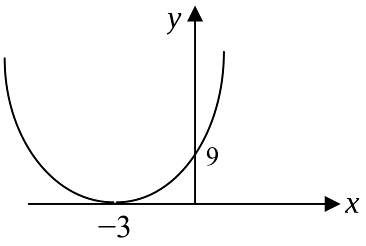
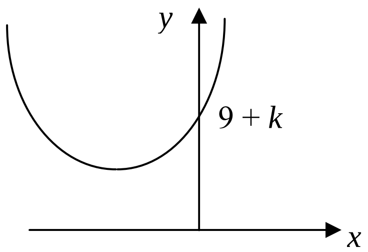
June 2006
6663 Core Mathematics C1
Mark Scheme

Question number	Scheme	Marks
	$2x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \quad (+c)$	
	$= 2x^3 + 2x + 2x^{\frac{1}{2}}$	
	for some attempt to integrate $x^n \to x^{n+1}$	**Total 4 marks**
	1^{st} A1 for either $\frac{6}{3}x^3$ or $\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$ or better	
	for all terms in x correct. Allow $2\sqrt{x}$ and $2x^1$. for + c, when first seen with a changed expression.	
	<u>Critical Values</u>	
	$(x \pm a)(x \pm b)$ with ab=18 or $x = \frac{7 \pm \sqrt{49 - -72}}{2}$ or	
	$\left(x - \frac{7}{2}\right)^2 \pm \left(\frac{7}{2}\right)^2 - 18$	
	$(x-9)(x+2)$ or $x = \frac{7 \pm 11}{2}$ or $x = \frac{7}{2} \pm \frac{11}{2}$	
	<u>Solving Inequality</u> $x > 9$ or $x < -2$ Choosing "outside"	
		Total 4 marks
	For attempting to find critical values. Factors alone are OK for M1, x = appearing somewhere for the formula and as written for completing the square 1^{st} A1 Factors alone are OK . Formula or completing the square need x = as written. 2^{nd} M1 For choosing outside region. Can f.t. their critical values. They must have two different critical values. $-2 > x > 9$ is M1A0 but ignore if it follows a correct version $-2 < x < 9$ is M0A0 whatever the diagram looks like. 2^{nd} A1 Use of $\geq$ in final answer gets A0	

ɛstion number	Scheme		Marks	
		U shape touching x-axis	B1	
		$(-3, 0)$	B1	
			B1	(3)
		Translated parallel to y-axis up		
		$(0, 9 + k)$	(2)	
			Total 5 marks	

2nd B1 They can score this even if other intersections with the x-axis are given.

2nd B1 & 3rd B1 The -3 and 9 can appear on the sketch as shown

M1 Follow their curve in (a) up only. If it is not obvious do not give it. e.g. if it cuts y-axis in (a) but doesn't in (b) then it is M0.

B1f.t. Follow through their 9

estion number	Scheme	Marks	
(a)	$a_2 = 4$		
	$a_3 = 3 \times a_2 - 5 = 7$	B1f.t.	(2)
(b)	$a_4 = 3a_3 - 5 (= 16)$ and $a_5 = 3a_4 - 5 (= 43)$		
	$3 + 4 + 7 + 16 + 43$		
	$= 73$	A1c.a.o.	(3)
		Total 5 marks	

2nd B1f.t. Follow through their a_2 but it must be a value.

$3 \times 4 - 5$ is B0. Give wherever it is first seen.

1st M1 For two further attempts to use of $a_{n+1} = 3a_n - 5$, wherever seen. Condone arithmetic slips

2nd M1 For attempting to add 5 relevant terms (i.e. terms derived from an attempt to use the recurrence formula) or an expression. Follow through their values for $a_2 - a_5$

Use of formulae for arithmetic series is M0A0 but could get 1st M1 if a_4 and a_5 are correctly attempted.

Question number		Scheme		Marks	
5.	(a)	$(y = x^4 + 6x^{\frac{1}{2}} \Rightarrow y' =)\ \ 4x^3 + 3x^{-\frac{1}{2}}$	or $\quad 4x^3 + \dfrac{3}{\sqrt{x}}$	A1	(3)
	(b)	$(x+4)^2 = x^2 + 8x + 16$			
		$\dfrac{(x+4)^2}{x} = x + 8 + 16x^{-1}$	(allow 4+4 for 8)		
		$(y = \dfrac{(x+4)^2}{x} \Rightarrow y' =)\ \ 1 - 16x^{-2}$	o.e.	M1A1	(4)
				Total 7 marks	

(a) M1 For some attempt to differentiate $x^n \rightarrow x^{n-1}$
1st A1 For one correct term as printed.
2nd A1 For both terms correct as printed.
$4x^3 + 3x^{-\frac{1}{2}} + c$ scores M1A1A0

(b) 1st M1 For attempt to expand $(x+4)^2$, must have x^2, x, x^0 terms and at least 2 correct
e.g. $x^2 + 8x + 8$ or $x^2 + 2x + 16$
1st A1 Correct expression for $\dfrac{(x+4)^2}{x}$. As printed but allow $\dfrac{16}{x}$ and $8x^0$.
2nd M1 For some correct differentiation, any term. Can follow through their simplification. N.B. $\dfrac{x^2 + 8x + 16}{x}$ giving rise to $(2x + 8)/1$ is M0A0

ALT Product or Quotient rule (If in doubt send to review)
M2 For correct use of product or quotient rule. Apply usual rules on formulae.
1st A1 For $\dfrac{2(x+4)}{x}$ or $\dfrac{2x(x+4)}{x^2}$
2nd A1 for $-\dfrac{(x+4)^2}{x^2}$

Question number	Scheme	Marks
6. (a)	$16+4\sqrt{3}-4\sqrt{3}-\left(\sqrt{3}\right)^2$ or $16-3$	M1
	$= 13$	A1c.a.o (2)
(b)	$\dfrac{26}{4+\sqrt{3}}\times\dfrac{4-\sqrt{3}}{4-\sqrt{3}}$	M1
	$=\dfrac{26(4-\sqrt{3})}{13}=\underline{8-2\sqrt{3}}$ or $8+(-2)\sqrt{3}$ or a= 8 and b = -2	A1 (2)
		Total 4 marks
(a)	M1 For 4 terms, at least 3 correct e.g. $8+4\sqrt{3}-4\sqrt{3}-\left(\sqrt{3}\right)^2$ or $16\pm8\sqrt{3}-\left(\sqrt{3}\right)^2$ or 16 + 3 4^2 instead of 16 is OK $(4+\sqrt{3})(4+\sqrt{3})$ scores M0A0	
(b)	M1 For a correct attempt to rationalise the denominator can be implied NB $\dfrac{-4+\sqrt{3}}{-4+\sqrt{3}}$ is OK	

Question number	Scheme		Marks
7.	$a+(n-1)d=k$	k = 9 or 11	M1
	$(u_{11}=)\ a+10d=9$		A1c.a.o.
	$\frac{n}{2}[2a+(n-1)d]=77$ or $\frac{(a+l)}{2}\times n=77$	l = 9 or 11	M1
	$(S_{11}=)\ \frac{11}{2}(2a+10d)=77$ or $\frac{(a+9)}{2}\times 11=77$		A1
	e.g. $a + 10d = 9$		
	$a + 5d = 7$ or $a + 9 = 14$		M1
	$a = 5$ and $d = 0.4$ or exact equivalent		A1 A1
			Total 7 marks

1st M1 Use of u_n to form a linear equation in a and d.
$a + nd = 9$ is M0A0

1st A1 For $a + 10d = 9$.

2nd M1 Use of S_n to form an equation for a and d (LHS) or in a (RHS)

2nd A1 A correct equation based on S_n.

For 1st 2 Ms they must write n or use $n = 11$.

3rd M1 Solving (LHS simultaneously) or (RHS a linear equation in a)

Must lead to a = … or d = …. and depends on one previous M

3rd A1 for $a = 5$

4th A1 for $d = 0.4$ (o.e.)

<u>ALT</u> Uses $\frac{(a+l)}{2}\times n=77$ to get $a = 5$, gets second and third M1A1 i.e. 4/7

Then uses $\frac{n}{2}[2a+(n-1)d]=77$ to get d, gets 1st M1A1 and 4th A1

<u>MR</u> Consistent MR of 11 for 9 leading to $a = 3$, $d = 0.8$ scores M1A0M1A0M1A1ftA1ft

Question number	Scheme	Marks
8. (a)	$b^2 - 4ac = 4p^2 - 4(3p+4) = 4p^2 - 12p - 16$ (=0)	M1, A1
	or $(x+p)^2 - p^2 + (3p+4) = 0 \Rightarrow p^2 - 3p - 4 (= 0)$	
	$(p - 4)(p + 1) = 0$	M1
	p = (-1 or) 4	A1c.s.o. (4)
(b)	$x = \frac{-b}{2a}$ or $(x+p)(x+p) = 0 \Rightarrow x = ...$	M1
	x (= $-p$) = <u>- 4</u>	A1f.t. (2)
		6

(a) 1st M1 For use of $b^2 - 4ac$ or a full attempt to complete the square leading to a 3TQ in p.

May use $b^2 = 4ac$. One of b or c must be correct.

1st A1 For a correct 3TQ in p. Condone missing "=0" but all 3 terms must be on one side.

2nd M1 For attempt to solve their 3TQ leading to p = …

2nd A1 For p = 4 (ignore p = -1).

$b^2 = 4ac$ leading to $p^2 = 4(3p+4)$ and then "spotting" $p = 4$ scores 4/4.

(b) M1 For a full method leading to a repeated root x = …

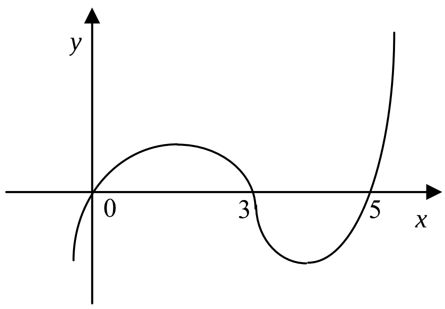
A1f.t. For x = -4 (- their p)

Trial and Improvement

M2 For substituting values of p into the equation and attempting to factorize.

(Really need to get to $p = 4$ or -1)

A2c.s.o. Achieve $p = 4$. Don't give without valid method being seen.

Question number	Scheme Marks	
9. (a)	$\text{f}(x) = x[(x-6)(x-2)+3]$ or $x^3-6x^2-2x^2+12x+3x = x($ M1	
	$\text{f}(x) = x(x^2-8x+15)$ *b*= - 8 or *c* = 15 A1	
= 1	both and *a* A1 (3)	
(b)	$(x^2-8x+15)=(x-5)(x-3)$ M1	
	$\text{f}(x) = x(x-5)(x-3)$ A1 (2)	
(c)	Shape B1	
	 their 3 or their 5 B1f.t.	

both their 3 and their 5 B1f.t. (3)
and (0,0) by implication

0 3 5 x

8

(a) M1 for a correct method to get the factor of x. $x($ as printed is the minimum.

1st A1 for b = -8 or c = 15.

-8 comes from -6-2 and must be coefficient of x, and 15 from 6x2+3 and must have no xs.

2nd A1 for a =1, b = -8 and c = 15. Must have $x(x^2 - 8x + 15)$.

(b) M1 for attempt to factorise their 3TQ from part (a).

A1 for all 3 terms correct. They must include the x.

For part (c) they must have at most 2 non-zero roots of their f(x) =0 to ft their 3 and their 5.

(c) 1st B1 for correct shape (i.e. from bottom left to top right and two turning points.)

2nd B1f.t. for crossing at their 3 or their 5 indicated on graph or in text.

3rd B1f.t. if graph passes through (0, 0) [needn't be marked] and both their 3 and their 5.

Question number	Scheme	Marks
10.(a)	$f(x) = \frac{2x^2}{2} + \frac{3x^{-1}}{-1} (+c)$ $\quad -\frac{3}{x}$ is OK	M1A1
	$(3, 7\frac{1}{2})$ gives $\frac{15}{2} = 9 - \frac{3}{3} + c$ $\quad 3^2$ or 3^{-1} are OK instead of 9 or $\frac{1}{3}$	M1A1f.t.
	$c = -\frac{1}{2}$	A1 (5)
(b)	f(-2) = 4 $+ \frac{3}{2} - \frac{1}{2}$ (*)	B1c.s.o. (1)
(c)	m = - 4 + $\frac{3}{4}$, = -3.25	M1,A1
	Equation of tangent is: $y - 5 = -3.25(x + 2)$	M1
	$4y + 13x + 6 = 0$ o.e.	A1 (4)

10

(a) 1st M1 for some attempt to integrate $x^n \to x^{n+1}$

1st A1 for both x terms as printed or better. Ignore (+c) here.

2nd M1 for use of $(3, 7\frac{1}{2})$ or (-2, 5) to form an equation for c. There must be some correct substitution. No +c is M0. Some changes in x terms of function needed.

2nd A1f.t. for a correct equation for c. Follow through their integration. They must tidy up fraction/fraction and signs (e.g. - - to +).

(b) B1cso If (-2, 5) is used to find c in (a) B0 here unless they verify f(3)=7.5.

(c) 1st M1 for attempting $m = f'(\pm 2)$

1st A1 for $-\frac{13}{4}$ or -3.25

2nd M1 for attempting equation of tangent at (-2, 5), f.t. their m, based on $\frac{dy}{dx}$.

2nd A1 o.e. must have a, b and c integers and = 0.

Treat (a) and (b) together as a batch of 6 marks.

Question number	Scheme	Marks
11.(a)	$m = \frac{8-2}{11+1} \quad (= \frac{1}{2})$	M1 A1
	$y - 2 = \frac{1}{2}(x - -1)$ or $y - 8 = \frac{1}{2}(x - 11)$ o.e.	M1
	$y = \frac{1}{2}x + \frac{5}{2}$ accept exact equivalents e.g. $\frac{6}{12}$	A1c.a.o. (4)
(b)	Gradient of $l_2 = -2$	M1
	Equation of l_2: $y - 0 = -2(x - 10)$ [$y = -2x + 20$]	M1
	$\frac{1}{2}x + \frac{5}{2} = -2x + 20$	M1
	$\underline{x = 7 \text{ and } y = 6}$ depend on all 3 Ms	A1, A1 (5)
(c)	$RS^2 = (10-7)^2 + (0-6)^2 (= 3^2 + 6^2)$	M1
	$RS = \sqrt{45} = 3\sqrt{5}$ (*)	A1c.s.o. (2)
(d)	$PQ = \sqrt{12^2 + 6^2}, = 6\sqrt{5}$ or $\sqrt{180}$ or $PS = 4\sqrt{5}$ and $SQ = 2\sqrt{5}$	M1,A1
	Area $= \frac{1}{2}PQ \times RS = \frac{1}{2}6\sqrt{5} \times 3\sqrt{5}$	dM1

$\underline{=45}$

A1 c.a.o. (4)

15

(a) 1st M1 for attempting $\frac{y_1 - y_2}{x_1 - x_2}$, must be y over x. No formula condone one sign slip, but if formula is quoted then there must be some correct substitution.

1st A1 for a fully correct expression, needn't be simplified.

2nd M1 for attempting to find equation of l_1.

(b) 1st M1 for using the perpendicular gradient rule

2nd M1 for attempting to find equation of l_2. Follow their gradient provided different.

3rd M1 for forming a suitable equation to find *S*.

(c) M1 for expression for *RS* or RS^2. Ft their *S* coordinates

(d) 1st M1 for expression for *PQ* or PQ^2.

$PQ^2 = 12^2 + 6^2$ is M1 but $PQ = 12^2 + 6^2$ is M0

Allow one numerical slip.

2nd dM1 for a full, correct attempt at area of triangle. Dependent on previous M1.
