

# Examiners' Report Summer 2008

GCE

**GCE Mathematics (8371/8374, 9371/9374)**

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# **Core Mathematics Unit C1**

## **Specification 6663**

### **Introduction**

The paper proved accessible to all the candidates and there was no evidence that they were short of time. The usual problems associated with a non-calculator paper were still evident: poor arithmetic, weaknesses when dealing with fractions and a failure to simplify and factorise expressions. A few candidates still rely far too heavily on using the quadratic formula and its use in Q4 and Q8 in particular was hardly appropriate and often led to unnecessary errors.

### **Report on individual questions**

#### **Question 1**

Most candidates showed a clear understanding of basic integration and many achieved full marks. Omitting the constant of integration was a common error as was a failure to integrate the 2. Some integrated  $5x^2$  and obtained  $\frac{5x^3}{2}$  or  $10x^3$  and a few differentiated thereby obtaining  $10x$ .

#### **Question 2**

The factor of  $x$  was usually extracted but a significant number of candidates stopped at this point and did not appear to recognize the difference of two squares. Some candidates who did identify the need to factorize  $x^2 - 9$  wrote  $(x+3)^2$  or  $(x-3)^2$  or sometimes  $(x-\sqrt{9})(x+\sqrt{9})$ . Some candidates do not appreciate the difference between “factorizing” and “solving” and there were a number of inappropriate attempts to “solve” the given expression but this was not penalised on this occasion.

#### **Question 3**

In part (a) most realized that a translation was required but some moved the curve horizontally or downwards. Some forgot to write the coordinates of the new turning point on their curve and a few placed the point  $(7, 3)$  on their horizontal axis. In part (b) the stretch was usually identified but curves often crossed at  $(0, 14)$  or touched at  $(14, 0)$ . The shape of the curve was usually “preserved” quite well but sometimes the vertex became too pointed or one end of the curve seemed to bend away towards an extra turning point. A minority of candidates still make errors by writing coordinates the wrong way round.

#### **Question 4**

Most candidates differentiated in part (a) and usually scored both marks. Sometimes the coefficient of the second term was incorrect ( values of  $2, \frac{1}{2}$  or  $\frac{1}{3}$  were seen). In part (b) most candidates were able to form a suitable equation and start to collect terms but a few simply evaluated  $f'(15)$ . There were a number of instances of poor algebraic processing from a correct equation with steps such as  $3x^2 = 12 \Rightarrow 3x = \sqrt{12}$  or  $3x^2 = 12 \Rightarrow x^2 = 9 \Rightarrow x = 3$  appearing far too often.

### **Question 5**

The notation associated with sequences given in this form still causes difficulties for some candidates and as a result parts (a) and (b) were often answered less well than part (c). A common error in the first two parts was to leave an  $x$  in the expression but most of those who could handle the notation gave clear and accurate answers. There were the usual errors in part (c), with  $a^2 - 3a - 4 = 0$  appearing quite often and it was encouraging to see most candidates factorising their quadratic expression confidently as a means of solving the equation. A few candidates still use a trial and improvement approach to questions of this type and they often stopped after finding just one solution and gained no credit.

### **Question 6**

Whilst many candidates gave clear and correct sketches in part (a) there were a number who failed to score all 3 marks here. The curve  $C$  caused the most problems: some thought that the 3 represented an upward translation of 3 and a few interpreted  $3/0$  as 0 and had  $C$  passing through the origin. Other thought  $C$  was a parabola and quite a number failed to include the branch for negative values of  $x$ . Most (but surprisingly not all) drew  $l$  as a straight line, usually with the correct gradient but they often omitted the intercept with the negative  $x$ -axis or labelled it as  $(2.5, 0)$ .

In part (b) most were able to start to solve the simultaneous equations, form the correct quadratic and factorize it. Some forgot to find the corresponding  $y$  values and a few substituted their  $x$  values into their quadratic equation rather than the equation of the curve or the line.

Those who made arithmetic errors did not check their answers against their sketch in part (a) to see if they made sense but, sadly, some of those with incorrect sketches did and rejected their negative solution of  $x$  instead of amending their sketch.

### **Question 7**

Most gave a convincing argument in part (a) but in part (b) some merely quoted the formula for the  $n$ th term and failed to substitute values for  $a$  and  $d$ . Many simplified their answer here to  $2n + 3$  and some gave the incorrect  $2n + 5$ . Part (c) was difficult for some and even those who started with a correct expression could not always complete the simplification with  $\frac{n}{2}(8 + 2n)$  being simplified to  $n(16 + 4n)$ . Apart from the candidates who tried to solve  $43 = S_n$  instead of  $u_n$  part (d) was usually answered correctly and often this was followed by a correct answer to part (e). A number of candidates though didn't appreciate that  $n$  had a value at this stage and they simply repeated their answer from part (c).

### **Question 8**

As usual many candidates floundered on this type of question or simply omitted it. Many though knew that the discriminant was required but some lost the A mark for failing to introduce an inequality before their final statement. Those who started with  $b^2 < 4ac$  tended to make fewer sign errors. In part (b) those who factorized were more successful in arriving at the two critical values than those who used the quadratic formula or tried to complete the square. A surprising number failed to give a final inequality and a few listed integers for their answer. The final mark was sometimes lost as the candidate switched the variable from  $q$  to  $x$ .

### Question 9

Most candidates could start this question and there were many fully correct solutions to part (a) although some weaker candidates were confused by the  $k$  and answers such as  $2kx^2$  or  $3k^2$  were seen. Part (b) though required some careful thought and proved quite discriminating. Many candidates identified the gradient of the line as 3.5 and sometimes they equated this to their answer to part (a). Those who realised that they needed to use  $x = -0.5$  in the resulting equation often went on to find  $k$  correctly but there were many who failed to give a convincing argument that  $k = 2$ . Some found  $f'(-0.5)$  but they set this equal to 7, -7 or 0 and a few, who confused tangents and normals, used  $-\frac{2}{7}$ . In part (c) there were attempts to substitute  $x = -0.5$  into the equation of the line or the differential and those who did substitute into the equation of the curve along with their value of  $k$  (even when this was correct) often floundered with the resulting arithmetic and so completely correct solutions to this question were rare.

### Question 10

Most candidates answered part (a) correctly. Diagrams were helpful and led to fewer mistakes than substitution into a formula especially when this was sometimes incorrect with the following versions being seen:  $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$  or  $\sqrt{(x_2 + x_1)^2 \pm (y_2 + y_1)^2}$ . There were few errors in simplifying  $\sqrt{45}$  although some had  $a = 9$ .

Those who had learnt the formulae for gradient, perpendicular gradients and the equation of a straight line usually had few problems here but some failed to quote a correct formula and then when their resulting expression was incorrect received no marks for that part. Others suffered from poor arithmetic with  $-\frac{3}{6}$  being simplified to  $-\frac{1}{3}$  or -2. More mistakes occurred with the use of the formula

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$
 than other approaches to finding the equation of a straight line.

Because the diagram was nearly to scale a significant minority of candidates “spotted” the gradient and intercept and wrote down the correct equation with no evidence to support this. Such a strategy is not recommended.

Part (c) was often correct although a number substituted  $y = 0$  into their line equation.

Finding the area of a triangle once again caused problems. Many failed to identify the correct triangle and took  $P$  to be on the negative  $x$ -axis and others assumed there were right angles in different, incorrect places. Most attempted to find  $PQ$  and used  $\frac{1}{2}PQ \times QR$  as intended but there were some successful attempts using composite methods or a determinant approach too.

### Question 11

Part (a) was usually answered correctly although there were a few errors seen:  $x^4 + 9$  for the expansion and partial or incorrect division being the common ones.

Part (b) was less well done. Some failed to realise that integration was required and others found the equation of a tangent. Those who did integrate sometimes struggled with the negative index and  $9x^{-3}$  appeared. Those who successfully integrated sometimes forgot the  $+c$  (and lost the final 3 marks) and others couldn't simplify  $9x^{-1}$  as it later became  $\frac{1}{9x}$ . Simple arithmetic let down a few too with  $3^3 = 9$  or  $9 \times 3^{-1} = 27$  spoiling otherwise promising solutions.

## **Core Mathematics Unit C2**

### **Specification 6664**

#### **Introduction**

This paper seemed to be a fair test for well-prepared candidates. Easy marks were available on most questions for those who coped with familiar demands, but there were several parts that proved challenging and discriminating. As in recent Core Mathematics C2 papers, candidates often had difficulty with trigonometry and logarithms. In most cases candidates showed sufficient working to justify method marks, but accuracy marks were frequently lost through carelessness in arithmetic and algebra. Occasional blank responses to the later questions suggested that some candidates might have run short of time, but in general all nine questions were attempted. Time was often wasted on question 4(b), where a great deal of confused logarithmic work was seen, even from good candidates.

#### **Report on individual questions**

##### **Question 1**

Part (a) of this question required the use of the factor theorem (rather than long division) and most candidates were able to show  $f(-4) = 0$ . As in previous papers, a simple conclusion was expected. Many candidates failed to provide this.

The most popular strategy in part (b) was to use long division, dividing the cubic expression by  $(x + 4)$  to find the quadratic factor. Some candidates stopped at that stage and so could only gain a maximum of two marks, but of those who reached  $2x^2 - 11x + 5$  and went on to factorise this, the vast majority gained full marks. Less formal approaches to the division, including 'division by inspection', were occasionally seen and usually effective.

Candidates who solved  $2x^2 - 11x + 5 = 0$  gained neither of the final two marks until they produced the relevant factors, and then one of the factors was often left as  $\left(x - \frac{1}{2}\right)$ , which lost the final mark unless the factor 2 was included.

Some candidates went on to give 'solutions'  $x = -4$ ,  $x = 5$ ,  $x = \frac{1}{2}$ , suggesting confusion over the meaning of 'factorise'.

##### **Question 2**

Part (a) was answered correctly by the vast majority of candidates. Where an error did occur it was frequently  $\sqrt{5^0 + 2} = \sqrt{0 + 2} = 1.414$ . The values were almost always given to the requested degree of accuracy.

Use of the trapezium rule in part (b) was often clear and accurate, but the common mistake in the value of  $h$  ( $h = \frac{2}{5}$  instead of  $h = \frac{2}{4}$ ) was frequently seen. Bracketing mistakes were less common but there were some candidates who left out the main brackets and only multiplied the first two terms by  $0.5h$ .

A few candidates used the equivalent method of adding the areas of separate trapezia, which was acceptable and was usually successful.

### **Question 3**

In part (a), most candidates were aware of the structure of a binomial expansion and were able to gain the method mark. Coefficients were generally found using  ${}^nC_r$ , but Pascal's triangle was also frequently seen. The most common mistake was to omit the powers of  $a$ , either completely or perhaps in just the simplified version of the answer.

Part (b) was often completed successfully, but a significant number of candidates included powers of  $x$  in their 'coefficients', resulting in some very confused algebra and indicating misunderstanding of the difference between 'coefficients' and 'terms'. Sometimes the wrong coefficient was doubled and sometimes the coefficients were equated with no doubling. Some candidates, having lost marks in part (a) due to the omission of powers of  $a$ , recovered in part (b) and achieved the correct answer.

### **Question 4**

Most candidates completed part (a) successfully (sometimes by 'trial and error'), but sometimes a mark was lost through incorrectly rounding to 3 decimal places instead of 3 significant figures.

Responses to part (b) varied considerably. Many candidates failed to appreciate that  $5^{(2x)}$  is equivalent to  $(5^x)^2$  and either substituted the answer to part (a) into the given equation or took logs of each separate term, resulting in expressions such as  $2x\log 5 - x\log 60 + \log 35 = 0$ . The candidates who managed to form the correct quadratic in  $5^x$  were usually able to proceed to a correct solution, but sometimes the final answers were left as 5 and 7. Notation was sometimes confusing, especially where the substitution  $x = 5^x$  appeared.

Some candidates wasted a significant amount of time on part (b), producing a number of different wrong responses with a variety of logarithmic mistakes.

### **Question 5**

In part (a), most candidates were able to gain the first two marks for attempting to find the radius. The form of equation for a circle was generally well known, but occasionally radius and diameter were confused. A few candidates felt that a mid-point calculation was required at some stage of the working and a few offered  $(x - 3)^2 + (x - 1)^2 = 29$  as the circle equation.

Part (b) was less well done than part (a) but most candidates made some progress. There were a few who could not accurately calculate the gradient of the radius, then others who did not seem to realise that the tangent was perpendicular to the radius. Candidates would have found a simple sketch beneficial here. Those who tried to find the gradient by differentiating the equation of the circle were almost always unsuccessful, since methods such as implicit differentiation were not known. The final mark was often lost through careless arithmetical errors or failure to understand the term integer. Good candidates, however, often produced concise, fully correct solutions to this question.

### **Question 6**

Parts (a) and (b) of this question were very well done and the majority of candidates gained full marks here. A few, however, found the sum of 20 terms in part (a) rather than the 20<sup>th</sup> term.

Only the very best candidates achieved full marks in part (c). The main difficulty was in dealing correctly with the inequality throughout the working. Often there were mistakes in manipulation and the division by log 0.8 (a negative value) rarely resulted in the required 'reversal' of the inequality sign. Another common mistake was to say that  $5 \times 0.8^k = 4^k$ . Despite these problems, many candidates were still able to score two or three marks out of the available four.

A surprising number of candidates made no attempt at part (d) and clearly did not realise they simply had to evaluate the expression in (c). Many failed to appreciate that  $k$  had to be an integer.

### **Question 7**

Many fully correct solutions to this question were seen. Those candidates who were unwilling to work in radians, however, made things more difficult for themselves (and sometimes lost accuracy) by converting angles into degrees.

In parts (a) and (b), those who knew the correct formulae scored easy marks while those who used formulae for the circumference and the area of a circle sometimes produced muddled working. A few thought that the angle should be  $0.8\pi$ .

In finding the perimeter in part (c), most candidates realised that they needed to find the lengths  $DC$  and  $BD$ . It was surprising to see 4.5 occurring occasionally for  $DC$  as half of 7. In finding  $BD$ , most made a good attempt to use the cosine formula, although calculation slips were not uncommon. Some assumed that  $BD$  was perpendicular to  $AC$  and worked with Pythagoras' Theorem or basic triangle trigonometry, scoring no more than one mark in this part. In part (d) some candidates tried, with varying degrees of success, to use  $\frac{1}{2}bh$  and some produced lengthy

methods involving the sine rule. Occasionally the required area was interpreted as a segment and the segment area formula was used directly. Without any method for the area of an appropriate triangle, this scored no marks. Some did use the segment together with the area of triangle  $BDC$ , and although this was lengthy, the correct answer was often achieved.

### **Question 8**

In general, this was very well done, showing that many candidates know when to differentiate and when to integrate.

In part (a), most candidates successfully differentiated the equation then presented evidence that they understood that  $\frac{dy}{dx} = 0$  at a turning point. A significant number went on to find the second derivative to establish the nature of the turning point (although this was not required here, given the graph).

Most candidates integrated correctly in part (b) and a majority subtracted the area of the triangle correctly. Those who found the equation of the straight line ( $y = 11x$ ) and then proceeded to find the area of the triangle by integration (or to subtract  $11x$  from the equation of the curve and then integrate) were more likely to make mistakes than those who simply used  $\frac{1}{2}bh$  for the triangle.

It was disappointing to see the final mark being lost by candidates who did not give an *exact* final answer, as specified in the question.

A few candidates who appeared to have no correct overall strategy for answering the question were still able to earn marks for their integration.

### Question 9

The most able candidates completed this question with little difficulty, sometimes using sketches of the functions to identify the possible solutions. Generally speaking, however, graphical approaches were not particularly successful.

In part (a), most candidates were able to obtain  $45^\circ$  and went onto get  $x = 65^\circ$  but it was disappointing to see  $115^\circ$  so frequently as the second answer ( $180 - 65$ ).

A surprising number of candidates subtracted 20 rather than adding, giving the answers  $25^\circ$  and  $115^\circ$ . A number of candidates gave their first angle in radians and then proceeded to get further solutions by mixing degrees with radians.

It was encouraging that few candidates thought  $\sin(x - 20)$  was equivalent to  $\sin x - \sin 20$ .

In part (b), the majority of candidates were able to obtain one or two correct solutions, but sometimes 'correct' answers followed incorrect working. Those with a good understanding of trigonometric functions produced very concise solutions, adding  $360^\circ$  and  $720^\circ$  to their values of  $3x$ , then dividing all values by 3. Weaker candidates often gave solutions with no clear indication of method, which were very difficult for examiners to follow.

As in part (a), disastrous initial steps such as  $\cos 3x = -\frac{1}{2} \Rightarrow \cos x = -\frac{1}{6}$  were rare.

# Core Mathematics Unit C3

## Specification 6665

### Introduction

The paper proved accessible to the majority of candidates and nearly all were able to attempt and gain some marks on all 7 questions. The general standard of presentation was good. Most candidates used their calculators sensibly and were able to produce proofs in which the steps of their reasoning could be followed. However a minority used calculators to obtain answers showing no working. This is always a dangerous procedure. It is rarely given substantial credit and, in questions which are worded “Hence solve the equation ...”, for example Q2(b) on this paper, candidates have to show a connection between the parts of the question to gain marks.

The standard of algebra was generally good but many candidates showed weaknesses in using brackets and they were often omitted. This can lead to a loss of marks. For example, in Q6(a)(i),  $3e^{3x} \sin x + 2 \cos x + e^{3x} \cos x - 2 \sin x$  cannot be accepted for  $3e^{3x} (\sin x + 2 \cos x) + e^{3x} (\cos x - 2 \sin x)$  unless there is some evidence to support this interpretation.

These papers are marked online and, if a pencil is used in drawing sketches of graphs, a sufficiently soft pencil (HB) should be used and it should be noted that coloured inks do not come up well and may be invisible. A number of candidates gave answers which went outside the area on the pages which is designated for answers and this caused the examiners some difficulties.

### Report on individual questions

#### Question 1

Part (a) was usually completed successfully and the great majority were able to take logs correctly to find  $x$ . In part (b), most could differentiate correctly and evaluate  $\frac{dy}{dx}$  to find the gradient of the tangent. A few failed to evaluate  $\frac{dy}{dx}$ , giving a non-linear equation for the tangent,

and this lost the last three marks. The majority demonstrated a correct method. However the final mark was often lost. Incorrect removal of the brackets, leading to  $y = 16x - 8\ln 2$ , was frequently seen and if, as here, the question asks for exact values of  $a$  and  $b$ , giving  $b = 10.45$  loses the final mark, unless the exact solution,  $18 - 8\ln 2$ , is also given.

#### Question 2

This question was a good source of marks for the majority of candidates. The method needed in part (a) was well understood, although many candidates failed to recognise the need to give

the angle to 3 decimal places and in radians. The condition in the question  $0 < \alpha < \frac{\pi}{2}$  implies

that  $\alpha$  is in radians. Although penalised here, degrees were accepted in the other parts of the question. Part (b) produced many good solutions with many scoring 3 or 4 marks out of 5. Only the best candidates, however, were able to produce a second solution within range. Commonly a second solution greater than  $2\pi$  was found or the second solution was ignored altogether. Most saw the connection between part (a) and (c) and there were few attempts to use differentiation. A few thought that the maximum of  $f(x)$  was  $5+12$  and that  $\cos(x-\alpha)$  had a maximum at  $x-a = \frac{\pi}{2}$  or  $\pi$ .

### Question 3

The principles of transforming graphs were well understood and part (a) was well done. Part (b) was less well done. Sketching  $y = -f(x)$ , instead of  $y = f(-x)$ , was a common error. In part (c), most candidates produced correct answers for the coordinates of  $Q$  and  $R$  but the coordinates of  $P$  proved difficult for many and, not infrequently, they were just omitted. Few realised that putting  $|x+1|=0$  gave the value of  $x$  and many made hard work of the question by finding equations of the two line segments and solving their equations simultaneously. This is a complicated method and many who tried it did not get very far. There were many fully correct solutions to part (d) but many simply replaced  $|x+1|$  by  $(x+1)$  and obtained just one solution. It is possible to solve the question by squaring but, to do this, it is first necessary to rearrange the equation as  $|x+1|=2-\frac{1}{2}x$ , and this was rarely seen.

### Question 4

The method of simplifying fractions was better known than in some previous examinations with many fully correct solutions. As noted in the introduction above, a failure to use brackets can lead to a loss of marks. Candidates should not assume that examiners will read  $2(x-1)-x+1$  as  $2(x-1)-(x+1)$ . Part (b) was one of the most testing parts of the paper

and very few obtained the fully correct answer  $0 < f(x) < \frac{1}{4}$ . Many had no idea at all how to

find the range and many defaulted to the answer “all real numbers”. The method of finding the inverse in part (c) was well known and, apart from the range “all real numbers”, the candidate’s answer to part (b) was followed through for the domain in part (c). The order of the functions needed for part (d) was generally well understood and there were many fully correct solutions. The answer  $-\sqrt{5}$  was frequently overlooked. Possibly candidates were

confused by the range of  $f$ ,  $x > 3$ , but as  $g$  is applied first,  $fg(-\sqrt{5}) = g(7) = \frac{1}{8}$  is a legitimate solution.

### Question 5

This was the best done question on the paper and full marks were very common. There were a great many concise solutions to part (a), the great majority started from  $\sin^2 \theta + \cos^2 \theta = 1$  and divided all the terms by  $\sin^2 \theta$ . Those who started from  $1 + \cot^2 \theta$  often produced longer solutions but both marks were usually gained. The majority of candidates saw the

connection between parts (a) and (b) and usually obtained both correct solutions. Candidates who substituted  $\tan \theta = \frac{\sin \theta}{\cos \theta}$  could achieve the same results by a longer method but

sometimes made errors in multiplying their equation by  $\sin^2 \theta$ . A significant number who used a quadratic in  $\operatorname{cosec} \theta$  obtained  $\operatorname{cosec} \theta = 5$  and got no further, seemingly deciding that it was not possible to solve this.

### Question 6

This was a very discriminating question and candidates who obtained full marks and very few marks were both quite common. When using the product or quotient rules, candidates should be encouraged to quote the rules as it is often very difficult for examiners to establish whether or not candidates are attempting a correct method. In part (a), many made errors differentiating  $e^{3x}$ ,  $\sin x$  and, especially,  $\ln(5x+2)$ . In part (b), a lack of bracketing, again, lead to confusion. On using the quotient rule, many candidates did not notice the factor  $(x+1)$  at the first stage and went on to expand two cubic expressions in the numerator. This caused extra work but the majority who used this method did complete the question correctly.

In part (d), those who wrote  $\frac{dy}{dx}$  as  $20(x+1)^{-3}$  and differentiated directly, usually completed the question quickly. However many used the quotient rule and, for many, this caused great difficulty. It was not uncommon to see  $\frac{d}{dx}(20) = 1$  written down and many,

who avoided this mistake directly, nevertheless produced a numerator which implied that  $(x+1)^3 \times 0 = (x+1)^3$ . This lead to an equation which is not solvable at this level and frequently much time was wasted in a fruitless attempt to solve it.

### Question 7

In parts (a) and (d), candidates need to be aware that showing that something is true requires them to give reasons and conclusions. In this part (a), it is sufficient to say that a change of sign in the interval  $(1.4, 1.45)$  implies that there is a root in the interval  $(1.4, 1.45)$ . In part (c), it would be sufficient to argue that a change of sign in the interval  $(1.4345, 1.4355)$  implies that there is a root in the interval  $(1.4345, 1.4355)$  and, hence, that  $x = 1.435$  is accurate to 3 decimal places. Part (b) was very well done but candidates must put all steps in a proof and not leave it to the examiners to fill in important lines. Part (c) was very well done. Some candidates attempted part (d) using repeated iteration but the wording of the question precludes such a method and no marks could be gained this way.



# **Core Mathematics Unit C4**

## **Specification 6666**

### **Introduction**

This paper proved to be accessible and there was little evidence of candidates being unable to complete the paper owing to time constraints. This paper afforded a typical E grade candidate the opportunity to gain some marks in many of the 8 questions. There were some testing questions, particularly those questions involving the use of integration, which gave an opportunity for more able candidates to demonstrate their ability. There were, however, a minority of candidates who had little knowledge of some topics in the specification.

The attempts at the vector question continue to improve upon the previous four examination sessions with about 75% of all candidates scoring at least 5 of the 12 marks available in Q6. Examiners were impressed with the variety of different strategies employed by a minority of candidates to find the position vector of  $B$ .

There was evidence that some candidates had done little or no revision for Q3 which tested the topic of “Connected Rates of Change”. Knowledge of basic formulae such as cross-sectional area or volume of a cylinder was lacking from a significant proportion of candidates. Statistics show that 25% of the candidature scored 0 marks, with another 25% of the candidature scoring only 1 mark and another 25% of the candidature scoring at least 6 of the 8 marks available. The paucity of good answers suggests that teachers may want to especially review this topic when revising Core Mathematics C4 in order to raise their students’ achievement.

In summary, Q1, Q2, Q4 and Q5 were a good source of marks for the average candidate, mainly testing standard ideas and techniques; and questions Q3, Q6, Q7 and Q8 proved effective discriminators. A significant proportion of candidates, however, were unable to make any progress with questions Q3, Q6(d), Q7(b) or Q8(d) with some candidates failing to offer any response to these questions.

### **Report on individual questions**

#### **Question 1**

A significant majority of candidates were able to score full marks on this question. In part (a), very few candidates failed to find either one or both of the  $y$ -coordinates required. In part (b), some candidates incorrectly used the formula  $h = \frac{b-a}{n}$ , with  $n = 6$  instead of  $n = 5$  to give the width of each trapezium as  $\frac{1}{3}$ . Many candidates, however, were able to look at the given table and deduce the value of  $h$ . A few candidates wrote down  $e^0$  as 0 instead of 1. Nearly all answers were given to 4 significant figures as requested in the question.

#### **Question 2**

In part (a), many candidates were able to use integration by parts in the right direction to produce a correct solution. Common errors included integrating  $e^x$  incorrectly to give  $\ln x$  or applying the by parts formula in the wrong direction by assigning  $u$  as  $e^x$  to be differentiated and  $\frac{dv}{dx}$  as  $x$  to be integrated.

Many candidates were able to make a good start to part (b), by assigning  $u$  as  $x^2$  and  $\frac{dv}{dx}$  as  $e^x$  and again correctly applying the integration by parts formula. At this point, when faced with integrating  $2xe^x$ , some candidates did not make the connection with their answer to part (a) and made little progress, whilst others independently applied the by parts formula again. A significant proportion of candidates made a bracketing error and usually gave an incorrect answer of  $e^x(x^2 - 2x - 2) + c$ .

In part (b), a few candidates proceeded by assigning  $u$  as  $x$  and  $\frac{dv}{dx}$  as  $xe^x$  and then used their answer to part (a) to obtain  $v$ . These candidates were usually produced a correct solution.

### Question 3

At the outset, a significant minority of candidates struggled to extract some or all of the information from the question. These candidates were unable to write down the rate at which this cross-sectional area was increasing,  $\frac{dA}{dt} = 0.032$ ; or the cross-sectional area of the cylinder  $A = \pi x^2$  and its derivative  $\frac{dA}{dx} = 2\pi x$ ; or the volume of the cylinder  $V = 5\pi x^3$  and its derivative  $\frac{dV}{dx} = 15\pi x^2$ .

In part (a), some candidates wrote down the volume  $V$  of the cylinder as their cross-sectional area  $A$ . Another popular error at this stage was for candidates to find the curved surface area or the total surface area of a cylinder and write down either  $A = 10\pi x^2$  or  $A = 12\pi x^2$  respectively. At this stage many of these candidates were able to set up a correct equation to find  $\frac{dx}{dt}$  and usually divided 0.032 by their  $\frac{dA}{dx}$  and substituted  $x=2$  into their expression to gain 2 out of the 4 marks available. Another error frequently seen in part (a) was for candidates to incorrectly calculate  $\frac{0.032}{4\pi}$  as 0.0251. Finally, rounding the answer to 3 significant figures proved to be a problem for a surprising number of candidates, with a value of 0.003 being seen quite often; resulting in loss of the final accuracy mark in part (a) and this sometimes as a consequence led to an inaccurate final answer in part (b).

Part (b) was tackled more successfully by candidates than part (a) – maybe because the chain rule equation  $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$  is rather more straight-forward to use than the one in part (a). Some candidates struggled by introducing an extra variable  $r$  in addition to  $x$  and obtained a volume expression such as  $V = \pi r^2(5x)$ . Many of these candidates did not realise that  $r \equiv x$  and were then unable to correctly differentiate their expression for  $V$ . Other candidates incorrectly wrote down the volume as  $V = 2\pi x^2(5x)$ . Another common error was for candidates to state a correct  $V$ , correctly find  $\frac{dV}{dx}$ , then substitute  $x = 2$  to arrive at a final answer of approximately 188.5.

About 10% of candidates were able to produce a fully correct solution to this question.

### Question 4

This question was generally well done with a majority of candidates scoring at least 6 of the 9 marks available.

In part (a), implicit differentiation was well handled with most candidates appreciating the need to apply the product rule to the  $xy$  term. A few candidates failed to differentiate the constant term and some wrote " $\frac{dy}{dx} = \dots$ " before starting to differentiate the equation. After differentiating implicitly, the majority of candidates rearranged the resulting equation to make

$\frac{dy}{dx}$  the subject before substituting  $\frac{dy}{dx}$  as  $\frac{8}{3}$  rather than substituting  $\frac{8}{3}$  for  $\frac{dy}{dx}$  in their differentiated equation. Many candidates were able to prove the result of  $y - 2x = 0$ . A surprising number of candidates when faced with manipulating the equation  $\frac{6x+y}{2y-x} = \frac{8}{3}$ , separated the fraction to incorrectly form two equations  $6x+y=8$  &  $2y-x=3$  and then proceeded to solve these equations simultaneously.

Some candidates, who were unsuccessful in completing part (a), gave up on the whole question even though it was still possible for them to obtain full marks in part (b). Other candidates, however, did not realise that they were expected to substitute  $y = 2x$  into the equation of the curve and made no creditable progress with this part. Those candidates who used the substitution  $y = 2x$  made fewer errors than those who used the substitution  $x = \frac{y}{2}$ .

The most common errors in this part were for candidates to rewrite  $-y^2$  as either  $4x^2$  or  $-2x^2$ ; or to solve the equation  $x^2 = 4$  to give only  $x = 2$  or even  $x = \pm 4$ . On finding  $x = \pm 2$ , some candidates went onto substitute these values back into the equation of the curve, forming a quadratic equation and usually finding “extra” unwanted points rather than simply doubling their two values of  $x$  to find the corresponding two values for  $y$ . Most candidates who progressed this far were able to link their values of  $x$  and  $y$  together, usually as coordinates.

### Question 5

This question was also generally well tackled with about 50% of candidates obtaining at least 8 of the 9 marks available. A substantial minority of candidates were unable to carry out the

first step of writing  $\frac{1}{\sqrt{(4-3x)}}$  as  $\frac{1}{2}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$ , with the  $\frac{1}{2}$  outside the brackets usually

written incorrectly as either 2 or 4. Many candidates were able to use a correct method for expanding a binomial expression of the form  $(1+ax)^n$ . A variety of incorrect values of  $a$  and  $n$ , however, were seen by examiners with the most common being  $a$  as  $\frac{3}{4}, 3$  and  $-3$  and  $n$  as

$\frac{1}{2}, -1$  and  $-2$ . Some candidates, having correctly expanded  $\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$ , forgot to multiply

their expansion by  $\frac{1}{2}$ . As expected, sign errors, bracketing errors and simplification errors were also seen in this part. A significant minority of candidates expanded as far as  $x^3$ , and were not penalised on this occasion.

In part (b), most candidates realised that they needed to multiply  $(x+8)$  by their expansion from part (a) although a small minority attempted to divide  $(x+8)$  by their expansion. A surprising number of candidates attempted to expand  $(x+8)$  to obtain a power series. Other candidates omitted the brackets around  $x+8$  although they progressed as if “invisible” brackets were there. The mark scheme allowed candidates to score 2 marks out of 4 even if their answer in (a) was incorrect and many candidates were able to achieve this.

### Question 6

In part (a), most candidates were able to set up and solve the three equations correctly. Some candidates either did not realise that they needed to perform a check for consistency or performed this check incorrectly. A surprising number of candidates did not follow the instruction in the question to find the position vector of the point of intersection. A few

candidates were unable to successfully negotiate the absence of the  $\mathbf{j}$  term in  $(-9\mathbf{i} + 10\mathbf{k})$  for  $l_1$  and so formed incorrect simultaneous equations.

In part (b), a majority of candidates realised that they needed to apply the dot product formula on the direction vectors of  $l_1$  and  $l_2$ . Many of these candidates performed a correct dot product calculation but not all of them wrote a conclusion.

In part (c), a majority of candidates were able to prove that  $A$  lies on  $l_1$ , either by substituting  $\lambda = 7$  into  $l_1$  or by checking that substituting  $(5, 7, 3)$  into  $l_1$  gave  $\lambda = 7$  for all three components.

There was a failure by many candidates to see the link between part (d) and the other three parts of this question with the majority of them leaving this part blank. The most common error of those who attempted this part was to write down  $B$  as  $-5\mathbf{i} - 7\mathbf{j} - 3\mathbf{k}$ . Those candidates who decided to draw a diagram usually increased their chance of success. Most candidates who were successful at this part applied a vector approach as detailed in the mark scheme. The easiest vector approach, adopted by a few candidates, is to realise that  $\lambda = 7$  at  $A$ ,  $\lambda = 3$  at the point of intersection and so  $\lambda = -1$  at  $B$ . So substitution of  $\lambda = -1$  into  $l_1$  yields the correct position vector  $-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}$ . A few candidates, by deducing that the intersection point is the midpoint of  $A$  and  $B$  were able to write down  $\frac{x+5}{2} = -3$ ,  $\frac{y+7}{2} = 3$  and  $\frac{z+3}{2} = 7$ , in order to find the position vector of  $B$ .

### Question 7

In part (a), many candidates realised that they needed to factorise the denominator to give two linear factors, and usually proceeded to give a fully correct solution. A few candidates, however, thought that  $4 - y^2$  was an example of a repeated linear factor and tried to split up their fraction up accordingly. Some candidates struggled with factorising  $4 - y^2$  giving answers such as  $(4+y)(4-y)$  or  $(y+2)(y-2)$ . The majority of candidates were able to write down the correct identity to find their constants, although a noticeable number of candidates, when solving  $4A = 2$  found  $A = 2$ .

A significant minority of candidates who completed part (a) correctly made no attempt at part (b). About half of the candidates in part (b) were able to separate out the variables correctly. Many of these candidates spotted the link with part (a). It was pleasing that candidates who progressed this far were able to correctly integrate  $\tan x$  and correctly find the two  $\ln$  terms by integrating their partial fraction. Common errors at this point were integrating  $\tan x$  to give  $\sec^2 x$  and the sign error involved when integrating  $\frac{K}{2-y}$ . A significant number of candidates at this point did not attempt to find a constant of integration. Other candidates substituted  $x = \frac{\pi}{3}$  and  $y = 0$  into an integrated equation which did not contain a constant of integration. A majority of candidates who found the constant of integration struggled to simplify their equation down to an equation with a single  $\ln$  term on each side. The most common error of these candidates was to believe that  $\ln A + \ln B = \ln C$  implies  $A + B = C$ .

Of all the 8 questions, this was the most demanding in terms of a need for accuracy. Fewer than 10% of candidates were able to score all 11 marks in this question, although statistics show that about half of the candidates were able to score at least 5 marks.

### Question 8

In part (a), many candidates were able to give  $t = \frac{\pi}{3}$ . Some candidates gave their answer only in degrees instead of radians. Other candidates substituted the  $y$ -value of  $P$  into  $y = 4 \sin 2t$  and found two values for  $t$ , namely  $t = \frac{\pi}{6}, \frac{\pi}{3}$ . The majority of these candidates did not go on to reject  $t = \frac{\pi}{6}$ . In part (b), many candidates were able to apply the correct formula for finding  $\frac{dy}{dx}$  in terms of  $t$ , although some candidates erroneously believed that differentiation of  $4 \sin 2t$  gave either  $-8 \cos 2t$ ,  $8 \cos t$  or  $2 \cos 2t$ . Some candidates who had differentiated incorrectly, substituted their value of  $t$  into  $\frac{dy}{dx}$  and tried to “fudge” their answer for  $\frac{dy}{dx}$  as  $\frac{1}{\sqrt{3}}$ , after realising from the given answer that the gradient of the normal was  $-\sqrt{3}$ . The majority of candidates understood the relationship between the gradient of the tangent and its normal and many were able to produce a fully correct solution to this part.

A few candidates, however, did not realise that parametric differentiation was required in part (b) and some of these candidates tried to convert the parametric equations into a Cartesian equation. Although some candidates then went on to attempt to differentiate their Cartesian equation, this method was rarely executed correctly.

Few convincing proofs were seen in part (c) with a significant number of candidates who were not aware with the procedure of reversing the limits and the sign of the integral. Therefore, some candidates conveniently differentiated  $8 \cos t$  to give “positive”  $8 \sin t$ , even though they had previously differentiated  $8 \cos t$  correctly in part (a). After completing this part, some candidates had a ‘crisis of confidence’ with their differentiation rules and then went on to amend their correct solution to part (a) to produce an incorrect solution. Other candidates differentiated  $8 \cos t$  correctly but wrote their limits the wrong way round giving

$$A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 4 \sin 2t \cdot (-8 \sin t) dt \text{ and after stating } \sin 2t = 2 \cos t \sin t \text{ (as many candidates were able}$$

to do in this part) wrote  $A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} -64 \sin^2 t \cos t dt$ . These candidates then wrote down the given answer by arguing that all areas should be positive.

Part (d) was unstructured in the sense that the question did not tell the candidates how to integrate the given expression. Only a minority of candidates spotted that a substitution was required, although some candidates were able to integrate  $64 \sin^2 t \cos t$  by inspection. Many candidates replaced  $\sin^2 t$  with  $\frac{1}{2}(1 - \cos 2t)$  but then multiplied this out with  $\cos t$  to give  $\frac{1}{2} \cos t - \frac{1}{2} \cos t \cos 2t$ . Very few candidates correctly applied the sum-product formula on this expression, but most candidates usually gave up at this point or went on to produce some incorrect integration. Other candidates replaced  $\sin^2 t$  with  $1 - \cos^2 t$ , but did not make much progress with this. A significant number of candidates used integration by parts with a surprising number of them persevering with this technique more than once before deciding they could make no progress. It is possible, however, to use a ‘loop’ method but this was very rarely seen. It was clear to examiners that a significant number of stronger candidates spent much time trying to unsuccessfully answer this part with a few candidates producing at least two pages of working.



# **Further Pure Mathematics Unit FP1**

## **Specification 6674**

### **Introduction**

The majority of the candidates had been well prepared for this examination, and most candidates were able to make a reasonable attempt at all of the questions. The paper proved to be a good test of the students' knowledge, with straightforward marks available, as well as testing sections for the more able students. The better candidates showed sufficient working to make their methods clear.

A number of candidates merely gave answers, with no working, possibly using their calculators to do so. They would be advised to follow the rubric to the paper and show their method of working. There were also a number of answers, which were very poor. Use of negatives was sometimes weak, factorising and multiplying brackets often incorrect. There appeared to be adequate time to complete the paper and for many to return to check answers, but there were a minority of candidates who left themselves with insufficient time for their solution to question 8.

### **Report on individual questions**

#### **Question 1**

In part (a) almost every-one earned the mark for the answer 4, but incorrect answers included 2, 8, and -4.

The most common approach to part (b) was to factorise  $(x^3 - 64)$  into  $(x - 4)(x^2 + 4x + 16)$ , and then to solve the quadratic equation  $(x^2 + 4x + 16) = 0$ . Most who did this were successful and obtained the answers  $-2 \pm 2\sqrt{3}$ . Mistakes included slips in long division with some attempts failing to reach the quadratic factor, and the solution of the quadratic equation (when obtained) caused further errors for some trying to complete the square. Other methods of solution began with  $(a + ib)^3 = 64$ , expanded with a binomial expansion and equated real and imaginary parts. Another method was to expand  $(x - 4)(x - a - ib)(x - a + ib) = 0$  and to compare with  $x^3 - 64 = 0$ . The latter two methods seemed to give rise to more errors than the first method described.

Part (c) usually resulted in full marks, as the mark scheme was generous here and did not require correct labelling on the axes. The second of the two marks was a follow through mark for having the third root in a conjugate complex position to the second. A few candidates strangely drew their real axis vertically and their imaginary axis horizontally. Candidates should be discouraged from this practice. As usual, the complex numbers were represented by points, line segments or vectors on the diagram.

#### **Question 2**

In part (a) it was expected that candidates would evaluate  $f(1.6)$ ,  $f(1.7)$  and declare a sign change, resulting in the conclusion that there was a root in the interval  $(1.6, 1.7)$ . Most candidates did this and earned both marks. There were a few errors evaluating the values, but almost all candidates worked in radians rather than in degrees. A number of candidates did not draw an adequate conclusion after doing the evaluation.

In solutions to part (b) there were some sign errors in the derivative  $f'(x)$ . Newton Raphson was usually stated correctly but there were some numerical slips in using the procedure and this frequently resulted in

the wrong answer 1.58. The answer here was required to be accurate to 3 significant figures and needed to be 1.62 (not 1.620). In this question the answer 1.62, with no working, resulted in zero marks, as did the answer 1.62 following incorrect work.

### Question 3

This question discriminated between those who understood complex numbers and those who just followed the algorithms that they had been taught. Most understood that they were required to express the quotient as a single complex number by multiplying the numerator and the denominator by the conjugate of the denominator. They then needed to find the real part and to put it equal to a half. However a minority did not realise that this stage was required and merely put the complex expression that had been given equal to a half, thus showing little understanding of the question and gaining no credit. Common errors were  $a^2-1$  on the denominator, and algebraic slips leading to  $a=\sqrt{3}$ . Some lost the final mark in (a) by including the negative root. The alternative solution given on the mark scheme was rarely seen.

Part (b) was usually done well. Most found their imaginary part and the “follow through” enable most to gain this mark. Some candidates did not make their working clear for finding the argument and a few used  $\tan^{-1}\left(\frac{\text{imaginary part}}{\sqrt{5}}\right)$  confusing their value of  $a$  with the real part of the complex number. A

small minority found the modulus rather than the argument, and some used tan rather than arctan. Almost all gave their answer in radians to 2dp. Again, very few used the alternative method on the Mark Scheme.

### Question 4

This question discriminated very well. There were some excellent complete and accurate solutions, but there were also muddled answers where the variables  $x$  and  $t$  were confused with the standard  $y$  and  $x$  and where the form of the particular integral was taken to be  $kt + c$ . Candidates were forced to think carefully about the letters that were used in the differential equation and the algebraic demand was different from the standard second order differential equation question. There were some bad errors in part (a) with answers +1 and +3 appearing regularly as solutions to the auxiliary equation, and some gave their complementary function as  $t^{-t} + t^{-3t}$  while others gave it as  $e^{-1} + e^{-3}$ . There were also algebraic errors finding the particular integral. Some candidates prematurely took  $k = 6$  which made the working easier and did not give them full credit.

Part (b) required some insight and an understanding that for large values of  $t$  the negative exponentials tended to zero.

### Question 5

On the whole, part (a) was correctly done and by the expected method. A number of candidates obtained the solutions  $-3 \pm \sqrt{17}$ , but did not go ahead to find -4 and -2. Only rarely did candidates square both sides and use the alternative solution printed on the scheme. These candidates had a more difficult problem to solve than those who considered  $\frac{x}{2} + 3 = \pm \frac{4}{x}$ . Most obtained four solutions in part (a), though a substantial minority realised that only three of these values were valid.

The sketch drawing in part (b) was usually well answered with some good clear sketches. Part (c) was also answered well and it was quite common for candidates to obtain 9 of the ten marks on question 5. Inevitably some had their inequalities the wrong way round as in  $-2 < x < -4$ , and some gave an extra solution set in their answer, losing the final mark.

### Question 6

Most found the partial fractions correctly in part (a) and went on to use them to tackle part (b). Candidates who made an error would have benefited from checking carefully, particularly if they did not have a negative sign. In part (b) the majority were able to reach the expression  $\frac{5}{6} - \frac{1}{n+2} - \frac{1}{n+3}$ , for their answer to the sum of the series. Only a very few included  $r = 0$  or  $r = n+1$  in their differences. Sign errors and slips in working with algebraic fractions led to some being unable to reach the correct answer that  $a = 5$  and  $b = 13$ .

Part (c) is a fairly standard question and most understood what was required, substituting 30 and 20 into the answer to part (b) and subtracting to obtain 0.02738. Errors arose where 30 and 21 were used as limits, or more strangely 30 and 9. Full credit was given to those who used calculator methods in part (c) adding ten appropriate terms, but this was unusual in this cohort as most used their previous answer to calculate the answer to part (c). A few reused the method of differences calculating  $1/22 + 1/23 - 1/32 - 1/33$ , but there was often an error when this method was used. Some tried to use  $u_{30} - u_{20}$  demonstrating lack of understanding. Overall this was a well answered question.

### Question 7

This was by far the most challenging question on the paper. Most candidates gained the method mark in (a) by substitution into the RHS of the DE. Many however treated  $v$  as a constant rather than a variable although this substitution is a standard one and should have been recognised as such. Those who appreciated the need for implicit differentiation usually went on to gain full marks in (a).

Part (b) was a good discriminator. Candidates needed to separate the variables to produce the neatest solution. Many, who tried this, had the LHS as  $2v + 1/v$  rather than  $1/(2v + 1/v)$ . They then needed to simplify their algebraic fraction in  $v$  and to recognise that both sides of their equation resulted in logs. Inclusion of an appropriate constant with sound use of log laws, followed by a return to the variable  $y$ , enabled better candidates to obtain the solution. The last method mark in part (b) was available to most candidates for replacing  $v$  with  $y/x$ .

Part (c) then followed with a requirement to find the arbitrary constant in a specific case. Most obtained the method mark, even where they had made errors earlier. A number tried to use an integrating factor, but usually inappropriately and with no success.

### Question 8

Part (a) required candidates to consider  $r \cos \theta$  and to differentiate to find a maximum value. This was implied as the tangent was perpendicular to the initial line and so  $\frac{dx}{d\theta} = 0$ . A number of candidates chose to consider  $r \sin \theta$  instead and little credit was given for this.

In part (b) most realised that they needed to find  $\frac{1}{2} \int (1 - 2 \cos \theta + \cos^2 \theta) d\theta$ . There were a number of sign slips but the methods were understood, and most tried to use double angle formulae. To find the shaded area they needed to use the limits  $\frac{\pi}{2}$  and  $\frac{\pi}{3}$  and to subtract. This gave the finite area enclosed by the arc  $AP$  and the straight lines  $OP$  and  $OA$ . They then needed to find the area of triangle  $OPN$ . The sum of these two areas gave them the total area required. A number of candidates found the area of a rectangle or used a triangle where the sides had incorrect lengths, particularly where the base was 2 units instead of 1 unit. Many answers for the triangle area included a pi term instead of a root 3.

Full marks in question 8 was indicative of a good grade A candidate.



# **Further Pure Mathematics Unit FP2**

## **Specification 6675**

### **Introduction**

The first three questions proved accessible and the last four questions proved good discriminators.

Most of the questions contained a ‘show that’ section. Candidates must be reminded that this means they must show all pertinent steps in their working. This was particularly true in questions such as Q1 Q6b, Q8a and Q8d where some key steps were sometimes omitted, and particularly in Q7a where some candidates quoted a general formula for the equation of the normal and then substituted values.

The paper revealed weaknesses in integration and/or trigonometric manipulation from some candidates. It was clear throughout the paper that those who set their work out in a clear, methodical fashion fared better than others. Many candidates lost terms, factors and most of all minus signs due to carelessness.

### **Report on Individual questions**

#### **Question 1**

This was well done, especially by those candidates staying with hyperbolic functions, with only the weakest failing to remember the  $\operatorname{sech} x$  squared. Some candidates did not make the steps showing the transition into double angles clear. In ‘show that’ questions candidates must make the steps in their method clear. Those who changed into exponentials or logarithms were often less successful.

#### **Question 2**

This proved the easiest question on the paper for most candidates. Errors largely consisted of slips in signs and a few factorisation errors. Those who changed into exponentials early found this very straightforward. The few who squared both sides and stuck with hyperbolic functions found themselves using a much longer and harder method.

#### **Question 3**

Those who split the integral into two separate integrals were generally successful. The candidates found the hyperbolic integral  $\int \left( \frac{3}{\sqrt{x^2 - 9}} \right) dx$  easier than the inverse square root  $\int \left( \frac{x}{\sqrt{x^2 - 9}} \right) dx$ .

Some attempted integration by parts and rarely made much progress since this involved integrating  $\cosh^{-1}\left(\frac{x}{3}\right)$ .

#### **Question 4**

Most candidates completed part (a) well, the printed answer assisting with the method. A few omitted the  $3x^2$  term from their derivative, or misapplied the chain rule, then ‘fudging’ the gradient of 2. In part (b) it was rare that candidates were able to progress beyond the first two marks, not being able to progress beyond obtaining  $4a^6 - 9a^4 + 4 = 0$ .

### Question 5

Part (a) proved very tricky for all candidates. Most knew they needed to use integration by parts but the derivative of  $\sin^n x$  as a multiple of  $\cos^{n-1} x$  was seen too often. Many candidates lost track of their  $\cos x$  terms and it was rare to see  $\cos^2 x = 1 - \sin^2 x$  used to set up the reduction formula. Many made sign errors and many lost track of the (n-1) and n coefficients. Those who set their work out neatly and tidied up their expressions were often more successful. Part (b) was well done, but  $\frac{2}{3}$  was

often seen instead of  $\frac{2}{5}$  and  $I_0$  was sometimes seen as  $\int e^x \sin x dx$  or just  $\int 1 dx$  and the value was often given as just  $e^\pi$ .

### Question 6

Most candidates were prepared to tackle this question and direct integration by parts was the preferred method. Many candidates omitted the  $\cosh x$  term when differentiating  $\arctan(\sinh x)$  and some integrated  $\tanh x$  as  $\operatorname{sech}^2 x$ . Some used the substitution  $t = \sinh x$  but then did not integrate  $\arctant$ . Once again, since the answer was printed in part (b) the ‘show that’ indicated that the candidates were required to show all their working - a number simply copied the question and its answer from the question paper.

### Question 7

The vast majority of candidates were able to complete part (a) successfully. A few found the tangent but more differentiated incorrectly – failing to differentiate the ‘1’ (or the 144) being by far the commonest error. Some made sign errors in multiplying out the bracket and others made life difficult for themselves by not simplifying, using  $-\frac{48 \tan t}{36 \sec t}$  as the gradient in the equation of the normal. In part (b) most candidates were able to find the eccentricity, or  $ae$ , and went on to find the coordinates of P correctly, usually substituting  $x=5$  into the given equation of H. Right at the start of this section some misquoted/misused the formula  $b^2 = a^2(e^2 - 1)$ , which is given in the formula booklet and others used  $y = \tan t$  and got  $(5, \frac{3}{4})$ . Some good attempts at (c) were seen, but only from the better candidates. Those who sketched the triangle usually fared better. A number found the area of triangle OPR and others did not attempt to find a value for t, or else stated, incorrectly, that  $t=0$ .

### Question 8

Candidates are reminded that in ‘show that’ questions they must make the steps in their working clear, many ‘jumped’ from  $\frac{\sin t}{1 + \cos t}$  to  $\tan \frac{t}{2}$  losing the final mark. A number of candidates differentiated  $3(1 - \cos t)$  as  $3 + 3\sin t$  and careless presentation often led to lost ‘3’s en route. The most successful route through part (b) was use of  $\sqrt{\dot{x}^2 + \dot{y}^2}$ , as shown in the mark scheme, the commonest error being getting integral as  $\sec \frac{t}{2}$ . A number used the  $\frac{dy}{dx}$  version but then integrated the answer to (a) with respect to t. The final mark in (b) proved the most often missed on the paper, with candidates not establishing that  $C=0$ . The correct intrinsic equation was often seen in part (c). Part (d) proved challenging for candidates. Many

substituted their answer to (b),  $12 \sin \frac{t}{2}$  into the integral rather than  $6\cos \frac{t}{2}$ . Some candidates did not convert into half angles correctly and many got to the integral of  $\cos^3\left(\frac{t}{2}\right)$  and did not proceed further. Having integrated others converted  $\sin^3\left(\frac{t}{2}\right)$  in terms of  $\sin\left(\frac{t}{2}\right)$  and  $\sin\left(\frac{3t}{2}\right)$ , and tried to work from there. Once again candidates are reminded that ‘show that’ means they must make the steps in their method clear.



# **Further Pure Mathematics Unit FP3**

## **Specification 6676**

### **Introduction**

Although some of the questions on this paper were easily accessible to the majority, the paper as a whole proved quite demanding for many candidates. Most candidates were able to complete Q1 quickly and accurately, but inefficient and lengthy methods were often seen for question Q2, leading undoubtedly to a shortage of time for some of the later questions. Performances from weaker candidates were often 'patchy', showing good understanding of some topics but lack of confidence in other parts of the specification. Most managed to show methods and working clearly, although presentation of proofs often left much to be desired. Work of a very high standard was seen from many candidates, but the discriminating demands of some questions meant that it was rare to see a total mark in excess of 70 out of 75.

### **Report on individual questions**

#### **Question 1**

This standard question was very well answered, proving an easy starter for the vast majority of candidates. A very common error, however, resulted from candidates approximating values prematurely and hence obtaining 0.6839 rather than 0.6838 for the second value. Occasionally, 0.6838 was thought to be the value of  $y$  at 0.05 rather than 0.1 and unnecessary further working was seen. There were instances of candidates working in degrees but this was rare.

#### **Question 2**

In this question many candidates wasted time on unnecessarily lengthy methods, particularly in part (b), where it was not uncommon to see several attempts. In part (a), most candidates identified the equation  $\mathbf{M}\mathbf{v} = \lambda\mathbf{v}$  and then used the resulting components to eliminate  $\lambda$ , usually obtaining the given relationship between  $p$  and  $q$ . Some, however, effectively used  $\lambda = 1$  or  $\lambda = 0$ . Even though the eigenvalue corresponding to the given eigenvector was not given, some candidates tried to use the characteristic equation to derive the given relationship.

In part (b), with  $\lambda = 5$  now given, the most efficient approach was to use the characteristic equation, although some candidates made it more difficult for themselves by not substituting the value for  $\lambda$  until they had ploughed through some algebra. Those who started with the equation  $\mathbf{M}\mathbf{v} = 5\mathbf{v}$  sometimes muddled the resulting algebra, perhaps having all of  $x$ ,  $y$ ,  $z$ ,  $p$  and  $q$  in their equations, but this should at least have given them a lead into part (c). A significant minority of candidates did not use the given fact that both  $p$  and  $q$  were negative, and finished in part (c) with the same eigenvector that had been given in part (a).

A minority trivialised part (b) by simply using the eigenvector from part (a).

In part (c), there was clear evidence that the standard technique for finding eigenvectors was well understood by the majority of candidates.

### Question 3

The majority of candidates performed very well on this question. In part (a), implicit differentiation and the use of the product rule were usually sound, enabling most candidates to achieve the given result with few problems. A small minority seemed unaware how to proceed in part (b), but otherwise the only mistakes were numerical slips or perhaps omitting the first term (1) of the series. A surprising number of candidates, having obtained the correct series in part (b), were unable to achieve the answer 0.77 in part (c).

### Question 4

Most candidates knew the correct method for establishing the cartesian equation of the locus in part (a), although a few forgot to square the 2. Those who included  $i$  in the modulus calculation usually failed to notice that the equation they produced was not that of a circle. Some candidates had no idea how to proceed and simply played around with  $z$ 's before giving up or creating an equation of a circle.

In part (b), there were many well-drawn diagrams, usually adequately labelled. Occasionally, however, there was insufficient evidence of the coordinates of intersections with the axes. Many candidates identified the locus of  $Q$  as a perpendicular bisector and many good sketches were seen. The most common error in this part was a lack of an appreciation that the locus of  $P$  and the locus of  $Q$  intersected on the negative imaginary axis.

In part (c), most candidates knew that the shading should be inside the circle but many were unable to determine the correct shading for the other inequality.

### Question 5

Candidates often found parts (a) and (b) of this question more difficult than parts (c) and (d), so it was helpful that the four parts could be tackled independently of one another.

Part (a) of the question was poorly answered. Candidates applied the matrix to a variety of different vectors including  $\mathbf{i} + 2\mathbf{j}$ ,  $2\mathbf{i} + \mathbf{j}$ ,  $x\mathbf{i} + y\mathbf{j}$ ,  $2x\mathbf{i} + y\mathbf{j}$ , etc. Many correctly performed the multiplication on  $x\mathbf{i} + 2x\mathbf{j}$  but then failed to use the fact that the line was mapped onto itself. By far the most common mistake was to suppose that  $x\mathbf{i} + 2x\mathbf{j}$  was mapped onto itself, an assumption that gave contradictory values  $k = 5$  and  $k = 1$ . Most candidates seemed unconcerned by this contradiction.

In part (b), to show that the matrix  $\mathbf{A}$  was non-singular for all values of  $k$ , most candidates knew that the determinant was involved and found it correctly. Although some simply established non-singularity for the specific value of  $k$  they had found in part (a), most were able to produce a convincing proof, either by completing the square for the quadratic function or by using the quadratic formula. Conclusions relating to non-singularity were sometimes omitted. The inverse

matrix in part (c) was often correct, although it was common to see  $\frac{1}{k^2 - 2k + 2} \begin{pmatrix} k & 1-k \\ -2 & k \end{pmatrix}$ , or

other variations, instead of  $\frac{1}{k^2 - 2k + 2} \begin{pmatrix} k & 2 \\ k-1 & k \end{pmatrix}$ .

In part (d), many candidates opted to use the original matrix and solve simultaneous equations rather than to use the inverse matrix. Although there were many good solutions, a common mistake was to map  $Q$  onto  $P$  instead of  $P$  onto  $Q$ .

In part (c), where a 'hence' method was required, candidates clearly struggled to find the link between the given equation and the result from part (b). Those candidates who substituted  $x = 2 \cos \frac{\pi}{10}$  into the given equation were more successful in recognising this link and many who used this technique thus managed to obtain full marks, providing their conclusion was clear. A significant minority of candidates made use of the calculator and clearly thought that simple substitution was enough, while others made very little real progress or completely omitted this part.

### Question 6

Good candidates had no difficulty with either part (a) or part (b) and many solutions to these parts were efficiently and clearly presented. From average and weaker candidates, however, the presentation of the induction proof was poor, often having steps missing or unclear. In particular, there was often insufficient working to justify the correct use of the compound angle formulae. Sometimes the understanding of the basic principles of the induction process seemed to be lacking, but the justification of the trivial case for  $n = 1$  was rarely missing.

In part (b), the majority of candidates identified the need to use both De Moivre's Theorem and the binomial theorem to efficiently obtain the printed identity. There were other ways to prove this identity (some more efficient than others), the most common of which involved the use of both the addition formulae and the double-angle formulae. Candidates using these alternative methods, however, often became muddled and made algebraic mistakes.

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### Question 7

The overall performance in this question was disappointing, with many candidates making little progress beyond parts (a) and (b).

In part (a) most were able to calculate the vector product, although arithmetic slips were surprisingly common. Part (b) was also generally well answered, with follow through from part (a) enabling most candidates to score both available marks for the equation of the plane.

For those who attempted part (c), the most popular method of finding the line of intersection of the two planes was to solve the cartesian equations of the planes. Success with this method, as with other methods, was variable and many candidates did not proceed to find the required cartesian equations of the line. A very concise method for this part was to use the appropriate cross product to obtain the direction vector of the line, but this was seen much less frequently.

Success in part (d) was largely dependent on some level of success in part (c), so no marks were available to those candidates who resorted to making assumptions about the coordinates of  $S$ .

In part (e), the overwhelming majority of candidates used  $1/6$  rather than  $1/3$  in their formula for the volume of the pyramid, although many did manage to use appropriate vectors for the required triple product. The alternative method of finding the distance of  $T$  from the plane  $PQRS$  and then multiplying by  $1/3$  of the area of  $PQRS$  was occasionally seen.



# **Mechanics Unit M1**

## **Specification 6677**

### **Introduction**

The paper proved to be quite a challenging test of the specification. It allowed weaker candidates to demonstrate some success while at the same time the more able were suitably stretched. The paper seemed to be of a suitable length for the vast majority and where there was evidence of a lack of time it was usually because candidates had used longer than necessary methods on earlier questions. The questions which provided the best source of marks were questions Q1, Q3 and Q4. The final question on the paper, Q8, was by far the most challenging and even better candidates found it difficult to score more than half marks. Overall, candidates who used large and clearly labelled diagrams and who employed clear and concise methods were the most successful.

In calculations the numerical value of  $g$  which should be used is 9.8, as advised on the front of the question paper. Final answers should then be given to 2 (or 3) significant figures – more accurate answers will be penalised. If a candidate runs out of space in which to give his/her answer than he/she is advised to use a supplementary sheet – if a centre is reluctant to supply extra paper then it is crucial for the candidate to say whereabouts in the script the extra working is going to be done.

### **Report on individual questions**

#### **Question 1**

This was done well by the majority of candidates. Part (a) was a straightforward opening question, almost always correctly answered. A few candidates wrote  $3 = 0.4(0 - v)$ , thus only gaining the method mark. In the second part most knew and could apply appropriately the conservation of momentum principle, with only occasional sign errors. Drawing a clear velocity diagram would have helped candidates who confused ‘before’ and ‘after’ velocities. Since it was a ‘show that’ question it was important that full working was seen in order to achieve full marks. Wordy explanations involving impulses with no equation, tended to achieve no marks.

#### **Question 2**

There were various approaches that could be applied successfully to answer this question. Those who fully understood the implications of projecting from above ground level could achieve full marks by the most direct method although sign errors were not uncommon. Another popular approach was to split the motion into two stages (to and from the highest point) in both part (a) to find the initial velocity, and in part (b) to find the whole time. Although this required more working, there tended to be fewer sign errors. Premature approximation occasionally led to inaccuracy in the final answer. The weakest candidates sometimes only considered motion to or from the highest point. It should be noted that the rubric requires  $g = 9.8$  to be used and not 9.81, which was penalised.

### **Question 3**

This question was done well by the vast majority of candidates. Most used trigonometry appropriately in part (a) to find the required angle. In the second part some used  $F=ma$  correctly but failed to find the magnitude, whereas others found the magnitude of the given acceleration vector (sometimes labelling it as the force) but did not go on to multiply by the mass. Many used the relevant vector constant acceleration formula to achieve a correct velocity in the final part, although occasionally candidates multiplied the velocity rather than the acceleration by 5, or they tried to convert it all into scalars.

### **Question 4**

The speed-time graph in part (a) was well drawn with sufficient detail although a few candidates went beyond 90 on the  $t$ -axis. In the second part most tried to equate the area under their graph to 1410 but there were occasional errors and inconsistent use of the unknown time within the equation. Some found the area of the triangle to be 60 (correctly) but then used this as a distance in a constant acceleration formula to find the deceleration. The final two marks required the use of an appropriate value of  $t$  (i.e. not 38) and this was usually achieved although some, using the correct time value of 8, lost a mark by giving the deceleration as a negative value. More inexplicably, a few calculated the hypotenuse of a right-angled triangle rather than the gradient. Those who tried to use constant acceleration formulae for more than one stage of the motion at a time received no credit; such attempts were only very rarely seen.

### **Question 5**

This question, involving a resultant force, was not well answered by many candidates, although there were also a fair number of full marks seen. There were two possible approaches (resolving and sine/cosine rule). Some treated it as an equilibrium problem and used correct terms in resolving but with sign errors. Some who used the triangle approach used a triangle with  $\mathbf{R}$  (the resultant) opposite an angle of  $150^\circ$  rather than  $30^\circ$ . This enabled them to find the correct numerical value for the magnitude of  $\mathbf{R}$  by the sine rule but then it was difficult to achieve any more marks since it was impossible to find a third angle. A number of candidates made no significant progress in answering this question; there was a lot of crossed out working seen.

### **Question 6**

Part (a) was well done by many who produced and solved an appropriate moments equation, although a minority thought that the two tensions were equal and ‘solved’ the problem by a vertical resolution. The second part proved to be more demanding, with those candidates who tried to use the value of the tension from part (a) scoring very little. Most resolved vertically and took moments about a point. There were occasional errors in the relevant lengths or omission of  $g$  and some candidates interpreted the tensions as  $T$  and  $10T$  rather than  $T$  and  $T+10$ . Errors were more prevalent in the working of those who took moments about two points giving them simultaneous equations to solve.

### **Question 7**

It was good to see so many fully correct solutions to this question which was best solved by resolving parallel and perpendicular to the plane. Only the weakest candidates failed to include all the relevant forces. Those candidates who attempted vertical and horizontal resolution often fell victim to inaccuracies in angles or more costly, to missing forces. Since  $g = 9.8$  had been used, the final mark in part (a) was often lost for an answer of 68.42. Virtually all tried to use  $F = \mu R$  appropriately in part (b) although occasionally  $F$  was acting in the

wrong direction. Other errors in both parts included incorrect signs, confusion over which angles to use and sine/cosine applied the wrong way round.

### Question 8

Full marks were rarely achieved in this question. Some made a poor start by using  $F=ma$  in part (a) rather than an appropriate constant acceleration formula. In the second part many used separate equations of motion for the two particles (sometimes with extra or omitted terms) but then not uncommonly solved them as simultaneous equations with the same  $F$  (friction term), showing a lack of understanding of the problem. Only a minority used the more straightforward ‘whole system’ approach. There was some recovery in part (c) where follow through marks were available as long as the ‘appropriate’ terms were included in the equation of motion of one particle. A significant number of candidates knew that an inextensible string implied that the accelerations of the two particles were the same in part (d), but some of those went on to incorrectly mention the tension as well and so lost the mark. Many candidates who reached part (e) seemed to know they had to find the new deceleration but lost marks by including a tension or the 30N in their equation of motion.



# **Mechanics Unit M2**

## **Specification 6678**

### **Introduction**

In many respects the candidates found this paper very accessible. Most offered solutions to all of the questions, and many demonstrated a strong understanding of the topics examined. As ever, the best solutions tended to be those accompanied by appropriately sized and clearly labelled diagrams. Some candidates use methods involving energy to good effect at every opportunity, but many still display relatively low confidence with this topic. Disappointingly, it is still common to find candidates giving final answers to an inappropriate level of accuracy following the use of an approximate value for  $g$ . Candidates with weaker algebraic skills tended to take longer to deduce the given answers in questions 2 and 5. All candidates need to be reminded to read the question very carefully and make sure that they find what they have been asked for – in both question 4 and question 7 the final part of the question asks for a speed, yet many candidates stopped as soon as they had found the velocity.

The most challenging question for candidates turned out to be question 5, where relatively few candidates took the shortest possible route to the required answer and many demonstrated little understanding of the forces acting on the plank.

### **Report on Individual questions**

#### **Question 1.**

This was a straightforward question for many candidates. However, as usual, some were not sure how to deal with power and velocity in a question involving forces and the inclined plane. A surprising number of candidates felt that the lorry had to work against gravity when it was going down the hill. Too many candidates failed to realise that using 9.8 for gravity could not give answers to a high degree of accuracy. Only answers to 2 or 3 significant figures were accepted.

#### **Question 2.**

(a) Very few candidates had a problem with the two equations required. The most common errors were still inconsistency in signs and getting the ratio the wrong way round for the impact law. This is a very costly error leading to the loss of most marks for this part of the question.

(b) Some candidates struggled with the algebra here. The correct values for the speeds of  $A$  and  $B$  immediately after the collision were usually used, but, especially when candidates found the change in kinetic energy of the particles separately and then combined their answers, they failed to subtract energies correctly. A significant number managed to make mass and velocity disappear from their final answer having successfully worked through the algebra to find the kinetic energy lost.

#### **Question 3.**

(a) Many candidates lost marks here. This was usually because they found only the loss in kinetic energy rather than using both the kinetic energy and the potential energy terms

(b) Those candidates who chose to use their answer to part (a) and use the work-energy principle tended to be successful in finding a value for  $\mu$ , but a significant number did not realise the connection between parts (a) and (b). Those who chose to use constant acceleration and then  $F=ma$  generally found  $\mu$  correctly. The most common errors were to omit the weight component in their equation for  $F=ma$  or to confuse the two methods and include extra energy terms in their energy equation.

Many candidates lost marks through giving answers to inappropriate accuracy following the use of an approximate value for  $g$ .

#### **Question 4.**

This was generally well answered. The mechanical principles were well understood by nearly all candidates.

(a) Many fully correct answers were seen. The most frequent errors were due either to problems in dividing by 0.5, or because candidates integrated  $\mathbf{F}$  rather than  $\mathbf{a}$ .

The use of  $v = u + at$  or differentiation instead of integration were occasionally seen.

Another, less common, error was to substitute  $t = 0$  first. Also the constant of integration was very occasionally written with ‘-1’ instead of ‘+1’ in the  $\mathbf{i}$  term.

The great majority of candidates remembered the initial conditions although a few lost the final A mark for omission of  $\mathbf{i} - 4\mathbf{j}$ .

(b) In many cases correct use of the impulse equation resulting in a velocity of  $15\mathbf{i} + 20\mathbf{j}$  was seen but then too many candidates did not go on to find the speed of the particle. Use of the initial velocity as  $\mathbf{i} - 4\mathbf{j}$  was a common error.

Less common, but costly, was to start by finding the magnitude of the impulse and attempt to use this. Also, some candidates attempted an impulse equation but ignored the mass of the particle.

Candidates with errors early in the question were often able to gain a mark at the end for finding the speed from their velocity.

#### **Question 5.**

This proved to be the most challenging question on the paper. It turned out to be a question which differentiated well between those who had a thorough understanding of the principles and those with a relatively poor grasp. As usual, too many solutions were spoiled by candidates who were not able to find the correct components of their forces, confusing  $\cos \alpha$  and  $\sin \alpha$ .

(a) Candidates who took moments about  $A$  and resolved vertically were the most successful with many producing compact and complete solutions. Others taking moments about  $C$  or  $B$  invariably omitted a force or were not able to deal with the extra unknowns.

Common errors were in mistakes with the reactions at  $A$  and  $C$ , e.g. a vertical reaction at  $C$ , the reaction at  $A$  perpendicular to the rod, and use of horizontal and vertical components at  $C$  with no method for combining them. Several candidates had the weight of the plank acting at  $C$ . Also, some introduced a reaction at  $B$ , usually horizontal, perhaps through confusion with problems relating to a ladder against a wall.

(b) This was answered a little more successfully with good use of the given answer from part (a). Most candidates seemed confident in their use of  $F = \mu R$  although some confused the reaction at  $A$  and the reaction at  $C$ . This confusion was sometimes the consequence of poorly labelled diagrams.

There was some difficulty in finding and using the value for  $\sin \alpha$ .

#### **Question 6.**

This question was often answered very well. Most candidates took moments about  $Oy$  or  $Ox$ , although alternatives were seen. Weaker candidates did not seem to be very confident in dealing with a lamina plus a set of particles.

(a) Full marks were usually seen. The alternative method of taking moments about axes through the centre of mass was seen and was usually implemented successfully.

A small number were too casual in claiming the printed result from a correct moment’s equation – candidates need to remember that with a given answer a little more detail is required.

(b) This was usually correct. Many candidates calculated the  $y$  coordinate of the centre of mass of the three particles as  $\frac{8}{3}$  and then used that to calculate the centre of mass of the system.

Any errors were usually in the total mass (e.g. taken to be  $40m$  or equal to the total mass of only those which contributed to the moment). A few responses did not include the lamina and scored nothing. Some candidates treated  $OABC$  as a set of connected rods, rather than as a lamina.

(c) Candidates who answered part (b) incorrectly often picked up two marks here from the follow through. The majority of candidates recognised the correct triangle, although many of them then calculated the incorrect angle. A number of responses used 5 and / or 8, which scored nothing. A few candidates lost the final mark by using rounded values from part (b) and arrived at an incorrect value for the angle.

### Question 7.

This was a well answered question with almost all candidates working in appropriate vertical and horizontal directions. However, some candidates were confused by the fact that the ball was projected below the horizontal – many of these assuming that it was projected horizontally and then followed the path illustrated.

(a) Most candidates used  $s=ut + \frac{1}{2}at^2$  to produce a quadratic equation albeit often with a sign error. When marks were lost this was usually because of difficulties with positive and negative vertical terms. The alternative, longer method, of finding  $v$  then  $t$  was not infrequent.

(b) The method was clearly understood but the numbers involved were very sensitive to rounding errors. Too many candidates over-rounded their value for  $t$  and obtained the inaccurate value of 1.02.

(c) It was pleasing to see that most candidates adopted the correct approach of finding  $t$  then  $v_y$  but some failed to then combine  $v_x$  and  $v_y$  to obtain the speed. Again, sign errors with the acceleration resulted in errors in  $v_y$  being less than the original vertical component, which one would have hoped might ring an alarm bell.

Few candidates used the energy method and those that did often used the incorrect vertical displacement.



# Mechanics Unit M3

## Specification 6679

### Introduction

It is disappointing to note that there was some considerably “fiddling”, “fudging” or “good luck” involved in answering Q1, Q3 and Q5. In Q3, the significance of the inequality was ignored until the end of part (a). The equality was simply altered in their answer to the inequality given or one of their previous equations was altered to an inequality (usually incorrectly  $T \sin \theta \dots m r \omega^2$ ). For Q5, the fact that  $1 - \cos 60$  is the same as  $\cos 60$  helped so many candidates who had obviously learnt the form of an equation to use rather than understood the principle involved. Similarly, a correct form for mass as  $4m$ , in part (c) was often overlooked but luckily using  $m$  only lost 1 out of the 6 marks available. Square roots were needed for the answers to Q1(b), Q3(b) and Q5(b); sometimes these

“wandered” and answers such as  $u = \sqrt{g} L$  for 1(b),  $\omega \frac{\sqrt{2g}}{5h}$  for 3(b) and  $\sqrt{\frac{g}{8}} a$  for Q5(b) were not infrequent. Candidates must ensure that it is clear which terms the square root applies to.

### Report on individual questions

#### Question 1

There was little here to trouble most candidates and nearly all soon realised that the solutions to both parts came out easily using Conservation of Energy. Even those who may have started to use Hooke’s Law and force soon changed their minds and approach. Only rarely did anyone continue to use SHM from  $F = ma$  and get it correct. Most incorrectly used a constant thrust and then tried work done = change in energy or they simply quoted results for SHM without proving it was SHM. The few who did loose out in part (b) were those who tried to equate changes in energy rather than energy at a position. It sometimes led to a sign error on the GPE term.

#### Question 2

As the question stated that the particle was moving with SHM most candidates could make some attempt at a solution. Where SHM was well understood these solutions were concise and correct.  $T = 1.5$  was seen too often and a few candidates worked with an amplitude of 2.4. In part (b) most candidates used  $t = 2$  and the majority of those who used  $x = a \sin \omega t$  obtained the correct answer but some thought they had finished when  $x$  had been found. Those who used  $x = a \cos \omega t$  generally failed to appreciate that  $t = 1.25$ ; they did not understand the question and rarely obtained any marks. Identifying the correct  $x$  to use in part (c) caused some candidates problems as they used their final answer from (b) instead.

#### Question 3

It was disappointing to note how many candidates missed the point of the mechanics in part (a) of this question. Certainly having a reaction force from contact with the table was an irritation to them so much so that they either ignored it all together or put it in and then said  $N = 0$ . The actual condition of finding an expression for  $N$  and using  $N \geq 0$  was lost on too many.

Most preferred part (b) where they could simply find some set values and plug in results. The ones who did this part incorrectly were usually those who had insisted that  $T \cos \theta = mg$  in part (a) and hence it also applied in part (b). They used it to find an incorrect value of  $T$  to use in

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$T \sin \theta = mr\omega^2$ . The relating of 3:4:5 to a given side of  $h$  was often dubious with  $h$  being missed out of some lengths and some careless trigonometry was also seen. Candidates often stopped at  $\omega^2$  and so lost the final mark.

#### Question 4

Centres of mass were well understood and, apart from the initial formulations for volumes of hemispheres, a large number of candidates had fully correct solutions to parts (a) and (b). A few did not know the formula for the volume of a sphere and a few forgot to divide by two. The latter group obtained a correct answer for (a) (but scored only 4/5). However, their error was exposed in (b) as they could not obtain the given answer here. A few went back and found and corrected their error; others gave up or fudged the answer. Weaker candidates sometimes omitted part (c). Several methods for part (c) were seen – the most common were finding the critical angle and finding the point at which the vertical through the centre of mass cut the plane, showing that the distance from the axis of symmetry was less than  $2a$ . A small number had the plane face of the hemisphere on the plane and a few thought that the answer to (b) was the distance of the centre of mass from the plane.

#### Question 5

There were a lot of completely, or nearly completely, correct solutions to this question. Some lost the final A mark only for using  $m$  instead of  $4m$ . Most candidates who made an attempt at this question scored highly on it. They seemed well versed in the two significant principles of using Conservation of Energy and Newton's Second Law towards the centre. Unfortunately again it may have been a case of results learnt rather than mechanics principles understood. In part (a), either  $v^2 = u^2 + 2ag(1 - \cos 60)$  or  $v^2 = 2ag(1 - \cos 60)$  was used in the C of E equation rather than identifying a correct trigonometric expression to link the GPE at the two positions. This happened again in part (c)(i) when they used  $mgh$  as change in GPE and then had  $\cos\theta = 1/16$  as their result for finding  $\theta$ . Similarly in part (c)(ii) where  $v = 0$  was the condition, too many thought that meant no acceleration and hence resolved vertically. In both (c)(i) and (ii)  $4m$  was not used consistently.

#### Question 6

This was a genuine “applied” mathematics question in the sense that most of the work was pure mathematics following from a little mechanics setting up the initial equations. Part (a) was usually completed successfully although a few forgot to include the mass in their original equation. The point of the question in (b) was lost on most candidates. They showed that  $v$  was never  $\sqrt{6}$ , by indicating that  $x$  had to be infinite for  $v$  to equal  $\sqrt{6}$ . However, they were asked to show that  $v$  never reached  $\sqrt{6}$  so a correct inequality was needed to show that it was always less than  $\sqrt{6}$ . Part (c) was rarely completed correctly. Some candidates gave up at this point. Many candidates had no difficulty in seeing the necessity for replacing the acceleration in part (a) with  $v \frac{dv}{dx}$  but then could not see the necessity for replacing  $v$  with  $\frac{dx}{dt}$  in (c). Even if this hurdle was overcome separating the variables was beyond many – square root term by term and reciprocal term by term should not be seen at this level. Those who did the integration were fairly evenly divided between those who did it by inspection and those who used substitution.

# **Mechanics Unit M4**

## **Specification 6680**

### **Introduction**

This paper was accessible to candidates at all levels, with a number of familiar questions throughout, and some relatively straightforward questions at the start of the paper. Most candidates were able to offer solutions to all questions, but there were a number of blank responses. A significant quantity of high quality work was seen, but full marks were relatively rare. Some questions exposed basic lack of understanding on the part of many candidates. Candidates seem to rely on standard methods and any problem which does not follow a pattern with which they are familiar causes difficulties – this was particularly evident in question 2. Vector methods are not well understood and problems which have solutions which are easily expressed in the vector notation of the question were made much more difficult by the candidates' reluctance to use vectors. The standard of presentation of solutions was not always good, with unclear symbols and excessive crossing out – appropriately sized and clearly labelled diagrams give a candidate a definite advantage.

### **Report on Individual questions**

#### **Question 1**

This question was well answered. Candidates generally favoured the relative vector approach and those who adopted this method were usually successful in finding the bearing correctly. Some candidates were able to use the cosine rule and sine rule on a vector triangle to produce a correct solution but there proved to be many more opportunities for errors in this method. The most common error arose from confusion over the direction of the relative velocity, with some candidates selecting the opposite direction.

#### **Question 2**

Many candidates failed to recognise this as one of the most straightforward questions they are ever likely to meet at this level. Those candidates who did recognise the need for a simple application of conservation of momentum, using the vectors given, were few and far between. It was more common for candidates to apply conservation of momentum in the **i** and **j** directions separately, and such candidates generally did so successfully. Many candidates wanted this to be the more usual style of question, where they could work parallel and perpendicular to the line of centres of the two spheres. Candidates who tried to find the line of centres were generally unsuccessful. Some assumed it was in either the **i** or the **j** direction and made no headway as a result. A large number of candidates were content to regard the velocity as the speed and did not attempt to find the magnitude of their velocity.

#### **Question 3**

This was a standard question and candidates demonstrated a sound understanding of how to solve this sort of problem. Most were able to obtain a differential equation in  $v$  and  $t$ , recognised the need to separate the variables and integrated to find an expression for  $t$  in terms of  $v$ . A few attempts considered upwards as positive but then failed to use  $-U$  and  $-2U$  as the values of  $v$ , but most were competent and successful. Most attempts to solve the problem by writing the equation of motion as a second order differential equation in  $x$  and  $t$  and then going on to differentiate  $x$  failed because they ignored the need for a particular integral as part of the general solution.

#### **Question 4**

This was another standard problem, which most candidates managed easily. The majority of candidates recognised the need to apply conservation of momentum parallel to the wall and Newton's experimental law perpendicular to it. Those that did so often went on to score full marks on this question.

There were many ways through the trigonometric manipulation including working with  $\tan$  throughout, or solving for  $\cos$  first and then finding  $\tan$ . Errors involving the incorrect use of  $e$  were rare but sign errors in the use of Newton's Experimental Law were more common.

#### **Question 5**

Establishing the equation of motion of the system was usually quite well done, although fuller and more convincing explanations should have been given in many cases.

Solving the differential equation proved more taxing. There were frequent errors in the complementary function, the addition of the particular integral, and the stage in the solution at which arbitrary constants should be found. Few candidates paid attention to the dimensional consistency of their equations. The realisation that  $x$  is a length and that the particular integral could also be expressed as a length only ( $-l/2$ ) would have simplified many solutions and enabled  $x$  to be found in terms of  $l$ ,  $w$  and  $t$  only. Those who did obtain the correct general solution tended to be successful in using the initial conditions to find the constants. Some candidates with complicated expressions for  $x$  were able to simplify their final answer to

$\frac{2}{3\omega}$  but many were content to give a final answer that was dimensionally inconsistent with a time.

#### **Question 6**

Candidates needed to apply Pythagoras' Theorem to a correct vector triangle and a great many did so successfully to obtain the given result in part (a). Part (b) was answered rather less well, with many candidates not able to apply implicit differentiation correctly. The use of the function of a function rule or demonstration of standard relations between the various expressions for acceleration was not well done. Very few candidates recognised the resulting equation as SHM. Generally candidates choose to solve this second order differential equation in a traditional way, finding the auxiliary equation and hence the general solution. Those that found a correct general solution, tended to be able to complete the problem correctly. A much smaller number of candidates used the result in part (a) to obtain a differential equation in  $x$  and  $t$  and solved it by separating the variables and using the standard result for arcsine.

#### **Question 7**

This standard problem about equilibrium positions and their stability was well done by most candidates. The vast majority were able to obtain the potential energy of the rod and most could also find  $BP$  and hence obtain a correct expression for the potential energy of the mass, leading to the given result. Generally the methods required for finding a position of equilibrium and determining its stability were well understood by candidates. Parts (b) and (c) were therefore answered well in most cases. In part (d) most candidates appreciated the need to find the second differential and consider its sign for the two values of  $\cos \theta$ . There were a great many mistakes in calculating the values of the second differential, however, although most candidates did make correct inferences about stability from the signs of their answers.

# **Mechanics Unit M5**

## **Specification 6681**

### **Introduction**

Overall the paper proved to be accessible with candidates of all abilities given the opportunity to demonstrate what they could do. Moreover, the vast majority were able to complete it in the time allowed. The questions which seemed to cause most difficulty were question 6 and question 7 i.e. those involving parts of the specification dealing with rotation. Despite comments in previous reports, some candidates continue to ignore the examiners' advice (and indeed the question!) when dealing with the period of a compound pendulum – see the comment on question 5. See also the comment under question 2, regarding the solution of vector differential equations. Candidates should note that methods which involve division by a vector and/or taking the logarithm of a vector will receive no credit, despite the fact that often the correct answer will emerge at the end.

In calculations the numerical value of  $g$  which should be used is 9.8, as advised on the front of the question paper. Final answers should then be given to 2 (or 3) significant figures – more accurate answers will be penalised.

If a candidate runs out of space in which to give his/her answer than he/she is advised to use a supplementary sheet – if a centre is reluctant to supply extra paper then it is crucial for the candidate to say whereabouts in the script the extra working is going to be done.

### **Reports on Individual questions**

#### **Question 1**

Many candidates answered this correctly, using a scalar product approach. Some were, however, unable to deal with  $\mathbf{P}$  being parallel to  $6\mathbf{i} + \mathbf{j}$ . A substantial minority tried to work with magnitudes rather than components and made little progress.

#### **Question 2**

There were three approaches used to tackle this question: treating it as a second order differential equation in  $\mathbf{r}$ , which was the neatest and safest way, working in  $\mathbf{v}$  and multiplying by an integrating factor, which involved solving two differential equations, or lastly, separating the variables and again solving two differential equations. Since the last approach involved dividing by a vector, no marks were awarded for the first stage of this process. Some candidates tried to get round this obstacle by replacing the vector differential equation by two scalar differential equations. These candidates sometimes came to grief if they tried to find the values of constants at too early a stage, as they were faced with  $\ln(-8)$ .

#### **Question 3**

Careful candidates found this question an easy source of marks. A minority were very careless with signs when evaluating cross products and this could lead to a heavy loss of marks in the worst cases. Some candidates either ignored the moment of  $\mathbf{R}$  about the origin or added it to the other moments. Otherwise excellent answers sometimes omitted to find the magnitude of the couple.

#### **Question 4**

In order to gain all of the marks in part (a), it was necessary to consider the change in momentum in a time  $\delta t$ . Some candidates obviously consider this to be a standard piece of bookwork, whereas others are clearly trying to work it out in the exam, in some cases considering  $M_0$  to be the mass at a general time  $t$ . Candidates trying to use the given answer to correct their signs should beware of altering the wrong ones!

In part (b), many candidates gave the very simple explanation involving the acceleration at time  $t = 0$  being positive. Others considered the general expression for acceleration and struggled to produce the given inequality. Most candidates were successful with part (c).

### Question 5

The vast majority of candidates successfully reached the required answer in part (a). In part (b), many candidates took moments as was intended. However, there were as usual candidates who ignored the instruction to write down an equation of motion for  $P$ , instead quoting a formula for the period, which gained no credit. Candidates should be reminded that the SHM formula should contain a minus sign. A few candidates interpreted “equation of motion” as an instruction to use Newton's second law.

### Question 6

Most candidates tried to use an increment of volume and then integrate, but a substantial minority did not use the parallel axes theorem to change to an axis through one of the ends of the cylinder, thus only gaining a possible 3 marks out of 10. A surprising number of candidates appeared to be considering a cone or a sphere. Some got their constants and variables mixed up, both in creating an expression for the moment of inertia of the increment and in integration.

### Question 7

Most candidates were successful with part (a), but often had little success with the later parts. Candidates who used moments in part (b) often seemed to assume that the moment of the weight had the opposite sign to  $\ddot{\theta}$ , as is often the case. An omission of a 2 from  $2mg$  could easily lead erroneously to the printed answer. A selection of invalid methods were used in trying to find the impulse in part (c). These included impulse = change in angular momentum (rather than moment of impulse) and impulse = change in linear momentum.

# **Statistics Unit S1**

## **Specification 6683**

### **Introduction**

There was a higher standard of responses than in previous Statistics S1 papers and candidates showed that they knew which techniques to select and also how to use them. There were relatively few scripts with very low marks and good candidates were able to score high marks and with the singular exception of Q6, the paper was well answered. However some candidates continue to lose marks because they do not do the simple things sufficiently well. It was not uncommon to observe that candidates had miscopied numbers from either their calculator display or their own work, e.g. 0.0375 sometimes became 0.375. Some candidates failed to review the reasonableness of their answers and were seemingly unaware of obvious errors e.g. probabilities greater than 1 and a mean outside the range of given data. There was also confusion between *skewness* and *correlation* in Q2 and Q4.

### **Report on individual questions**

#### **Question 1**

Most candidates were able to draw a six-branched tree diagram correctly, although a number of candidates had incorrect or missing labels. From a correct diagram most gained full marks in part (b). The conditional probability in part (c) once again caused difficulty for many of the candidates. Many of the responses in part (d) were, incorrectly, referring to the importance of testing people for a disease rather than referring to the probability in part (c).

#### **Question 2**

Parts (a) to (d) represented a chance for all students who had an average grasp of statistics to score highly. The median in part (b) was incorrectly identified by a significant number of candidates, but the standard deviation was often correct.

Part (e) was done surprisingly well with students appearing to have a much greater understanding of what is required for a comparison than in previous years. Often numbers were stated without an actual comparison. Confusion was evident in some responses as skewness was often referred to as correlation. A small minority of candidates had failed to take note of the 'For the Balmoral Hotel' and had done some correct statistics for all 55 students.

#### **Question 3**

This proved to be a good question allowing the better students to show their understanding of the topic. It was generally done well with the majority of students aware of what they were trying to achieve. Of those who did less well, many failed to realise the significance of the 0.55 and others only came up with one equation in part (a) and thought that  $q$  was therefore equal to 0. A very large number of candidates incorrectly squared -1 in part (b) affecting their calculation of variance. Part (c) was generally well answered.

#### **Question 4**

This was done well by all but the weakest students with most using sufficient accuracy to score highly. Many candidates demonstrated an understanding of the use of the formulae to achieve full marks in part (a) and part (b). By far the main reason for loss of marks was premature approximation. Part (c) and part (d) were done well by good candidates. Only the more able candidates had a correct reason why  $t$  was the explanatory variable. Many called  $v$  the explanatory variable but gave a correct reason for  $t$ . The written parts were not universally done correctly, although the ability of students to deal with this topic has improved considerably in recent examinations. Rounding once again caused issues in part (e), but usually did not have an effect on part (f).

#### **Question 5**

A lot of fully correct Venn Diagrams were seen in part (a) although it was surprising the number who resorted to decimals rather than just using straightforward fractions; this often led to loss of many accuracy marks. A significant minority had negative numbers in their Venn diagram and saw nothing wrong in this when converting them to probabilities later in the question. Fewer candidates forgot the box this time. Part (c) proved to be the only difficult part, as many candidates struggled with the concept of conditional probability, and many denominators of 300 were seen.

#### **Question 6**

This question was an excellent example of why students should revise the syllabus and not just from past papers. Only a minority of candidates tackled this question effectively; some candidates seemed to have no idea at all as to how to tackle the question. Those who gave correct solutions often made many incorrect attempts in their working. The vast majority showed an understanding of discrete random variables but most missed or did not understand the word “cumulative” and consequently spent a lot of time manipulating quadratic expressions trying to make them into a probability distribution. The majority view was that  $F(1) + F(2) + F(3) = 1$  which led to a lot of incorrect calculations.

#### **Question 7**

Part (a) was answered with the highest degree of success with all but the weakest students not gaining 3 marks. Many candidates incorrectly interpreted the sign of the inequality in part (b) and went on to calculate a score above the mean. Many of those who arrived at an answer got circa 54 kg and failed to consider the reasonableness of their answer i.e. if the mean is 50kg, then 99% of the packets cannot weigh more than 54kg. Candidates often did not use the percentage points table quoting 2.33 instead of 2.3263 and this was reflected in the very small proportion of B marks awarded in part (b). Few candidates equated their standardised equation to a z-score with a consistent sign. Part (c) really sorted out those who really understood what was going on and who were hazily following the rules. Many failed to recognize that they only required the answer from part (a), a marked number specifically calculating the individual probabilities anew. The number who recognized that there was a factor of three involved was small although a significant number scored 2 marks by using  $p^2 (1 - p)$ . Quite marked was the significant number who multiplied 0.0688 by two rather than squaring it. There was also a fair number who added the probabilities rather than multiplying.

# **Statistics Unit S2**

## **Specification 6684**

### **Introduction**

Candidates would appear to have had adequate time to do this paper. There were few questions where no attempt had been made to produce an answer. The level of work was very mixed. There were a number of candidates who had little idea about significance testing and quite a few who had problems with selecting the correct distributions. The standard of presentation was very weak – it was often very difficult to follow the work. Presentation of working on a page was often disorganised, this was particularly the case on question 7. Handwriting was sometimes very poor. Candidates did not always label the part of the question they were doing and parts ran into each other often with no indication of what their answer referred to.

### **Report on individual questions**

#### **Question 1**

Part (a) was answered well. A minority of candidates had difficulty with parts (b) and (c). In (b) they generally looked at  $P(X > 2)$  rather than  $P(X \leq 2)$ .

In part (c)  $\frac{1}{5} \times 5$  or  $\left(\frac{4}{5}\right)^5$  was used even when part (b) was correct.

The work on part (d) was disappointing. Candidates did not understand the concept of conditional probability.

Only the very best candidates are likely to have got this correct. Those that did used the  $U[0,5]$  or  $U[5,10]$  route.

#### **Question 2**

Candidates did this well on the whole, with the main error being the absence of a continuity correction (which was penalised appropriately.) The question worked well although the 42% mentioned in the question caused some confusion as many took this to be the percentage of students that were female rather than 58%. Very few candidates were unable to make a reasonable attempt at a Normal approximation.

#### **Question 3**

Whilst many candidates knew what they were doing in part (a) they lost marks because they left their answers as  $P(X \leq 3)$  etc and did not define the critical regions. A few candidates were able to get the figures 0.0212 and 0.0220 but then did not really understand what this meant in terms of the critical value. A critical region of  $X \geq 15$  was common.

Part (b) was poorly answered. The wording “incorrectly rejecting  $H_0$ ” confused many candidates. They often managed to get to 0.432 but then they took this away from 0.5 or occasionally 1. It was not uncommon for this to be followed by a long paragraph trying to describe what they had done.

#### **Question 4**

This question was generally answered well. A few candidates put the Poisson for (a) and then used Variance = Mean to get 5.5 for the variance. Some candidates rounded incorrectly giving an answer of 5.49 for the variance.

Part (c) was generally answered correctly although a minority of candidates used the normal approximation – most used 2.5 in their standardisation and so got 1 mark out of the 4.

#### **Question 5**

Most candidates recognised that this was a Binomial for part (a) but quite a few did not define it completely by putting in the 15 and 0.5. Parts (b) and (c) were answered well. Most candidates are getting much better at the layout of their solutions and it was good to see that candidates are identifying one-tail tests, although some candidates did not get the hypotheses completely correct; omitting  $p$  altogether or using  $\lambda$  was not uncommon. The working for finding 0.0037 or a critical region  $\geq 13$  was often done well. Those who used a critical region method required more working and seemed to make more errors. It is recommended that this method is not used in Hypothesis testing at this level. There was the usual confusion in interpreting the test. In particular candidates often did not interpret correctly in context. ‘The coin is biased’ was a common inadequate answer.

#### **Question 6**

Part (a) was done well generally although a reference to ‘calls’ was not made by a few candidates. Weak candidates talked about the Poisson needing large numbers and others seemed to not understand what was required at all; writing ‘quick and easy’. Part (b) was done correctly by the majority of candidates. A few did not use  $P(X > 8)$  in (i) and a number used  $P(X > 8) = 1 - P(X \leq 7)$  in (ii). In part (c) weaker candidates did not use  $\lambda$  for their hypotheses nor did they use Po(9). Some hypotheses had  $\lambda = 3.5$  and  $\lambda \geq 3.5$ . Two tail tests were often suggested. Most candidates got to 0.0739 and only a few candidates used the critical value route. Only the able candidates got the interpretation of the significance test correct. Weak candidates generally only considered whether it was significant or not, with mixed success. They rarely managed to interpret correctly in context.

#### **Question 7**

A minority of candidates achieved a high rate of success on this question. Part (a) was often badly done with lots of fiddling of figures involved in getting to  $\frac{1}{5}$ . The work was often very poorly

organised with lots of crossing out so that candidates did not really know what they had got. Part (b) was often at least half correct. Quite a few candidates stopped at 1.24 but many managed to gain the correct answer. Part (c) was completed correctly by a minority of candidates. Many candidates did not put in limits although they subsequently used them. An incorrect answer of

$\frac{1}{20}x^4 + \frac{1}{20}$  was common and  $\frac{1}{20}x^4 - \frac{1}{5}$  cropped up in a several cases showing an inability to deal with fractions correctly. Part (d) was answered well by the more able but many candidates equated  $\frac{1}{4}x^2$  to a half rather than  $\frac{1}{20}x^4 + \frac{1}{5}$ .

In part (e) if a candidate attempted this part they generally got 1 mark and often 2 marks.

# **Statistics Unit S3**

## **Specification 6691**

### **Introduction**

The paper proved accessible to all the candidates and there was no evidence that they were short of time. The work on hypothesis tests was generally better with most candidates using population parameters and giving correct conclusions in context. The work on linear combinations of normal random variables (Q4) continues to prove challenging for some candidates but generally the standard of work seen on this paper was good.

### **Report on individual questions**

#### **Question 1**

The mean in part (a) was nearly always correct but premature approximation meant that the answers for  $s^2$  were often not accurate to 3sf. In part (b) the correct formula for a confidence interval was usually used but the full  $z$  value of 2.5758 was sometimes truncated or an approximate value used from the “large” table.

#### **Question 2**

This question was answered very well. A few candidates did not give their expected frequencies to 1 decimal place but the numerical work was usually carried out very proficiently. The hypotheses were mostly correct and given in context as was the final conclusion. A few were caught out by the 1% significance level but for most of the candidates this question was a good source of marks.

#### **Question 3**

The majority of candidates could draw a correct diagram for  $r = 1$  but far fewer managed to do so for (a)(ii), a set of points lying on a line of negative gradient was a common error. In part (b) the ranking caused some to stumble and a value of 4 for  $\sum d^2$  was fairly common but most could use the formula for  $r_s$  correctly. The hypotheses in part (c) were sometimes given in words or in terms of  $r_s$  rather than  $\rho$  and a number of candidates used a two-tailed test. The appropriate critical value was usually given and the conclusions were nearly always correct and in context.

#### **Question 4**

In part (a) most found the mean of 336 but some thought the variance was  $44^2$  however many did obtain the correct values here and went on to complete this part successfully. Part (b) though proved more challenging. Some never defined a suitable random variable such as  $M - 1.5W$  and made no progress. Those who did make a suitable start were usually able to obtain the mean of -9 but the variance calculation caused difficulties with many using 1.5 instead of  $1.5^2$ . The standardisation was usually carried out correctly but a diagram would have helped some get their final probability the right way round.

### **Question 5**

Questions of this type are usually quite challenging for candidates and examiners alike. Candidates should be aware of how many marks the examiner is seeking to award in each part of the question and try and ensure that they make that many independent points. The use of bullet points rather than continuous prose might help both candidates and examiners.

In part (a) a number of candidates missed the fact that the sample contained no managers. Most knew how to take a systematic sample and explained the need to label the employees and pick every 11<sup>th</sup> one but a mechanism for selecting the first one at random was often not mentioned. The stratified sampling procedure was well known too and usually applied to this situation quite well. Some lost a mark for failing to label the managers and cleaners or for not using random numbers when selecting the samples from each strata. Part (c) proved an easy two marks for those who knew how to use the random number tables and most did know!

### **Question 6**

Part (a) was a “Show that...” and many candidates failed to provide sufficient evidence. Some clearly showed where the 223 came from but an alarming number thought that  $\frac{223}{100} = 0.223$ , others assumed the 223 but did show that the denominator was  $10 \times 100$  with only the most careful students explaining how both numerator and denominator were found and securing both marks. Part (b) was well done although some did not give answers to 2 decimal places as in the table. Most stated the hypotheses correctly but some failed to include the value of  $p = 0.2$ . Part (d) caught out a few candidates who failed to realise that the final two classes needed merging leading to the calculation  $5 - 1 = 4$ , the common error was to assume that  $p$  had been estimated and  $4 = 6 - 1 - 1$ . A number of candidates decided to calculate the test statistic themselves rather than using the value of 4.17 given in the question but they usually made a correct comparison with the critical value although few remembered to mention the “cuttings” in their final conclusion.

### **Question 7**

Part (a) was answered well with only a few candidate confusing standard deviations with variances in the test statistic or using a critical value of 1.6449 instead of 1.96. Most had the hypotheses correct too with some giving them in terms of words and the population parameters. The conclusion was usually correct and in context. Part (b) exposed some candidate’s weak understanding of the central limit theorem with several lengthy answers failing to mention “mean” at all.

# **Statistics Unit S4**

## **Specification 6686**

### **Introduction**

Overall the paper worked well enabling nearly all candidates to demonstrate what they knew but also allowing the stronger candidates to show their true potential.

Most students found the first 4 questions very accessible and many scored highly here. Questions 2(b), 4(c), 5(b), and 6 proved to be good discriminators and only the better candidates made significant progress through these. Generally candidates were able to carry out calculations but showed a lack of understanding when they had to use or interpret the answer to their calculations.

### **Report on Individual Questions**

#### **Question 1**

This proved to be a good starter question and most candidates gave good solutions. A minority of candidates did not state the bias.

In part (b) many candidates did not know that the Variance of  $X_2, X_3$  etc was  $\sigma^2$  not  $\frac{\sigma^2}{n}$

#### **Question 2**

This question was answered well with a large proportion of the candidates getting full marks. In part (b) many candidates stated that the marks were normally distributed which was not an assumption they had made as it had been stated in the question.

In part (c) the pooled estimate was worked out correctly by many candidates but they then failed to use the square root of it in their calculations of  $t$ . Many candidates were able to draw a conclusion in the context of the question.

#### **Question 3**

Full marks were gained on this question by many candidates with a large proportion of them using a paired  $t$ -test. A minority used a 2 sample test. It is important that candidates learn when to use which test as the test required will not always be stated in the question

#### **Question 4**

Most candidates knew the method to use and realised that the  $t$ -value was required but a significant number used the normal distribution value of 1.6449.

In part(c) many candidates thought that because the value was in the confidence interval that the council should be worried showing a lack of understanding of what confidence intervals are.

#### **Question 5**

The vast majority of candidates gained full marks for part (a). In part (b) many candidates simply used 0.07 and stated it was not in the interval and a minority of those who did realize they needed to use 0.0049 did not state at any point that there was evidence that the standard deviation of the volumes was 0.07.

### **Question 6**

This question proved to be the most challenging for many candidates. In part (a) they did not understand what was required and stated  $B(20, 0.35)$  as their answer. In part (b) the most common error was to use  $P(X \leq 4) + P(X \geq 10)$ . Parts (c) and (d) were generally answered correctly but candidates showed a lack of understanding in part (e). Many thought this test was suitable and did not realise that the power of the test gives an indication of its suitability.

### **Question 7**

Candidates found this a nice last question. Many of them were able to gain full marks. The most common error was not finding the unbiased estimate of the population variance and using the variance of the sample instead.

# **Decision Mathematics Unit D1**

## **Specification 6689**

### **Introduction**

This paper proved accessible to the candidates. The questions differentiated well, with each giving rise to a good spread of marks. All questions contained marks available to the E grade candidate and there also seemed to be sufficient material to challenge the A grade candidates also.

Some candidates wasted time in Q1 and Q5 using a sorting algorithm to order the numbers. Unless candidates are directed to use a sorting algorithm in the question they may re-order numbers by inspection for First –Fit decreasing and Kruskal.

Candidates are reminded that they should not use methods of presentation that depend on colour, but are advised to complete diagrams in (dark) pencil.

Candidates are also reminded that this is a ‘methods’ paper. They need to make their method clear, ‘spotting’ the correct answer, with no working, rarely gains credit. This was particularly true in Q4b where some candidates just drew the MST with no method visible.

Some candidates are using methods of presentation that are too time-consuming. The space provided in the answer booklet and the marks allotted to each section should assist candidates in determining the amount of working they need to show.

Centres are reminded that Flows in Networks (Q5) and Simplex (Q6) have now made their farewell appearance on D1 as they move to D2 from January 2009.

### **Report on individual questions**

#### **Question 1**

This proved an accessible first question and was well answered by many candidates. Some candidates probably spent too long on this question, drawing out very neat and accurate bar graphs in (b) and (c), where numbers in bins were perfectly acceptable. Most candidates calculated the lower bound correctly in part (a) although some attempted a full bin solution and others divided by 9, some having calculated 5.02 rounded down to 5. Apart from the usual omissions of data, part (b) was usually well answered, the most common slips being to swap the 38 and the 41, or to use the 52 to start off the second bin. First –Fit **increasing** was disappointingly often seen in part (c), but those who used the correct algorithm were usually successful with the only common error being misplacing the 38.

#### **Question 2**

This was a good source of marks for well-prepared candidates. Most are remembering to show the ‘change status’ step and are making the alternating paths they used clear. Alternating paths must go from an unmatched node to an unmatched node, a number of candidates added A–2=E onto the end of the alternating path found in part (a) for example. Candidates are reminded that they are instructed to list their alternating paths, which makes them clearer for the examiners to give full credit. Many candidates got involved in long explanations in part (c) usually referring to A, E, G,

W, 2, 3 and 5 rather than the simpler 1R4 answer. Candidates scoring full marks in (a) and (b) were usually successful in part (d) too.

### **Question 3**

Most candidates found part (a) straightforward and gained full marks, but candidates are reminded that it is the order of the working values that is key for examiners to determine if the algorithm has been applied correctly. Many gained full, or nearly full marks in (b) with errors mostly due to stating an incorrect inspection route (C was frequently omitted). Some wasted time finding the shortest route from A to H from scratch, rather than using their working for part (a), Dijkstra's algorithm finds the length of the shortest route from the start to all intermediate points. Others wasted time finding the weight of the network even though it was given to them.

### **Question 4**

Part (a) caused problems for some candidates. Candidates were asked for differences between the two algorithms and many simply made statements such as Prim uses nodes without going on to say the Kruskal uses edges. A popular answer was to mention ordering the arcs into ascending order for Kruskal; if this used as a difference candidates must indicate why this is important when using the algorithm. There was the usual muddled use of basic technical terminology. Some candidates muddled the two algorithms. Part (b) as usually well-answered although some just drew their MST and did not list the arcs, in order, as instructed. Some wasted time in Prim by converting to the matrix form. Some did not make their rejections clear when applying Kruskal's algorithm.

### **Question 5**

Candidates often answered parts (a) (c) and (e) correctly. Part (b) was well answered by most but some omitted at least one arc. Part (d) proved the most challenging, although good answers were seen many used the flow numbers rather than the capacity numbers. In part (f) candidates needed to state a minimum cut and refer to the max flow-min cut theorem, few did both.

### **Question 6**

This was the most challenging question for some on the paper and some omitted the question entirely. Of those who attempted the question most, although not all chose the first pivot correctly. More are remembering to change the basic variable. Many selected a negative pivot, with a consequent negative 'theta' value, for their second pivot choice, which is not acceptable. Candidates are reminded that negatives in the value column (apart from the P row) are a certain sign that something has gone wrong. Basic variables can never be negative at any point in the algorithm. Most candidates were able to gain the mark in part (b), but some referred to the profit column or profit equation.

### **Question 7**

This was a challenging question with few scoring full marks, even in part (a), although most were able to gain some credit often in part (b) and (c). Most candidates were able to score at least one mark in (a), although full marks were rarely seen. Part

(b) was usually successfully attempted, although some included E as a critical

(b) was usually successfully attempted, although some included E as a critical activity. The floats were often calculated correctly in part (c). Many omitted, or made only a token attempt at part (d) others drew a scheduling diagram. Of those who made full attempt most handled the critical activities correctly, but activities A and B were often incorrectly drawn with float being added to B and removed from A. Part (e) was answered well by some but others focussed on activities H I and J etc, rather than just E's total float of 1. Relatively few candidates gained both marks in (f). When using a cascade chart to determine a lower bound, candidates need to state both the time they are looking at and the activities that must be happening then.

### Question 8

Many candidates omitted ‘maximise’ here, other common errors were omitting the non-negativity constraint on  $a$ , and getting the 2 on the wrong side of the second inequality. A number of candidates tried to combine several conditions into one inequality, a frequently seen one being  $2a + b \leq 800$ . A number of candidates wasted time by starting to solve the LP problem.



# **Decision Mathematics Unit D2**

## **Specification 6690**

### **Introduction**

The paper proved accessible. Q2 proved a good early question and questions 4 and 6 in particular discriminated well. The candidates did not seem to have any time problems and most were able to attempt all parts of all questions.

Good use was made of the tables in the answer book in general with far fewer ‘over using’ one table to show three or four steps at once, see particularly Q2. Candidates generally presented their solutions clearly. The general quality of presentation was good and much improved on recent years with candidates using clear but more efficient styles. Centres should be congratulated on this, it has helped candidates with their time management. Most candidates demonstrated that they had plenty of time for the paper.

Candidates are reminded that they should not use methods of presentation that depend on colour, but are advised to complete diagrams in (dark) pencil.

Centres are reminded that as from June 2009, Flows in Networks and Simplex, both formerly in D1, are moving to D2.

### **Report on individual questions**

#### **Question 1**

The definition of ‘walk’ was less successful than the definition of ‘tour’. Some candidates wrote the same definition for both parts. Part (a) proved quite challenging with poor use of technical terminology, but most were able to gain credit in (b) with many gaining both marks.

#### **Question 2**

This was a good early question for the candidates, and a good source of marks for most. Almost all the candidates gained some credit in part (a) for saying that supply was not equal to demand, but fewer stated that supply was greater than demand. Parts (b) and (c) were well done. Most found the correct first set of 5 shadow costs and 2 improvement indices, although some, often those who used a table to display them, included extra (zero) improvement indices. Although there were plenty of tables printed a few candidates tired to show the initial solution, improvement indices and stepping stone route all on just one diagram, making it very tricky to read. Most candidates found the correct stepping stone route and improved solution, although once again an extraneous zero sometimes appeared in the emptying square. A few candidates failed to recalculate shadow costs for the improved solution and some did not evaluate the new improvement indices. Most were able to find the correct cost.

#### **Question 3**

This was often a rich source of marks for most. It was rare to see candidates that gained both marks in part (a), with poor use of terminology seen, especially route/arc/path/ but also imprecise use of vertex, distance and value. Part (b) was often well done with many fully correct solutions seen. All the ‘usual’ misreads were also seen with candidates finding the minimax, minimum or maximum routes. A much smaller number of candidates than usual reversed the states or worked forwards, and centres should be congratulated in this.

#### **Question 4**

Most candidates were able to give a reasonable definition of dominance, although some confused column and row dominance. Many made rather sweeping statements such as ‘the values in the x column are less than the values in the y column’, rather than preceding this by ‘for each row’. The majority of candidates answered part (b) well, but some found the row maximin and column minimax or did not identify the two key values from their row minimums and column maximums. Algebraic errors were pleasing rare in part (c) this year and most were able to find the correct three equations and draw them accurately. Examiners are still disappointed by the number of candidates who do not use a ruler to draw straight lines, and a number of candidates continued their lines beyond the domain for a probability. The optimal point was less regularly correctly identified with many choosing the intersection of the lines  $3 + 3p$  and  $6 - 2p$  and giving a popular incorrect solution of  $3/5$ . A significant number of candidates did not state the probability for each of Liz’s three options and/or the value of the game. Part (d) was usually well answered, but a few wasted time setting up a simplex tableau.

#### **Question 5**

This algorithm was generally well understood. A few candidates minimised, but most of these, realising that the solution was optimal after the row and column subtractions, restarted and maximised. Otherwise this question was extremely well done with only a few slips. Candidates should try to take care that their lines are not so thick that they obscure the numbers below.

#### **Question 6**

Many candidates found the MST correctly but a significant minority used Prim’s rather than Kruskal’s algorithm. Most were able to use their answer to find a correct initial upper bound. Candidates did not always make their shortcuts clear in part (c). Candidates are reminded that this is a ‘methods’ paper and they must make their method clear. The most successful candidates were those who stated their short cuts clearly and then listed the route and its length. Some candidates evidently just ‘spotted’ a route and gained no credit. Most candidates were able to find the NN route in (d), but some did not add the final arc to B, others doubled their route from B to D. In part (e) most candidates found the correct residual minimum spanning tree, although BD was often incorrectly included. The vast majority added in the two leaf arcs to C. Full marks in (f) was only possible for those gaining full credit in (c) – (e).

# **Grade Boundaries: May/June 2008 GCE Mathematics Examinations**

The tables below give the lowest raw marks for the award of the stated uniform marks (UMS).

<b>Module</b>	<b>80</b>	<b>70</b>	<b>60</b>	<b>50</b>	<b>40</b>
6663 Core Mathematics C1	62	53	44	36	28
6664 Core Mathematics C2	63	54	45	36	27
6665 Core Mathematics C3	64	57	50	43	37
6666 Core Mathematics C4	57	50	44	38	32
6674 Further Pure Mathematics FP1	61	54	47	40	33
6675 Further Pure Mathematics FP2	54	47	40	33	27
6676 Further Pure Mathematics FP3	55	49	43	37	31
6677 Mechanics M1	57	49	41	33	25
6678 Mechanics M2	64	56	49	42	35
6679 Mechanics M3	57	50	43	37	31
6680 Mechanics M4	65	58	51	44	38
6681 Mechanics M5	56	48	41	34	27
6683 Statistics S1	60	54	48	42	37
6684 Statistics S2	65	58	51	44	37
6691 Statistics S3	63	55	47	39	32
6686 Statistics S4	66	57	48	39	30
6689 Decision Maths D1	58	51	44	37	30
6690 Decision Maths D2	63	55	47	40	33



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