M1M1 but for A1* there must be a convincing further step.</p> <p>(b) $\frac{dy}{dx} = \frac{1}{1 - \sin^2 x} \cdot \cos x$; $= \frac{\cos x}{\cos^2 x} = \sec x$</p> <p>Note: If $\tanh y = \sin x$ is differentiated M1 requires $\frac{dy}{dx} = f(x)$</p> <p>(c) Attempt at by parts and use of result in (b)</p> <p style="text-align: center;">$= -\cos x \operatorname{ar\,tanh}(\sin x) + \int \cos x \sec x dx$</p> <p style="text-align: center;">$= -\cos x \operatorname{ar\,tanh}(\sin x) + x$</p> <p>Using limits correctly : $= -\frac{1}{\sqrt{2}} \ln(1 + \sqrt{2}) + \frac{\pi}{4}$ or exact equivalent</p>	<p>M1</p> <p>M1</p> <p>A1* (3)</p> <p>M1A1 (2)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1A1 (5) [10]</p>
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9	<p>(a) Correct method for finding $\frac{dy}{dx} \left[\frac{1}{P} \right]$</p> <p>Gradient of normal = $-p$</p> <p>Equation of normal: $y - 2ap = (-p)(x - ap^2)$</p> $y + px = 2ap + ap^3 \quad \text{AG}$ <p>(b) Using both equations and eliminating x or y</p> <p>$(p - q)x = 2a(p - q) + a(p^3 - q^3)$ may be unsimplified</p> $x = 2a + a(p^2 + pq + q^2)$ <p>Finding the other coordinate</p> $y = -apq(p + q)$ <p>(c) Using $pq = 3$ in both x and y (in any form)</p> <p>$[x = a(p^2 + q^2 + 5), y = -3a(p + q)]$</p> <p>Complete method for relating x and y, independent of p and q</p> <p>A correct equation, in any form</p> <p>[e.g. : $y^2 = 9a^2(p^2 + q^2 + 2pq) = 9a^2 \left\{ \left(\frac{x - 5a}{a} \right) + 6 \right\}$]</p> $y^2 = 9a(x + a)$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1*</p> <p>(4)</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>(5)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1√</p> <p>(4)</p> <p>[13]</p>
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