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## Contents

<table>
<thead>
<tr>
<th>Subject</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core Mathematics C1</td>
<td>5</td>
</tr>
<tr>
<td>Core Mathematics C2</td>
<td>9</td>
</tr>
<tr>
<td>Core Mathematics C3</td>
<td>13</td>
</tr>
<tr>
<td>Core Mathematics C4</td>
<td>17</td>
</tr>
<tr>
<td>Pure Mathematics FP1</td>
<td>21</td>
</tr>
<tr>
<td>Pure Mathematics FP2</td>
<td>25</td>
</tr>
<tr>
<td>Pure Mathematics FP3</td>
<td>29</td>
</tr>
<tr>
<td>Mechanics M1</td>
<td>33</td>
</tr>
<tr>
<td>Mechanics M2</td>
<td>35</td>
</tr>
<tr>
<td>Mechanics M3</td>
<td>37</td>
</tr>
<tr>
<td>Mechanics M4</td>
<td>41</td>
</tr>
<tr>
<td>Mechanics M5</td>
<td>43</td>
</tr>
<tr>
<td>Statistics S1</td>
<td>45</td>
</tr>
<tr>
<td>Statistics S2</td>
<td>47</td>
</tr>
<tr>
<td>Statistics S3</td>
<td>51</td>
</tr>
<tr>
<td>Statistics S4</td>
<td>53</td>
</tr>
<tr>
<td>Decision Mathematics D1</td>
<td>55</td>
</tr>
<tr>
<td>Decision Mathematics D2</td>
<td>59</td>
</tr>
<tr>
<td>Grade Boundaries</td>
<td>63</td>
</tr>
</tbody>
</table>

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Introduction

This paper was accessible and nearly all the students were able to make some progress on most questions and there was no evidence that the candidates were short of time. Fluent simple arithmetic without a calculator is a great asset when tackling a paper of this type and far too many candidates are losing marks for careless or ineffective arithmetic and algebraic manipulation.

It is generally good practice in mathematics to simplify equations where possible. Working with un-simplified expressions often leads to arithmetical errors and the examiners reported many examples of this for example on Q7.

Candidates and centres should realize that a sketch does not require points to be plotted and certainly shouldn’t be drawn on graph paper. An unlined space in the answer book will be provided, as in Q3 and Q9 and a free hand drawing with a pen or soft pencil is all that is required.

Candidates should be reminded to state the formula that they were using before substituting values into it. This was particularly important in Q11.

Report on Individual Questions

Question 1

This was a successful starter to the paper and nearly all the candidates were able to make some progress. Most were able to integrate the first two terms successfully but some treated the third term as $x^\frac{1}{2}$ rather than $x^{-\frac{1}{2}}$. Simplifying the terms did cause some difficulties though and $\frac{6}{3} = 3$ or $\frac{1}{\frac{1}{2}} = 1\frac{1}{2}$ were common errors. Most remembered to include the $+c$.

Only a small minority of candidates tried differentiating the expression which suggests that the notation was understood well.

Question 2

It was encouraging to see most candidates factorizing the quadratic expression in order to find the critical values for the inequality. Sometimes the critical values had incorrect signs, despite the factorization being correct, but the most common error was still a failure to select the outside region. Some candidates struggled with the correct symbolic notation for the answer and $-2 > x > 9$ was occasionally seen.

A few candidates chose to use the formula or completing the square to find the critical values, these approaches are less efficient in this case and often gave rise to arithmetical errors.

Question 3

This question was answered well with most candidates having a good idea of the shape and identifying the need for a vertical translation. In part (a) most drew a parabola but some common errors were $y = (x - 3)^2$, $y = x^2 + 9$ and $y = (x - 3)(x + 3)$. Some candidates thought that the curve should be asymptotic to the y-axis. Attempts at part (b) were sometimes less successful. Some could not deal with a general number $k$, choosing to give it a value, and vertical stretches rather than a translation were quite common. In some cases it was not clear what transformation had been applied.
As is usual with this type of question there were cases of confusing coordinates with the intersection on the $y$-axis for example being labelled $(9, 0)$.

**Question 4**

Most candidates knew how to start this question and full marks for part (a) was common, however some lost out due to poor arithmetic such as $3 \times 3 - 5 = 6 - 5 = 1$ for $a_2$. A minority of candidates though had no idea how to interpret the recurrence relation notation with a significant number interpreting $3a_n - 5$ as $3 \times n - 5$. In part (b) many candidates were convinced that the series had to be arithmetic and they gained no further marks. Some did find $a_4$ and $a_5$ correctly but then used the arithmetic formula $\frac{n(a + l)}{2}$ with $l = 43$. Clearly students are familiar with the work on arithmetic series but in some cases this seems to have overshadowed their understanding of recurrence relations.

**Question 5**

Many students answered this question very well but there were the usual crop of errors as well as some unusual misinterpretations of the notation.
In part (a) the first term was nearly always differentiated correctly but some interpreted $6\sqrt{x}$ as $x^{\frac{3}{2}}$ and some integrated one or both terms. In part (b) most were able to multiply out the numerator correctly, although a few still gave it as $x^2 + 16$, but then problems arose. A number simply differentiated numerator and denominator but many did attempt some sort of division. A common mistake though was to multiply by $x$ instead of divide whilst others simply added or subtracted $x^{-1}$ to their expression. Those who did complete the division correctly often went on to complete the problem but some forgot to differentiate the $x$ term and the $1$ was missing from their answer. Some, presumably A2 candidates, used the product or quotient rule to answer part (b) with varied degrees of success.

**Question 6**

This question was generally answered well although there was the usual crop of arithmetic and sign errors especially in part (a) where some candidates struggled to simplify $(\sqrt{3})^2$. In part (b) most knew how to start the problem, although a few multiplied by $\frac{4 + \sqrt{3}}{4 + \sqrt{3}}$. It was disappointing to see how many candidates multiplied out the numerator first and then divided by $13$, often forgetting to divide one of the two terms by $13$.

**Question 7**

There were many good responses to this question and candidates who used the correct formulae for arithmetic equations were often able to solve two simultaneous equations and reach the answers quickly. Some candidates showed weaknesses in algebraic processing e.g. $11(10 + 10d)$ leading to $110 + 10d$ or $10d = 4$ followed by $d = 10/4 = 2.5$. There were the inevitable arithmetic errors too e.g. $2 \times 77 = 144$ was common. Careful simplification of equations at each stage would avoid the need for such difficult calculations and the subsequent risk of errors.
Candidates should be encouraged to see that \( \frac{11}{2} (2a + 10d) = 77 \) can be simplified to
\[
\frac{1}{2} (2a + 10d) = 7 \text{ or even better } a + 5d = 7.
\]
Inevitably some candidates used a trial and improvement approach to this question, this is not recommended, wastes time and is particularly susceptible to errors. By contrast some candidates found very slick and efficient solutions to the question.

**Question 8**

Candidates who equated the discriminant \( b^2 - 4ac \) to zero were often successful although \( b^2 \) was often given as \( 2p^2 \). Sometimes a second error in multiplying out \(-4(3p + 4)\) as \(-12p + 16\) led to the incorrect equation \( 2p^2 - 12p + 16 = 0 \). This of course led to \( p = 4 \) and although accuracy marks were lost in part (a) full marks could be gained in part (b). Some tried to complete the square usually with little success but others did find \( p = 4 \) by trial and improvement. A few candidates spotted that \( 3p + 4 \) had to be a square number and indeed had to be \( p^2 \) and this often lead to a correct solution.

In (b) the majority simply solved to find equal roots, very few candidates seemed to appreciate that the root will simply be \( x = -\frac{b}{2a} \).

**Question 9**

Part (a) was answered well although some candidates forgot the \( 3x \) term and the double negative caused problems for some leading to a value of 9 instead of 15. In part (b) they usually factorized their quadratic but often failed to write the complete expression including the \( x \). Many thought they had to solve an equation possibly suggesting some confusion between “solving” and “factorizing”. In part (c) most of the candidates knew the shape of the cubic graph although some were inverted and others failed to appreciate that \( x = 0 \) was a root and they simply sketched a quadratic or a cubic with a repeated root.

**Question 10**

This question was not always answered well. Most candidates knew that integration was required in part (a) and usually they scored both the marks but many forgot to include a constant of integration. Many still did not realize their error even when they obtained \( f(-2) = 5.5 \) and this proved to be a costly mistake.

The candidates could still complete part (c) even if they had made mistakes in part (a) and most attempted this part. The most common error was to find the equation of the chord between the points \((-2, 5)\) and \((3, 7.5)\). Those who realized that \( f'(-2) \) was required often had trouble with the arithmetic and some thought that the gradient of the normal was required. A few candidates used \((3, 7.5)\) instead of \((-2, 5)\) in part (c). Those candidates who successfully negotiated these pitfalls usually gave their answer in the required form but there were few fully correct solutions to this part.
Question 11

The formulae for the gradient and the equation of a straight line were well known and many candidates scored well here. There were a number of candidates though who made errors and it was not always clear if a correct formula was being used. A few failed to give the equation in the correct form.

In part (b) candidates generally used a correct method but arithmetic slips, sign errors or weak algebraic manipulation led a significant number to obtain coordinates for $S$ that were clearly inappropriate or unlikely on a non-calculator paper. Some candidates used a graphical approach to find $S$ but they gained no credit as the question asked them to “calculate”.

Most candidates used a correct method in part (c) but those with mistakes in earlier parts often seemed unperturbed that their answer was not as printed. Once again part (d) could still be completed using the given result in the previous part. Far too many candidates had poor diagrams (or none at all) but for those who saw the connections with the rest of the question it was a simple move to find the length of $PQ$ and use the usual half base times height formula to obtain the answer but some were unable to cope with the manipulation of the surds. Many assumed that $PR$ was perpendicular to $QR$, often based on a poor diagram. Others though, with more carefully drawn diagrams, used an enclosing rectangle, or trapezium, and by subtracting the areas of simple right angled triangles were able to arrive at the answer of 45 quite easily.
Core Mathematics Unit  C2  
Specification 6664

Introduction

This paper was a fair test of the specification, giving most candidates ample opportunity to demonstrate their knowledge and understanding. Standard, familiar questions were a good source of marks for many, while some parts proved more demanding and discriminating. Weaker candidates often had difficulty with some aspects of trigonometry and logarithms. Poor algebra was commonly seen throughout the paper and particularly in question 9. In general, candidates appear to have had time to attempt all ten questions. Unfortunately, although the solution space provided on the paper was more than adequate for most questions, a significant number of candidates ran out of space for question 9 and had to continue their solutions on other pages or on supplementary sheets. Standards of presentation varied, as usual, with some candidates inevitably penalising themselves by not showing their methods clearly.

Report on Individual Questions

Question 1

This question was well answered by many candidates. However, those who attempted to take out a factor of 2 before expanding often failed to realise that they needed then to multiply by $2^6$. Sometimes the ‘2’ was completely ignored and the expansion of $(1 + x)^6$ was given. In general, candidates fared better if they used a formula for $(a + b)^n$ rather than $(1 + x)^n$. Some candidates failed to simplify their terms or made careless mistakes in their attempts to simplify.

Question 2

This standard test of definite integration was handled well by the vast majority of candidates. Mistakes, where made, tended to be in the integration of $x^{-2}$, although errors in simple arithmetic sometimes spoilt otherwise correct solutions. Candidates who differentiated were only able to pick up one method mark, for the substitution of limits.

Question 3

In part (i), some candidates thought that $\log_6 36$ was equal to 6, but there were many correct answers, sometimes following ‘change of base’ and the use of a calculator. Part (ii) caused more problems, the most common mistakes being to express $2 \log_a 3 + \log_a 11$ as either $2 \log_a 33$ or $2 \log_a 14$. Sometimes a correct first step ($\log_a 9$) was followed by the answer $\log_a 20$. In general, responses from weaker candidates suggested a poor understanding of the theory of logarithms.

Question 4

Many candidates unnecessarily used long division in part (a) to find the remainder. The correct remainder $-6$ was often achieved, but sometimes the answer 6 followed correct working. Careless algebraic and arithmetic mistakes spoilt some solutions. Candidates who used long division rather than the factor theorem lost the marks in part (b) of this question, and those who obtained $f(-3) = 0$ but failed to give a conclusion lost the second mark.
There were many good solutions to the factorisation in part (c). Candidates usually found the quadratic factor by long division (which was generally well understood) or by ‘inspection’ and went on to factorise this quadratic, obtaining the correct linear factors. Some of the weaker candidates failed to recognise that \((x + 3)\) from part (b) was one of the factors and tried to use \((x + 2)\) from part (a). A few attempted to use the formula to find the roots of the quadratic and then to use the roots to find the factors. This was not always successful, as it tended to lead to the loss of a factor of 2 in the final answer.

**Question 5**

Well-prepared candidates often scored full marks on this question. Although many good sketches were seen in part (a), some candidates had little idea of the shape of the curve, others omitted this part completely and a significant number failed to show the curve for \(x < 0\). Even those who were unable to sketch the curve correctly usually managed to complete the table accurately in part (b), their sketch sometimes contradicting the \((0,1)\) found in the table. Use of the trapezium rule in part (c) was often clear and accurate, but the mistake of misunderstanding \(h\) was again all too frequently seen, with \(h = \frac{1}{6}\) instead of \(h = \frac{1}{5}\) being common. Bracketing was also a problem for some candidates, who wrote

\[
\frac{1}{2} \times 0.2(1 + 3) + 2(1.246 + 1.552 + 1.933 + 2.408)
\]

and then often performed the calculation as written. Occasionally candidates opted to calculate areas of separate trapezia, but were still usually able to proceed to a correct answer.

**Question 6**

Finding the value of \(\tan \theta\) in part (a) proved surprisingly difficult for some candidates. Some, not recognising the link between the two parts of the question, failed in part (a) but went on to find a correct value in part (b) before solving the equation. Those who had a value for \(\tan \theta\) were usually able to attempt a solution to the equation. Most candidates achieved an acute value for \(\theta\) but then some omitted or used wrong methods to find the second solution. Another, almost invariably unsuccessful, method seen in part (b) was the use of \(\sin^2 \theta + \cos^2 \theta = 1\), or a false variation such as \(\sin \theta + \cos \theta = 1\).

**Question 7**

There were many correct solutions to part (a), with most candidates realising that the required straight line \(PQ\) had to be perpendicular to the tangent. Inappropriately, a few looked ahead to the given information for part (b), immediately taking \(Q\) as \((5, 1)\) and scoring no more than 2 marks out of 4 for parts (a) and (b) combined. For most, part (b) provided useful verification of the accuracy of their equation for \(PQ\).

Those who failed in the first two parts of the question were still able to attempt the equation of the circle in part (c), but this part was not particularly well done, only about half the candidates being able to produce a completely correct equation. Some did not realise that they needed to calculate the radius of the circle, while others were unsure of the significance of \(a, b\) and \(r\) in \((x - a)^2 + (y - b)^2 = r^2\). Some used \((2, 2)\) as the ‘centre’, some used \((1, 5)\) instead of \((5, 1)\) as the centre, and some confused radius and diameter.
Question 8

There were many excellent solutions to the first two parts of this question, with most candidates sensibly using the formulae $r \theta$ and $\frac{1}{2} r^2 \theta$ rather than trying to convert to degrees. Conversion to degrees was rather more popular in part (c), however, and while many candidates did so correctly and converted their answer back into radians at the end, this was an inefficient method, likely to produce errors. Common wrong methods in part (c) were $\pi - 0.65$ and $1 - 0.65$. It was disappointing that, in part (d), many candidates were unable to obtain the correct area of triangle $ACD$. Some unnecessarily calculated a perpendicular height for the triangle (giving a greater risk of error), and some, using $\frac{1}{2} ab \sin C$, took both $a$ and $b$ to be 1.86. A few, having found the area of the triangle, forgot to add it to the area of the sector to give the required answer.

Question 9

While most candidates were able to produce $ar = 4$ and $\frac{a}{1-r} = 25$, either in part (a) or elsewhere, many had difficulty in establishing the given result $25r^2 - 25r + 4 = 0$. Often the solution to the quadratic equation was seen in part (a) rather than part (b), but candidates usually acknowledged that what they had found was, indeed, the answer for (b). Careless (and sometimes very bad) algebra was not uncommon, but otherwise many correct solutions to parts (b) and (c) were seen. The main difficulties in this question came in the last two parts. In part (d), justification of $S_n = 25(1 - r)^n$ was often omitted or unconvincing, and the general proof of the sum formula for a geometric series occasionally appeared. It was common to see the result obtained falsely by adding the $r$ values to get 1, adding the $a$ values to get 25, substituting these hybrid values into the correct formula and ignoring the zero denominator.

In part (e), although some candidates used logarithmic or ‘trial and improvement’ methods very efficiently, others were inclined to produce algebra such as $25(1 - 0.8)^n = 25 - 20^n$. Some successfully obtained the value $n = 14.4$, but failed to realise that $n$ had to be an integer, or chose $n = 14$ instead of $n = 15$. Some wasted time on lengthy, inaccurate algebra that was leading nowhere.

Question 10

In general, candidates scored well on parts (a) and (c) of this question, usually managed part (b), but struggled with part (d). Most knew the method for part (a), and were able to differentiate correctly and solve the appropriate quadratic equation. Although part (b) asked for the value of the second derivative at $A$, some candidates equated the derivative $6x - 16$ to zero, solved this equation and then tried to use this result to justify the maximum. The vast majority of candidates were successful in part (c), performing the indefinite integration.

Many marks were lost, however, in part (e), where candidates often had little idea how to calculate the required area. A common approach was to use limits 0 to $-\frac{10}{3}$ (rather than 0 to 2), and those who continued often seemed confused as to which area they should subtract. Some supported their arguments with reference to the diagram, but more often than not triangles (or trapezia) being used were not clearly identified. For some, working was further complicated by their decision to find the area of a triangle by using the equation of a straight line, and integrating. A few produced very clear, concise and accurate methods, which were a pleasure to mark amidst the convoluted efforts of the majority.
Core Mathematics Unit  C3
Specification 6665

Introduction

This paper was accessible to the majority of candidates, with a relatively small number of non-attempts at questions seen. Candidates appeared to have had sufficient time to attempt all eight questions. In particular, the first four questions proved accessible to many candidates and a large number of correct solutions were seen. Q5, 6 and Q8 all had parts which required candidates to prove or show a given result. As has been found in previous C3 papers, candidates often revealed a lack of understanding and precision in their answers to such questions. Candidates should be advised that, in order to provide a correct and convincing solution to a proof, they must ensure that they show every step of their argument. Candidates also need to understand the difference between working with an expression and an equation. Although many showed good algebraic manipulation, missing brackets and inaccurate use of common denominators led to some candidates struggling with several pages of complicated or incorrect working.

Report on Individual Questions

Question 1

For many candidates this was a successful start to the paper with completely correct solutions being the norm. A few candidates used long division in part (a) but did not often succeed using this method. In part (b) candidates who chose a denominator other than the lowest common denominator often failed to gain the final two marks because they did not factorise the resulting cubic in the numerator. Some candidates ignored the hint given in the phrase ‘hence or otherwise’ and repeated the work they had already completed in part (a).

Question 2

Part (a) was done well by many candidates. However, as was noted in the reports on both of the previous C3 papers, some candidates have difficulty in differentiating \( \ln(ax) \); the most common errors on this occasion being \( \frac{1}{2x} \) or \( \frac{2}{x} \). The chain rule was well understood and many candidates scored full marks in part (b), although a few lost the final mark because they did not fully simplify their solution. Inappropriate applications of the product rule were occasionally seen in both parts of this question.

Question 3

This question proved accessible and largely successful for many candidates. Most candidates reflected the appropriate part of \( y = f(x) \) in the x-axis in part (a) but some lost marks through drawing a turning point rather than a cusp at \( Q \) or having an incorrect curvature in the first quadrant. In part (b) many candidates drew the line \( y = x \) on their diagram and attempted to reflect \( y = f(x) \) in this line. Many correct solutions were seen. Errors usually came from incorrect curvature in the second quadrant or incorrectly stating the required points as \((0, -3)\) and \((2, 0)\). Most candidates scored at least one mark in part (e) for a curve in the correct quadrants. However, candidates were not so successful with the points of intersection with the coordinate axes. Common errors included \((0, -4)\) and \((9, 0)\).
Question 4

Calculator work was generally accurate in this question and it was encouraging to see most candidates give their answers to the required degree of accuracy. The vast majority of candidates gave the correct answer of $425^\circ C$ in part (a). Many candidates were able to substitute $T = 300$ in part (b) and correctly change an equation of the form $e^a = b$ to $a = \ln b$. Weak candidates showed a lack of understanding of logarithms by failing to simplify their initial equation to the form $e^a = b$ and using an incorrect statement of the form $a = b + c \Rightarrow \ln a = \ln b + \ln c$. Not all candidates understood the need to differentiate in part (c) and found the gradient of a chord instead of finding $\frac{dT}{dt}$. The most common error made by candidates who did differentiate was to give the differential as $-20te^{-0.05t}$. Candidates often had difficulty giving precise explanations in part (d). Although many referred to the $+25$ term in their answers, far fewer gave adequate reasons as to why this meant that the temperature could never fall to $20^\circ C$, particularly with regard to $e^{-0.05t} > 0$. Lack of understanding of the concept of limit led some to write (in words or symbols) $T \geq 25$ rather than $T > 25$.

Question 5

The product rule is well known and was accurately applied by many candidates in part (a). Rather than changing $\tan 2x$ to $\frac{\sin 2x}{\cos 2x}$ and $\sec^2 2x$ to $\frac{1}{\cos^2 2x}$, some candidates used the identity $1 + \tan^2 2x = \sec^2 2x$. These candidates were rarely able to make progress beyond a few more lines of manipulation and such solutions were often abandoned. Algebraic manipulation was a problem for some candidates. Others never set $\frac{dy}{dx}$ equal to zero and incorrectly multiplied only one side of their equation by $\cos^2 2x$ rather than using a common denominator or stating that $\frac{dy}{dx} = 0$ before multiplying by $\cos^2 2x$. This part of the question asked candidates to show a given result and candidates did not always show sufficient steps in their work. Full marks were not awarded unless the $\sin 4k$ part of the equation came from an intermediate result of $2\sin 2k \cos 2k$ somewhere in the solution. Many correct solutions were seen in (b), although a few candidates were inaccurate when giving their answers to 4 decimal places. By far the most common error came from candidates using their calculators in degree mode rather than radian mode. Part (c) was generally done well. Some candidates chose an unsuitably large interval and some worked in degrees. Candidates who performed further iterations gained the marks provided they showed sufficient accuracy in their answers.

Question 6

In part (a) most candidates took the given identity, divided by $\sin^2 \theta$ and correctly manipulated their equation to obtain the required result. Correct solutions were also given by those who started with the expression $\cosec^2 \theta - \cot^2 \theta$ and used the given identity to show that this expression came to 1. However, those candidates who assumed the result (i.e. $\cosec^2 \theta - \cot^2 \theta \equiv 1$) and manipulated this to obtain the given identity were not given the final mark unless they drew at least a minimal conclusion from this (e.g. hence result). Candidates who understood the link between parts (a) and (b) and used the difference of two squares completed part (b) easily. Other, more lengthy, solutions were also seen. Weaker candidates tended to produce circular arguments or use incorrect statements such as $\cosec^4 \theta - \cot^4 \theta = 1 \Rightarrow \cosec^4 \theta - \cot^4 \theta = 1$. The first method mark for linking parts (b)
and (c) was gained by most candidates. Many were also able to use the result in part (a) to obtain a quadratic in \( \cot \theta \). Candidates who did not spot these links were usually unsuccessful. For those candidates who obtained a quadratic equation, factorising was generally done well although less proficiency was seen in giving solutions to the resulting trigonometric equations in the correct range.

**Question 7**

Most candidates scored the first mark in part (a) for a correctly shaped curve. However, many candidates appeared to ignore the information given that \( k > 1 \) and so it was common to see graph of \( f \) incorrectly crossing the positive \( x \)-axis and negative \( y \)-axis. Candidates were more successful with their sketches of \( g \). Part (b) was not well done with many giving a range which contradicted their sketches. Incorrect answers implied a lack of understanding of the difference between the domain and range of a function. In part (c), most candidates showed good understanding of the required order of operations. However, the modulus sign was often omitted completely or treated as a bracket leading to a frequent incorrect answer of \( \ln \left( \frac{k}{2} \right) \). Part (d) was sometimes misinterpreted and it was not uncommon to see candidates using \( y - y_1 = m(x - x_1) \) and attempting to find the equation of a tangent. Those candidates who found \( f'(3) \) and equated it to the gradient of the line were usually successful in finding the correct value of \( k \), although the gradient of the line was sometimes given as 2 and sometimes as \( \frac{1}{9} \).

**Question 8**

Marks were only given in part (a) if a method was seen, so no credit was given to answers obtained entirely from a calculator. Many candidates were able to find the exact value of the sine of an acute angle whose cosine is \( \frac{3}{4} \) and were also able to use the double angle formula for \( \sin 2\theta \). However, candidates found it a greater challenge to work out in which quadrant \( 2A \) would appear and relate this to the correct sign. Candidates who incorrectly used a 3, 4, 5 triangle seemed unperturbed by producing a value for \( \sin 2A \) outside the range \(-1 \leq \sin 2A \leq 1 \). Many correct solutions were seen to part (b)(i) by candidates who used either of the relevant trigonometric identities given in the formulae book. A few candidates spent unnecessary time deriving results which are given in the formulae book and some were not able to evaluate \( \cos \frac{\pi}{3} \). Weaker candidates tended to ignore trigonometric identities and write incorrect statements of the form \( \cos(A + B) = \cos A + \cos B \). In part (b)(ii), those candidates who attempted to find \( \frac{dy}{dx} \) from the form given in the question were rarely able to continue beyond the differential to find the given answer. However, it was often done successfully by those candidates who used the link between parts (i) and (ii). Some candidates used various trigonometric identities to rearrange their expression for \( y \) before differentiating. Although these solutions were sometimes long-winded, they were quite often successful. This was another occasion on which candidates needed to be careful to show all the steps in their work to reach a convincing conclusion to a given answer. As in similar questions, some candidates tried to fool the examiner by inserting the given answer following several previous lines of incorrect working.
Core Mathematics Unit C4
Specification 6666

Introduction

This paper proved to be accessible to and there was no evidence of candidates being unable to complete the paper owing to time constraints. There was, however, a minority of candidates who had no knowledge of some topics in the specification. It was pleasing to note that teachers have taken advice from previous Pure reports and have encouraged candidates to use exact values in their solutions. This was borne out by candidates’ responses to questions 3(b), 4(a) and 6(d). Also encouraging, was the increasing number of candidates who were able to separate variables and correctly solve the differential equation in question 7(c).

The first two questions gave a good introduction to the paper with candidates showing good skills in differentiating implicitly and in applying binomial expansions correctly. From question 3 onwards, there were some testing questions, particularly those questions involving the use of integration, which allowed the paper to discriminate well across all ability ranges.

There was evidence that candidates were not well prepared in tackling more demanding vector material with parts (b) and (c) of question 5 being badly answered even by some stronger candidates. Examiners suggest that teachers may want to review their planning for the delivery of the Core 4 specification to take account of this.

Questions 1, 2, 4(a), 6(a) and 6(b) were usually a good source of marks for many of the candidates. In question 6(b), only about 10% of candidates were able to gain the explanation mark. In question 3(c), it was found that there were many incorrect ways that candidates could arrive at the correct answer of $9\pi^2$. The mark scheme, however, was designed to ensure that only those candidates using a correct method would be appropriately credited.

Report on Individual Questions

Question 1

This question was successfully completed by the majority of candidates. Whilst many demonstrated a good grasp of the idea of implicit differentiation there were a few who did not appear to know how to differentiate implicitly. Candidates who found an expression for $\frac{dy}{dx}$ in terms of $x$ and $y$, before substituting in values of $x = 0$ and $y = 1$, were prone to errors in manipulation. Some candidates found the equation of the tangent and a number of candidates did not give the equation of the normal in the requested form.

Question 2

In part (a) candidates needed to start with the correct identity; although correct solutions were seen from a good proportion of the candidature, a significant number of candidates started with the wrong identity and thus gained no marks. The most common wrong starting point was to use $3x - 1 = A(1 - 2x)^2 + B(1 - 2x)$, but $3x - 1 = A(1 - 2x)^2 + B$ and $3x - 1 = A(1 - 2x)^2 + Bx$ also occasionally appeared. Candidates using the first identity often produced answers $A = \frac{1}{2}, B = -\frac{3}{2}$, the same values but for the wrong constants. Candidates
using the second identity could produce the ‘correct’ answers $B = \frac{1}{2}$, $A = -\frac{3}{2}$ (eg by setting $x = 0$ and $x = \frac{1}{2}$) but this is fortuitous and clearly gains no marks.

Generally candidates showed a good understanding of the work on expanding series in part (b) and most were able to gain some credit. The mark scheme allowed four marks to be gained for the correct unsimplified expansions, as far as the term in $x^3$, of $(1 - 2x)^{-1}$ and $(1 - 2x)^{-2}$. This helped some candidates who went on to make numerical or sign errors when simplifying their expansions and errors in part (a) only affected the final two accuracy marks.

Candidates who multiplied $(3x - 1)$ by the expansion of $(1 - 2x)^{-2}$ gave solutions that were not dependent on their answers in part (a) and it was not uncommon to see a score of zero marks in part (a) followed by a score of six marks in part (b).

Question 3

In part (a), most candidates realised that to find the shaded area they needed to integrate $3\sin\left(\frac{x}{2}\right)$ with respect to $x$, and the majority of them produced an expression involving $\cos\left(\frac{x}{2}\right)$, so gaining the first method mark. Surprisingly a significant number of candidates were unable to obtain the correct coefficient of -6, so thereby denying themselves of the final two accuracy marks. Most candidates were able to use limits correctly, though some assumed that $\cos 0$ is zero.

In part (b), whilst most candidates knew the correct formula for the volume required, there were numerous errors in subsequent work, revealing insufficient care in the use or understanding of trigonometry. The most common wrong starting point was for candidates to write $Y^2$ as $3\sin^2\left(\frac{x}{2}\right)$, $9\sin^2\left(\frac{x}{2}\right)$ or $3\sin^2\left(\frac{x}{2}\right)$. Although some candidates thought that they could integrate $\sin^2\left(\frac{x}{2}\right)$ directly to give them an incorrect expression involving $\sin^3\left(\frac{x}{2}\right)$, many realised that they needed to consider the identity $\cos 2A = 1 - 2\sin^2 A$ and so gained a method mark. At this stage, a significant number of candidates found difficulty with rearranging this identity and using the substitution $A = \frac{x}{2}$ to give the identity $\sin^2\left(\frac{x}{2}\right) = \frac{1 - \cos x}{2}$. Almost all of those candidates who were able to substitute this identity into their volume expression proceeded to correct integration and a full and correct solution.

There were, however, a significant minority of candidates who used the method of integration by parts in part (b), but these candidates were usually not very successful in their attempts.

Question 4

Part (a) was surprisingly well done by candidates with part (b) providing more of a challenge even for some candidates who had produced a perfect solution in part (a).

In part (a), many candidates were able to apply the correct formula for finding $\frac{dy}{dx}$ in terms of $t$, although some candidates erroneously believed that differentiation of a sine function produced a negative cosine function. Other mistakes included a few candidates who either cancelled out “cos” in their gradient expression to give $\frac{t + \frac{x}{6}}{t}$ or substituted $t = \frac{x}{6}$ into their $x$ and $y$
expressions before proceeding to differentiate each with respect to \( t \). Other candidates made life more difficult for themselves by expanding the \( y \) expression using the compound angle formula, giving them more work, but for the same credit. Many candidates were able to substitute \( t = \frac{\pi}{6} \) into their gradient expression to give \( \frac{1}{\sqrt{3}} \), but it was not uncommon to see some candidates who simplified \( \frac{1}{\sqrt{3}} \) incorrectly to give \( \sqrt{3} \). The majority of candidates wrote down the point \( \left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right) \) and understood how to find the equation of the tangent using this point and their tangent gradient.

Whilst some candidates omitted part (b) altogether, most realised they needed to use the compound angle formula, though it was common to see that some candidates who believed that \( \sin(t + \frac{\pi}{6}) \) could be rewritten as ‘\( \sin t + \sin \frac{\pi}{6} \)’. Many candidates do not appreciate that a proof requires evidence, as was required in establishing that \( \cos t = \sqrt{1 - x^2} \), and so lost the final two marks. There were, however, a significant number of candidates who successfully obtained the required Cartesian equation.

**Question 5**

The vast majority of candidates could answer part (a), mostly gaining all three marks available. Many were able to find that \( \lambda = -6 \) and substituted this into their \( y \)-component to find the correct value of \( a \). A few, however, used the same parameter to incorrectly find that \( b = -13 \).

In part (b), the many candidates realised that \( \mathbf{a} \cdot \mathbf{b} = 0 \) could be used but had very little idea of what \( \mathbf{a} \) and \( \mathbf{b} \) represented. Some candidates could quote \( x + 4y - 2z = 0 \), but many of them could get no further than this. On the other hand, those who could get beyond this point mostly arrived at the correct position vector of \( \mathbf{P} \). It was not uncommon, however, to see some candidates who had correctly written the correct equation \( 21\lambda + 81 = 0 \) to go on to solve this incorrectly to find that \( \lambda = 4 \). There were a few correct ‘non-standard’ methods seen by examiners that gained full credit. They included some candidates who either began their solutions by solving the equation \( \mathbf{AP} \cdot \mathbf{OP} = 0 \) or finding the value of \( \lambda \) that minimises an expression for \( \mathbf{OP}^2 \).

In part (c), there were two main approaches used by candidates in proving that the points \( A, P \) and \( B \) were collinear. The most popular approach was for candidates to find any two of the vectors \( \mathbf{AP}, \mathbf{PB} \) or \( \mathbf{AB} \) and then go on to prove that one of these vectors was a multiple of the other. The second most popular approach was for candidates to show that \( B \) lay on the line \( l \) when \( \lambda = -1 \). Some candidates were able to state the correct ratio, but it was not uncommon to see the square of the ratio \( \mathbf{AP}:\mathbf{BP} \) instead of the ratio itself.
Question 6

In part (a), the first mark of the question was usually gratefully received, although for $x = 1.5$ it was not uncommon to see $\frac{1}{2} \ln\left(\frac{1}{2}\right)$.

In part (b), it was not unusual to see completely correct solutions but common errors included candidates either stating the wrong width of the trapezia or candidates not stating their final answer correct to four significant figures.

Answers to part (c) were variable and often the mark in this part was not gained.

In part (d) all four most popular ways detailed in the mark scheme were seen. For weaker candidates this proved a testing part. For many candidates the method of integration by parts provided the way forward although some candidates applied this formula in the ‘wrong direction’ and incorrectly stated that $\frac{dv}{dx} = \ln x$ implied $\frac{1}{x^2}$. Sign errors were common in this part, e.g. the incorrect statement of $-\int \left(\frac{x}{2} - 1\right) dx = -\frac{x^2}{4} - x$, and as usual, where final answers have to be derived, the last few steps of the solution were often not convincing.

In summary, this question proved to be a good source of marks for stronger candidates, with 12 or 13 marks quite common for such candidates; a loss of one mark was likely to have been in part (c).

Question 7

A significant number of candidates found parts (a) and (b) difficult although other candidates answered these two parts of the question with ease. Those candidates who used $\frac{dx}{dt} = \frac{dx}{ds} \times \frac{ds}{dt}$ in part (a) and $\frac{dV}{dx} = \frac{dV}{ds} \times \frac{ds}{dx}$ in part (b) managed better than those candidates who worked with $V$ and $S$ or $V$, $S$ and $x$. The most common error in these parts was for candidates to incorrectly quote the surface area $S$, as $x^2$ or $4x^2$ instead of $6x^2$.

Part (c) was tackled better than the rest of the question with many candidates recognising the need to separate the variables, integrate, find the constant of integration and substitute for $V$. Many candidates were able to score full marks easily on this part. There was, nevertheless, plenty of scope for errors to occur at all stages in the solution. Those who separated out $\frac{1}{2V^{\frac{3}{2}}}$ frequently simplified this to $2V^{\frac{1}{2}}$. After integration incorrect expressions such as $V^{\frac{4}{3}}$, $V^{\frac{1}{3}}$ and $\ln V^{\frac{1}{3}}$ all regularly appeared. A number of candidates did not use a constant of integration. Other candidates found difficulty in working with $\left(16\sqrt{2}\right)^{\frac{3}{2}}$. 
Further Pure Mathematics Unit FP1
Specification 6674

Introduction

There was a substantial increase in the number of candidates taking this specification this summer. The work was generally of a good quality although there was some evidence that not all candidates had understood all of the topics in the specification. There were candidates who showed, for examples, no knowledge of the method of differences or of the use of integrating factors in solving differential equations. The standard of algebraic technique was very variable and, particularly in questions 1 and 8, the solutions of candidates who clearly understood the methods required were spoilt by, not infrequently, multiple algebraic errors. This was the first time that this specification was marked on line and the majority of candidates did produce solutions of sufficient clarity in the correct places in their scripts. The use of pencil for working should be discouraged. Good practice has always been that rough work should be worked in ink and crossed out, but not obliterated, in such a way that it can be read by the examiner. Many candidates have been awarded credit where their final solutions have contained some ambiguity but their working has shown clearly what was intended. The use of calculators was generally accurate and appropriate but it needs to be emphasized that in questions, like question 6 on numerical analysis, intermediate results need to be shown to a sufficient accuracy to establish that a correct method, with an appropriate accuracy, has been used.

Report on Individual Questions

Question 1

This question proved a very accessible start to the paper and almost all candidates could use an appropriate method to solve the pair of simultaneous equations. The majority used methods of elimination or substitution they had learnt for GCSE. Completely correct solutions to part (a) were common but, as noted above, many spoilt their work with inaccurate algebra. It was quite usual to see two or more mistakes in elementary algebra, particularly in signs. A significant minority used the alternative method of substituting, say, \( z = a + ib \) and \( w = c + id \) and, by equating real and imaginary parts, obtained 4 equations in 4 unknowns. Superficially this seemed a complicated method but, in practice, the equations came out quite easily and completely correct solutions using this method were not uncommon. In part (b), the majority could use a tangent to find an angle related to the argument but getting this angle into radians in the right quadrant proved demanding. It is very helpful to draw a diagram in such circumstances and candidates should be encouraged to do this.

Question 2

The method needed was well understood and most could use an appropriate formula and identify the correct double angle formula to carry out the indefinite integration. The limits, however, proved testing and only about 60% of the candidates used the correct limits of \( \frac{\pi}{4} \) and \( \frac{5\pi}{4} \).

Question 3

This question was manipulatively straight forward and those who read it carefully and understood what they had been asked often gained full marks in no more than 9 or 10 lines of working. However many seem to tackle differential equation questions on a kind of automatic pilot and set about answering the question they expect to be asked rather than the one that actually has been asked. In part (a), many did not recognise that they had been given a Particular
Integral and set about finding it. Those who started with \( y = a \sin 2x + b \cos 2x \) usually produced no solution. Some started with expressions as complicated as

\[
y = a \sin 2x + b \cos 2x + cx \sin 2x + dx \cos 2x
\]

and a few of these did get the question out but at the cost of losing a great deal of time. Part (b) was better done and it was not uncommon to see candidates gaining no marks in part (a) but full marks here. A common error was to take the particular integral as 12\( \cos 2x \). The majority could obtain the correct trigonometric form of the General Solution and use the boundary values to complete their solution.

**Question 4**

This proved to be the easiest question on the paper. A few had difficulty in finding the correct second quadratic factor but full marks were common. An unexpected source of error was that a number who had the correct second factor \( x^2 + 6 \) were unable to solve \( x^2 + 6 = 0 \) correctly. \( x = \pm \sqrt{6} \) and \( x = \pm 6i \) were both relatively common. Almost all could show their answers on an Argand diagram correctly.

**Question 5**

Part (a) was intended as an easy route into the question and, for the majority, it was. However about 15% of the candidates were unable to obtain both of the available marks and those who did sometimes took nearly a page of working to obtain them. As has been noted for a number of years, the method of differences remains an area of weakness and 40% of the candidates were unable to obtain any marks in part (b). However those who did know the method often produced excellent and clearly written out solutions. A few candidates had some idea of how terms cancelled but did not establish a method as they either did not, or were unable to, make any connection with the series they were asked to sum. When summing an expression of the form

\[
24r^2 + 2 = (2r + 1)^3 - (2r - 1)^3
\]

it is essential to produce a statement of the form

\[
\sum(24r^2 + 2) = (2n + 1)^3 - 1
\]

and the handling of the left hand side of this equation is as important as the right hand side. Most knew what to do in part (c) but it was disappointing to see errors in expanding \((3r - 1)^2\) at this level. Some having achieved a correct expression,

\[
\frac{9}{6} n(n + 1)(2n + 1) - \frac{6}{2} n(n + 1) + n
\]

attempted to simplify this before substituting. This is poor tactics as it leads to a possible source of error and, in practice, almost all who did this made one or more mistakes.

**Question 6**

The majority of candidates gained both marks in part (a). It is essential to realise that, when answering parts (a) and (b), statements like \( f(0.4) < 0 \) are inadequate. They have a fifty per cent chance of being accurate and offer no way that an examiner can evaluate a candidate’s response. Here one significant figure is enough \( f(0.28) \approx 0.09 > 0 \) is a sufficient statement but anywhere in a question on numerical analysis, intermediate results should be given which show that working is being done with sufficient accuracy to obtain the result required in the question. Interval bisection is not always well understood and, again, questions need to be read carefully. Some candidates having given completely correct working gave their answer as an approximation to the root, \( \alpha \approx 0.255 \), instead of giving, as asked, an interval of width 0.005 which contained \( \alpha \), (0.255, 0.26). Some candidates produced three linear interpolations instead of three interval bisections and the amount of time this took must have seriously affected their ability to complete the paper. In part (c), nearly all candidates knew how to use the
Newton-Raphson method but the majority of candidates were unable to differentiate $4\sin \sqrt{x}$ correctly.

**Question 7**

It is pleasing to report that this awkward inequality question was well done. Candidates, on average, scored more marks on this question than on any other on the paper, and, relative to the number of marks available, only question 7 proved easier. Part (a) was usually well done. The easier of the two quadratics obtained, $2x^2 - 2x = 0$ proved a source of error. $x = 0$ was sometimes dropped and $x = 2$ was not uncommon. A few squared both sides. This is a sound method but very few made progress this way. The curve proved to be the difficult part of the diagram. Many had the curvature the wrong way round for larger values of $x$ and it was not uncommon to see the curve bent back on itself so that there was more than one value of $y$ for a value of $x$. It is quite difficult to get all four intersections on the graph and, as long as the sketches were correct in other ways, the 3 marks available were awarded if three intersections were correct. Those who realised that parts (a) and (b) showed them that there were 4 critical values, usually completed the question correctly.

**Question 8**

This question proved very demanding. Some may have been short of time, possibly as a result of using inappropriate methods in, possibly, questions 3 and 6. However most had enough time to present solutions to both parts of the question. A substantial number of candidates attempted to separate the variables and no progress could be made this way. Those who knew how to use an integrating factor usually showed that they understood how to complete the question but only a minority of these could complete part (a) successfully. Sign errors in integrating both $\frac{2}{120-t}$ and $\frac{1}{4(120-t)^2}$ were almost as common as the correct answers. A particularly damaging error was to proceed from $S = \frac{1}{4(120-t)} + C$ to $S = \frac{120-t}{4} + C$ before evaluating the constant of integration. This was not heavily penalised in part (a) as a linear equation resulted. On differentiation this gives a constant and, hence, those making this error were unable to show a complete method in part (b). Those who had an expression for $S$ and realised that a stationary value had to be found could usually show a valid method in part (b) and it is notable that on this question, intended as a demanding, discriminating question at the end of a Further Mathematics module, 13% of the candidates gained full marks. A substantial number of candidates just substituted $t = 0$ or 120 (or even 119 or 119.9, presumably taking the view that the maximum was just before the end of the process) into their expression for $S$. 
Further Pure Mathematics Unit FP2
Specification 6675

Introduction

Although some of the questions on this paper were easily accessible to the majority, the paper as a whole proved quite demanding for many candidates. Generally, after spending too long on inefficient methods for some of the earlier questions, candidates were short of time when it came to question 9, where many incomplete and rushed attempts were seen. Candidates often seemed to spend too much time on efforts to obtain given answers, changing wrong (or even correct) working and sometimes creating more confusion for themselves. Poor use of notation and careless algebra often led to loss of marks. Standards of presentation varied considerably, but clear, concise work of a very high standard was seen from many candidates. It was pleasing to see how many were able to achieve a total mark in excess of 70 out of 75, suggesting a high level of mathematical talent.

Report on Individual Questions

Question 1

This standard question proved an easy starter for the vast majority of candidates, who almost invariably used the expected method, forming a quadratic equation in \( x \). The occasional careless error led to loss of marks, but most candidates were able to produce a fully correct solution.

Question 2

To complete part (a) of this question successfully, it was necessary to find the value of \( e \) (the eccentricity of the ellipse) or \( ae \). Most candidates used \( b^2 = a^2 (1 - e^2) \) here and were usually able to score at least the method marks, but some trivialised the question by simply using \( a = 2 \) in \( y^2 = 4ax \). A few inferred \( a = 2 \) and \( b = 0 \) from \( \frac{x^2}{4} + y^2 = 1 \).

In part (b), a follow-through mark was available for those who had obtained a cartesian equation for the parabola in the correct form. Most candidates scored this mark.

Question 3

Candidates who were able to see ‘by inspection’ that \( \int e^{\sin \psi} \cos \psi \, d\psi = e^{\sin \psi} + C \), or who were able to reach this result via a simple substitution, usually went on to find the correct value of \( C \), scoring full marks. Unfortunately, there were many who made no progress with the integration, typically attempting integration by parts, wasting time and achieving nothing useful. Weaker candidates sometimes omitted this question, or seemed to have little understanding of intrinsic coordinates, often failing to appreciate that the radius of curvature could be represented by \( \frac{ds}{d\psi} \).
Question 4

The differentiation of \( y = \arctan(x^2) \), whether performed directly or implicitly, was usually well done, although \( 1 + x^4 \) and \( 1 + x^2 \) were common wrong versions of \( \frac{dy}{dx} \). Finding the second derivative caused a few more problems: here it was common to see incorrect use of the chain rule or product/quotient rules. Although most candidates scored the method mark for substituting \( x = 1 \) into the correct radius of curvature formula, some wasted time by attempting to simplify a general version before substituting. Those who did this were inevitably more likely to make mistakes. There were, however, many completely correct solutions.

Question 5

In part (a), most candidates differentiated \(- x + \tanh 4x\) correctly. Those who went wrong here usually gave either \(-1 + \sec h^2 4x\) or \(-1 + 4 \sec h^2 x\). Proceeding from \( \cosh 4x = 2 \) (or equivalent), it was surprising how many resorted to exponential definitions and solved a quadratic equation in \( e^{4x} \) or \( e^{8x} \) rather than using the standard formula for \( \text{arcosh} \ x \). Many wasted a great deal of time in part (b) in trying to establish \( \tanh 4x = \frac{\sqrt{3}}{2} \) for

\[
x = \frac{1}{4} \ln(2 + \sqrt{3})
\]

The most efficient method, surprisingly rarely seen, was to use the identity \( \text{sech}^2 \theta = 1 - \tanh^2 \theta \), or \( \cosh^2 \theta - \sinh^2 \theta = 1 \), but again it was typical to see candidates trying to establish the result by working from \( e^{\ln(2+\sqrt{3})} - e^{\ln(2+\sqrt{3})^{-1}} \), or more complicated expressions. Those using this route were sometimes successful, but often floundered or jumped to the given answer without sufficient justification.

Question 6

There were many excellent solutions to this question. In part (a), apart from careless mistakes such as squaring \( 2t^\frac{1}{2} \) to give \( 2t^{-1} \), the only candidates who really struggled were those who failed to recognise \( \frac{1}{t^2} + \frac{1}{t} \) as the square of \( \frac{1}{t} \). At this level, it was disappointing to see a few candidates writing \( \sqrt{1 + \frac{2}{t} + \frac{1}{t^2}} = 1 + \sqrt{2} t^{-2} + t^{-1} \). Those who managed part (a) were usually able to score well in part (b), where most mistakes were numerical ‘slips’, either in the coefficients of integrated terms or in the substitution of limits.
Question 7

The unstructured nature of this question provided a major challenge for the average candidate. The most popular way to begin the integration was to use parts to get

\[ \int x^2 \text{arsinh} x \, dx = \frac{x^3}{3} \text{arsinh} x - \frac{1}{3} \int \frac{x^3}{\sqrt{x^2 + 1}} \, dx. \]

From there, however, candidates had to decide on an appropriate method for \( \int \frac{x^3}{\sqrt{x^2 + 1}} \, dx \).

Disappointing few opted for a simple algebraic substitution such as \( u = x^2 + 1 \), while the most popular approach was to use the substitution \( x = \sinh u \). Other methods, including a further application of integration by parts, were also possible, but many candidates abandoned their efforts at an early stage while others, sometimes unwisely, spent a great deal of time trying to obtain the given answer. Evaluation of terms such as \( \cosh(\text{arsinh} 3) \) was sometimes a problem for those who chose the hyperbolic substitution. Generally, the question proved to be a good discriminator. Candidates’ overall approach to this unstructured challenge was commendable.

Question 8

Solutions to this question were often very good, with many candidates scoring at least 9 marks out of 12. Most coped well with the standard reduction formula proof in part (a), but it was disappointing to see frequent sign errors (despite the given answer). Sign and factor errors in collecting terms were very much in evidence in part (b), showing perhaps that candidates were now beginning to rush. The method of using the reduction formula was well known, but the arbitrary constant was sometimes completely omitted, and a few candidates thought that \( I_0 = 1 \).

Arithmetic slips continued in part (c), but most candidates managed to earn method marks here, limiting the overall damage. Occasionally the link between parts (b) and (c) was missed and candidates wasted time by ‘starting again’.

Question 9

By now, there were very clear signs of shortage of time. Many candidates only tackled the first couple of parts or even omitted the question completely. Part (a) was usually well done, although the algebra was sometimes rather messy. In part (b), however, many failed to appreciate the fact that a zero discriminant was required for a ‘repeated root’. Typically, candidates started implicit differentiation on the equation of the ellipse and made no progress. Finding the area of triangle \( OAB \) in part (c) proved generally straightforward for those who attempted it.

Parts (d) and (e), however, were completed successfully only by a minority of candidates. In part (d), the usual approach was to differentiate the area of the triangle (from part (c)) with respect to \( m \), although a few able candidates used neat algebraic methods to justify the minimum area. By part (e), many appeared to have lost track of what was happening. It was necessary to find the (repeated) root of the quadratic equation from part (a), using the conditions for \( c \) and \( m \). Only the most able candidates managed to complete this successfully.
Further Pure Mathematics Unit FP3
Specification 6676

Introduction

This paper proved accessible to the majority of candidates, with the good spread of topics enabling them to demonstrate their abilities. It was encouraging to see significant progress being made into questions, clearly showing that much of the syllabus had been well covered by the candidates. However they found that a substantial amount of work was required within each question and this caused a lack of time for many to complete the paper. Those who attained high marks in the first six questions rarely had time to progress past question 7(a). Although some work was of an excellent standard, many candidates demonstrated a lack of basic skills in algebraic manipulation, use of calculator and graph sketching. In questions involving proof or "show that", they frequently neglected to produce the complete sequence of steps necessary to form a rigorous and precise argument. An awareness of detail would undoubtedly ensure all available marks are secured. Accuracy was a problem throughout the paper. At this level candidates should be practised in using sufficient digits from their calculator within their subsequent working. This was particularly relevant in Questions 2 and 4.

Report on Individual Questions

Question 1

Many candidates understood the requirements of the method of induction and were able to complete the arguments in full, including clear algebraic manipulation in moving from \[ A_k + 1 \] to \[ 2A_k + 2 \]. A significant number though were unable to produce any convincing arguments. In considering \( n = 1 \), they frequently omitted the in-between step of working \( \frac{1}{2}(1 + 3) = 2 \), to demonstrate their understanding. Far too many jumped directly from \( 2 + k + \frac{1}{2}(k^2 + 3k) \) to \( \frac{1}{2}((k + 1)^2 + 3(k + 1)) \) without explanation, possibly not appreciating the necessity to show each step clearly.

Question 2

Even the weaker candidates were able to score good marks on this question. Those who set out their working in a systematic way seldom made numerical errors in either parts (a) or (b). Others, however, produced muddled attempts; a few simply wrote down the series for \( \cos x \) up to \( x^4 \), or substituted \( x - \frac{\pi}{4} \), or evaluated their differentials at either \( x = -\frac{\pi}{4} \) or \( x = 0 \). In part (b), some used \( x = 2 \) to evaluate their series, others rewrote \( (1 - \frac{\pi}{4}) \) as \( (\frac{3\pi}{4}) \) before evaluating. Checking the value of \( \cos 2 \) on their calculator would have allowed candidates to know whether their answer was correct and possibly rethink!

Question 3

Most candidates completed part (a) successfully with neat concise solutions. Those starting with \( z^n = 2i \sin n\theta \) generally produced much lengthy working before abandoning this part of the question. Some did work methodically through, competently dealing with \( \sin 3\theta \) to reach the required form. The vast majority made a substantial attempt at part (b), including those unable to cope with part (a), gaining good marks. Few managed to find all five values of \( \theta \). Too often \( \sin \theta \) was crossed out on both sides of the equation and, in taking the square root, the negative solutions of \( \cos \theta \) were frequently omitted. Those working with \( \cos 2\theta \) were usually able to find all four values. Generally candidates scored well.
Question 4

Many candidates became engulfed in equations and values by not setting out their working in a clear form. Accuracy was a major problem in part (a); using values correct to one or two significant figures is not appropriate when working towards numerical approximations.

Finding $x_{0.1}$ caused few difficulties, but many assumed $\left(\frac{d^2x}{dt^2}\right)_{0.1} = 0$ before progressing with a correct method to $\left(\frac{d^2x}{dt^2}\right)_{0.2}$. Clarity in presentation is essential to exclude mis-copying.

Calculations were often made and re-made several times, occasionally from correct to incorrect values. Parts (b) and (c) were frequently not attempted. A few candidates used their approximations from part (a) to form their series in part (b). Only the most able managed to recognise and use the chain rule when finding $f^{\text{viii}}(t)$. Those who formed any series generally proceeded to evaluate it with $t = 0.3$ in part (c).

Question 5

Part (a) presented no problems and full marks were regularly awarded. Surprisingly candidates seemed unfamiliar with finding the inverse of a $2 \times 2$ matrix in part (b). Commonly the determinant was not found and/or M was used as the adjoint matrix. With such a simple matrix it may have been advisable for candidates to have evaluated their $M^{-1}M$ to confirm that they had successfully found the correct inverse matrix before continuing with the question. A few sensibly set up $M^{-1}M = 1$, evaluating four simultaneous equations to find $M^{-1}$. Part (c) was not well done. With the answers implied, many manipulated their working to turn their $\lambda^{-1}_1$ and $\lambda^{-1}_2$ into the required values. A correct start was needed to produce a valid method. A mixed response to part (d) divided candidates into those with a good understanding of eigenvalues being able to find the equations quickly and easily, those who thought $MX = X$, and those who made no attempt.

Question 6

The majority of candidates gave excellent solutions to part (a) although a variety of numerical and algebraic errors sometimes led to an incorrect centre and radius. In squaring the modulus of both sides of the equation, 3 was occasionally not squared on the right hand side leading to a loss of several accuracy marks. In simplifying their equation, $8x^2 + 8y^2 + 48x - 24y = 0$ sometimes became $x^2 + y^2 + 6x - 4y = 0$. Completing the square was the popular method to find the centre and radius and most were competent with this technique. Diagrams in part (b) varied widely in accuracy and precision. Although the position of the circle was recognised, it was rare indeed for candidates to consider where the circle might cross the x and y axes. Often three or four attempts at sketches were made, rarely with much improvement. This time could have been put to better use with a more analytical approach. The line was mostly correctly positioned but many did not investigate the true nature of the line and expected it to be a half line. Most candidates shaded within the circle but not always above the line in part (c). Although perfectly correct solutions were rare, candidates scored substantial marks on this question.
Question 7

It appeared that few candidates had time to make much headway past part (a). Almost everyone scored well here with only numerical errors arising, mainly in the sign of the first term. Those with time progressed into part (b) using their vector from part (a). Candidates were clearly not comfortable with the form of the vector equation of the line demanded in part (c). Indecision led to a promising start being spoilt by a lack of clear aim. Some found a direction vector then were unable to find a point on the line. Others used the cartesian equations of the two planes and attempted to find the intersection line by elimination but their work petered out when they failed to express each of $x$, $y$ and $z$ in terms of a parameter. Of those who did, some were unsure which part represented the point and which the direction. Few candidates attempted part (d) and the majority of these ran into difficulties. They did try to draw diagrams to simplify the situation, which was a sensible approach, but all too often the planes and lines became entangled with no real progress being made. The most successful candidates found the length $OP$ using their $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ from part (c), then used calculus or completed the square on $OP^2$ to find the $\lambda$ which defined the point $P$. Full marks were achieved by a small number of candidates.
Mechanics Unit M1  
Specification 6677

Introduction

The paper was found to be very accessible by candidates, with enough on the paper to enable candidates to demonstrate their abilities. There was no clear evidence of time being a problem. A number made no attempt at question 7 at all, but often this seemed to be weaker candidates who perhaps found the whole topic of vectors difficult.

As always, vector work was found to be difficult for many weaker candidates. Also the first parts of qu. 6, involving connected bodies, caused some problems for weaker candidates.

The issue of giving answers to an ‘appropriate’ degree of accuracy continues to be a problem for some though not as much on this paper as on some in recent years. The Edexcel policy remains that, when working with a problem where a value of \( g \) as 9.8 m s\(^{-2}\) has been used, candidates are expected to be aware that giving answers to large numbers of significant figures is ‘inappropriate’. The policy of the board is to accept answers to 2 or 3 s.f. in such cases, but answers to 4 or more s.f. will be penalised (by one mark per question). In other questions where approximate answers are to be given, an accuracy of 3 s.f. is encouraged.

The standard of presentation in answers was moderate, though there was quite a lot of rather scrappily presented work in many instances. Candidates should also be encouraged to use the answer space provided for each question; if they are forced to use extra pages, the use of an extra sheet is probably preferable to continuing elsewhere in the booklet in the space provided for answers to other questions. If candidates do adopt the latter policy, it is vital that they indicate on the pages allocated for that question that they are continuing their work for the question on another page. Otherwise, with online marking where each question is marked separately, there is a danger that the working may not be properly credited.

Report on Individual Questions

Question 1

The question was generally well answered and proved to be a reasonably friendly opening question. Most recognised the significance of the straight lines in parts (a) and (b), though some simply stated in part (a) that the cyclist was ‘accelerating’ (without mentioning the constancy). In part (c), most attempted to find the area under the graph. Some weaker candidates assumed that the whole area was that of a single trapezium; and some made errors in find the area of the rectangle on the right hand side (with e.g. 5 x 7 instead of 5 x 4 seen).

Question 2

The relevant principles here were well known: virtually all candidates could apply the law of conservation of linear momentum in part (a), and could find an impulse in part (b) by attempting to find the change in momentum of one particle. Problems usually arose in relation to the signs of the velocities in question and some weaker candidates evidently failed to realise that, in their standard formulae such as \( I = mv - mu \), the velocities concerned are velocities not speeds and hence could have negative values. A significant minority failed to give the units of the impulse correctly in part (b).
Question 3

A good number of fully correct solutions were seen here. The formulae for constant acceleration were generally well known and accurately used. Mistakes sometimes arose from confusing $B$ and $C$ in part (b). In part (c), quite a few chose to use a method involving a quadratic equation in $t$, though they were often successful and accurate in doing this, even though simpler solutions were available via other approaches. The most common error was to use a prematurely rounded answer for the speed at $C$, which then led to an inaccurate answer in part (c) (1.68 instead of 1.69).

Question 4

This was generally well done and many fully correct solutions were seen to part (a). However, a number of weaker candidates could not handle the angle in question (e.g. using 0.75 degrees); also some weaker candidates were evidently confused about what precisely ‘$F$’ was in the equations $F = \mu R$ and $F = ma$. In part (b) a number of candidates also lost marks by effectively omitting one of the two terms in the equation of motion, forgetting about either the friction or (more commonly) the component of the weight acting down the plane.

Question 5

Although this was well done by many, it appeared to provoke a lot of crossing out with often multiple attempts made to parts (b) and (c), with working then continued in the space for other questions. Most could do part (a) successfully. In part (b), a ‘forwards’ approach, taking moments, was adopted by many, though a number adopted a verification (‘backwards’) approach, showing that two moments were equal if the distance was as given. In part (c) a number of correct solutions were seen; the common error among weaker candidates was to assume that the answer gained in part (a) still applied to this new situation: this then required only one equation in one unknown and considerably shortened the work required.

Question 6

For those who could handle connected bodies, parts (a) and (b) proved to be straightforward; however, others found difficulty in sorting out the forces acting on each body, showing failure to understand the basic mechanics involved in the situation. In part (c) candidates could recover provided they realised that the situation was now different from that in part (a): however, those who simply carried their answer from (a) to use here, without attempting to find a new acceleration, gained no credit. Answers to part (d) were generally disappointing with very few apparently showing awareness of the implications of the fact that the rope was inextensible.

Question 7

As always the vector question proved to be challenging for many weaker candidates. A number did not attempt it at all. For those who did, most found the speed correctly in part (a), but answers to part (b) were very variable: many chose the wrong vector to use (the position vector, not the velocity); others could not find the correct bearing from the acute angle obtained from their diagrams. Most with any understanding of the topic could complete parts (c) and (d), though there was quite a lot of confusion about the 24 hour clock (with e.g. values for $t$ of 1600 or 200 being used [instead of 2 etc]). In part (e) some equated the $j$ component to zero, rather than to the value obtained from part (c); others obtained $t$ as 1.2 correctly but could not put it back into the context of the question as a time of day. Those who got as far as part (f) could usually make a good attempt (though a number used $t = 4$ rather than 2), and a number of correct final answers were seen.
Mechanics Unit M2
Specification 6678

Introduction

The paper proved to be accessible and there was little evidence of candidates being under time pressure. The standard of presentation was generally good but there were some scripts which were very untidy and also some which appeared to have been answered in pencil which causes considerable problems for the examiners. The two areas which generally seemed to cause difficulties were Statics (Q6) and the Work-Energy Principle (Q7). The first three questions were by far the best source of marks.

Report on Individual Questions

Question 1

This proved to be an easy starter and was generally very well answered with the vast majority of candidates scoring 5 or 6 marks. There were some errors in integration, with some candidates failing to include a constant and some unable to solve the required quadratic equation. Of those that could, some failed to reject the negative solution. A few candidates assumed constant acceleration and scored little.

Question 2

Most candidates scored full marks for this question. A few left the answer to part (a) in watts and some rounded their answer to 14 kW. In part (b) a few did not appreciate the difference between power and force and confused these when forming their equation using ‘\(F = ma\)’. Some forgot to include the force produced by the engine and others omitted the component of the weight down the plane.

Question 3

There were few errors on this question. Only a few candidates failed to use vectors to calculate the impulse in part (a) but some forgot to calculate the magnitude of their vector. The second part was mostly completely correct. Candidates need to read the question carefully and ensure that they answer the question asked. Those who forgot to calculate magnitudes lost four marks in this question!

Question 4

This question was well answered by many candidates and the method in part (a) was well known. Some, however, used “\(m\)” as mass per unit length for the framework, or counted the masses of the particles more than once in an attempt to consider each rod separately. Common sense often failed to prevail, with the mass of the whole system sometimes appearing as different values in the two equations. Most were able to attempt part (b), but many failed to use \((2a - y)\) in their ratio. The very few who decided to use sine instead of tangent were usually successful.
Question 5

The first part was reasonably well done although some candidates failed to appreciate that it was a 4 mark proof and therefore required a full explanation – those that simply equated the horizontal velocity components only scored two of the four marks. Part (b) was more discriminating and many didn’t appreciate that they needed two vertical distance-time equations and of those that did, a significant number were unable to combine them correctly.

Question 6

This was probably the worst attempted question of all. Some candidates clearly knew how to deal with questions of this type and had correct forces in their diagram and were able to demonstrate an efficient use of moments equations and resolution equations to achieve the correct answers. Too many, however, were unable to put the correct forces on their diagram. The normal reaction at the peg was often acting vertically or even horizontally and all too often extra forces at B were also present. The reaction on the rod at A was often acting perpendicular to the rod instead of perpendicular to the ground. These errors and others then either led to incorrect equations or to there being too many unknowns, or both. Many managed to form the moments equation required for part (a) although those who had a vertical force at C were then unable to prove the required result. Part (b) was very poorly done. Of those who formed correct equations many assumed limiting equilibrium instead of using an inequality and these candidates were unable to score full marks as a result. A significant number attempted to use moments instead of simply resolving horizontally and vertically, making the solution much more difficult. Almost all candidates answered the final part correctly.

Question 7

Many students were unable to apply the ‘work-energy’ principle efficiently or accurately. In part (a) there was sometimes confusion between work done against friction and work done against gravity and often the weight component was thought to be part of the friction force. Relatively few candidates realised that they could use their answer to part (a) in their solution to part (b) and either started again or else just ignored the friction altogether. Some just ignored the instruction to “use the work-energy principle”, and scored no marks. The final part was usually done better, as candidates could use a force-acceleration method rather than work-energy and there were many correct solutions.

Question 8

In part (a) candidates generally understood the methods involved and were able to produce momentum and restitution equations. Inconsistencies between directions in diagrams and equations were common and many were unable to obtain a correct expression for the speed of A after the collision.
In the second part, the rebound again caused problems with signs but most were able to set up the first inequality. The given double inequality was sometimes fudged or just stated without any attempt to justify $e > \frac{1}{4}$. In part (c), the correct method was usually adopted but accuracy errors were common.
Introduction

This was the first time that this specification was taken only by candidates studying for an award in AS or A Further Mathematics but the standard of work achieved was very similar to that seen in the previous two years. There were many excellent scripts, showing a thorough grasp of the principles of mechanics, but, for a relatively advanced module, there were also a surprising number of scripts submitted which showed very little knowledge of any of the mechanical ideas specific to this module. The paper proved very accessible to candidates and the majority scored well on the first 5 questions. There were many incomplete solutions to the last question and some may have been in time difficulty. Many had spent a good deal of time on question 5. Although, in general, candidates scored high marks on this question, the end of the question was very demanding and even the strongest candidates sometimes needed two or three attempts to attain the final result and this may have had a knock on effect with the later questions. The second part of the last question, however, proved very demanding and the many candidates probably achieved what they could on this question. The general standard of presentation was good and the use of calculators nearly always appropriate.

Report on Individual Questions

Question 1

This question proved to be, for the great majority, a test of memory. Those who could remember the correct formula for the centre of mass of a solid of revolution almost always gained full marks and those who could not gained very little. Very few candidates used the idea of breaking the solid up into elementary discs either as a method of demonstrating the formula or of checking that they had remembered the formula correctly.

Question 2

Part (a) was very well done and full marks were common. The best and clearest solutions reduced the masses to the ratio $8 : 1 : 7$ before starting the calculation for the centre of mass and it may be a good policy to encourage candidates to remove densities, radii and $\pi$ before they write down their moments equation. Part (b) proved more difficult but many completely correct solutions were seen. A common error was to fail to make the connection with part (a) and take the centre of mass of the bowl as being $\frac{3}{8}a$ from the surface of the liquid. Another source of error was including volumes in the mass ratios which, in this part, are simply $1 : k$.

Question 3

Those who had revised their SHM formulae did well on this question. and many gained full marks very quickly. The main source of error in part (a) was taking $T = 5$, which led to $\omega = \frac{24}{5}$. In part (b), a common error was to assume that, when the particle was struck, $x = 0$, whereas the conditions of the question implied that $x = 0.1$ had to be used in the equation $v^2 = \omega^2 (a^2 - x^2)$. The request for 3 significant figures was generally heeded but a few lost a mark by ignoring this request. Candidates are penalised a maximum of one mark in a question for such matters.
Question 4

This question was a very good discriminator. There were many excellent concise solutions leading quickly to full marks. However there was also a substantial minority of students who failed to get started. Inadequate diagram often led to candidates confusing an angle with its complementary angle and this led to errors in resolution. A common error was to assume that the line of action of the reaction went through the centre of the circular end of the cone. In resolving, the vertical equation was more often incorrect than the horizontal equation. $R = mg$, $R = mg \cos \alpha$ and $R = mg \sin \alpha$ were all common errors. An unexpected feature of the responses was, for the first time for some years in any numbers, to see some students using centrifugal or, even, centripetal forces. Such methods were not envisaged when this set of mechanics specifications were designed but, if used correctly, are accepted.

Question 5

Part (a) was well done. The only common error was considering the elastic potential energy in only one part of the string instead of in both parts. Most candidates realised that energy was involved and the few who attempted using Newton’s Second Law almost all failed to consider a general point of the motion and so gained no credit. Nearly all candidates could start part (b) by resolving vertically and writing down some form of Hooke’s Law. The manipulations required to obtain the required trigonometric relation, however, were demanding and even strong candidates often needed two or three attempts to complete this and the time spent on this was sometimes reflected in an inability to complete the paper. This was particular the case if candidates attempted to use or gain information by writing down an equation of energy. This leads to very complicated algebra and is not a practical method of solving questions of this type at this level. (Correctly applied it leads to a quartic not solvable by elementary methods.) For those who were successful in part (a), writing $T$ in terms of, say, the angle made by each part of the string with the vertical proved the critical step. If they obtained $T = \frac{49}{0.75} \left( \frac{0.75}{\sin \alpha} - 0.75 \right)$, or its equivalent, the majority of candidates had the necessary trigonometric skills to complete the question.

Question 6

This question was predominately a pure mathematics problem involving curve sketching and integration. Too many candidates saw it as just that and did not interpret their sketch in terms of the mechanics of the situation by identifying gradients with accelerations and areas with displacements, and not distances. Sketches in part (a) were often poor, with the join between the curves often incorrectly smoothed. This is, perhaps, understandable but the number of candidates who were unable to sketch correctly a factorised quadratic and a straightforward hyperbola, both GCSE topics, was disappointing. Surprisingly few used their graph to identify the time interval over which the acceleration was positive. However they were usually able to establish a method using differentiation even if there were errors of detail. For example, the acceleration at $t = 2$ is 0, which is not positive. In part (c), nearly all candidates knew that integration was involved and the majority realised that they had to integrate separately from $t = 0$ to $t = 4$ and from $t = 4$ to $t = 5$. However there was much adjustment of the signs to obtain the printed answer without any justification being given. In part (d), the use of indefinite integration was commoner than the use of definite integration but the latter was generally more successful. Those using indefinite integration often had difficulty with the boundary conditions. The use of $s = 25$, instead of $s = -25$, or $s = 0$ at $t = 0$, was common.
Question 7

Part (a) was very well done. The only error that was seen at all commonly was having the difference in kinetic energies the wrong way round. In part (b) the error of thinking that \( v = 0 \) at the highest point of the semi-circle was widespread. This lead to a very brief solution which could gain at most two of the nine marks. Those who did write down a correct equation for the tension in the string often arrived at \( AB = \frac{5l}{6} \) but had often assumed that \( T = 0 \) and were unable to give a justification for their result being a minimum. Inequalities remain unpopular with candidates in mechanics.

A considerable number of candidates appeared to rely on memorised formulae involving an angle \( \theta \) for both the energy and the tension without any clear idea of what \( \theta \) was in this question. The introduction of such an angle is an unnecessary complication in this question, where only the highest point needed to be considered.
Introduction

The paper proved demanding for many candidates, but there were a substantial number of marks above 60. Q2 and Q4 provided a good source of marks, even for weaker candidates. The questions which caused the most problems were Q1, Q3, Q4(b) and Q7. The standard of presentation was very variable, particularly amongst home centres. Poor diagrams and notation cause difficulties for the candidates as well as for the examiner. Candidates should be encouraged to introduce their notation – often this can be done succinctly on diagrams that they have already drawn. Some candidates gave the impression of having rote learned standard methods and were intent on forcing problems to fit their model solution, particularly when the orientation of the diagram given was not to their liking.

Report on Individual Questions

Question 1

This relative velocity question was not well done. Most candidates attempted to draw a vector triangle though the orientation of the components was often wrong, so that although a correct angle in the triangle was found, the correct bearing could not be deduced. A significant number of candidates had Q’s velocity directed towards P’s initial position rather than that of Q relative to P.

Question 2

Completely correct solutions to this question were common. Even those who initially decided that momentum might be conserved perpendicular to the wall usually realised the error of their ways and corrected their solutions. A few candidates correctly used conservation of momentum and Newton’s experimental law but then were unable to proceed to an expression for the kinetic energy.

Question 3

(a) This part was well answered, either with the relevant vector triangle drawn and the cosine rule applied or working with the velocities in component form.

(b) There was more success for those using their vector triangle to find the direction of the relative velocity than for those who, working with components, minimised \( \sqrt{c_{\text{D}}^2 + c_{\text{V}}^2} \) or used the condition \( c_{\text{rD}} = \frac{c_{\text{rV}}}{c_{\text{rD}}^2} \). Calculation of the time taken was frequently incorrect due to the use of 4 (km) rather than 4000 (m). Quite a lot of candidates found the time taken but did not continue to give the actual time at which the closest approach occurred, thus losing the final mark.

Question 4

(a) This was a routine type of question that most candidates were happy to tackle. However the zero position for GPE was occasionally chosen as the ‘variable point’ Q, instead of using a fixed level (PR), which then resulted in a sign error – with the benefit of the printed answer, many making this error adjusted their solutions either legitimately (without penalty) or by faking with a loss of 2 marks. It was pleasing to note that nearly all candidates found the EPE correctly and followed this with accurate algebra.
(b) Candidates who do not reach a printed answer should be instructed to proceed with the printed answer, not with their incorrect expression. This part was less routine, and although most candidates differentiated $V$ and put $V' = 0$, the convincing use of $\cos \theta < 1$ to obtain the given result was rarer.

Question 5

(a) Many candidates were not very convincing in their application of Newton’s second law to produce the required equation. The symbol $F$ frequently had two different meanings within the solution; it would be very much clearer if teachers were to encourage their pupils to use a different symbol such as $D$ for the driving force.

(b) The differential equation was usually solved successfully, although some weaker candidates were unable to separate the variables. A few candidates tried to use an integrating factor, but often failed to realise that $v^2$ should be used rather than $v$ for that method to work.

Question 6

(a) This was a standard question that many candidates completed successfully. Many however forced the problem to conform to their preconceived idea of this type of question. A diagram, preferably in the orientation given in the question, with velocity components clearly shown, would have helped candidates to get the signs correct in their equations. Some candidates used the velocities in their calculation of impulse, rather than the components along the line of centres.

(b) Despite problems with part (a), most candidates used a correct method with often only a single mark being lost due to errors from part (a).

Question 7

(a) Most candidates were able to make a reasonable attempt although there were some sign errors in Newton’s second law. Some weaker candidates missed out the component of the weight.

(b) The auxiliary equation was usually solved correctly. Subsequently a common error was either not to find a particular integral or to attempt to find it having already used the initial conditions on the complementary function – this was heavily penalised.

(c) Although the straightforward method of equating the velocity to zero was usually known, those candidates who had not simplified their answer to part (b) were often unable to complete this part. There were a very small number who attempted to use an energy method, occasionally correctly.
Mechanics Unit M5
Specification  6681

Introduction

The paper proved to be accessible and there was little evidence of candidates being under time pressure. Whilst there were some excellent scripts, it was surprising to see, at this level, that some of the candidates seemed to be poorly prepared, particularly for questions 6 and 7 which were testing the parts of the specification dealing with the rotation of rigid bodies about a fixed axis. Question 7, in particular, was found to be challenging, even for some of the better candidates. The first part of question 5 also proved to be demanding for many candidates. The first parts of questions 1, 3 and 4 proved to be the best source of marks.

Report on Individual Questions

Question 1

The first part proved to be an easy starter and was generally well answered. Part (b) was more demanding but nevertheless was well done by many, either by using the perpendicular axes rule directly or by starting from the centre of the square and using both the perpendicular axes and the parallel axes rules. A few candidates tried using integration but usually without success.

Question 2

Most candidates found the vector \( \mathbf{AB} \) and then equated the Work done to the KE gained, using the dot product to calculate the work done. Many candidates, failing to realise that the resultant force, acceleration and final velocity all had to be a scalar multiple of \( \mathbf{AB} \), made little further progress. Any of these quantities could be used to complete the question. A few candidates tried to use a cross product to find the work done by a force and a few thought that \( \mathbf{F}_2 \) had to be a multiple of \( \mathbf{AB} \).

Question 3

Many candidates found part (a) a source of easy marks. The majority used the standard auxiliary equation method for solving a second order differential equation and some of those wasted a lot of time and energy using components. Others successfully started from a first order equation in \( v \) and some integrated both sides with respect to time first. A few candidates (mostly overseas) tried to divide a vector by a vector, which was not accepted. In the second part many candidates gave the equation \( x = 3 \) but many did not go on to give any description implying that the particle moved in a straight line.

Question 4

This was a good source of marks for many candidates, with almost all getting part (a) correct although a few thought that \( \sum \mathbf{F} + \mathbf{R} = \mathbf{0} \). In part (b) the method was well known but some gave up, however, when they reached the stage where they had one equation in two unknowns, not realising that any pair of values that satisfied the equation would do. A few candidates tried to use dot products and some used \( \mathbf{F} \times \mathbf{r} \) but this could lead to full marks if they were consistent.
**Question 5**

Candidates who attempted part (a) of this question from first principles were often successful, with the possible exception of their signs. Those who tried to fit it to a solution that they had met before had problems, often caused by the inclusion of an impulse term which should not have been there. Many candidates gained four marks for the second part of the question, starting from the printed answer but missing out the subtraction of the final mass from $M$. Some however worked with the answer that they had obtained for part (a). Others got confused over which speed went with which time and often introduced a speed of zero.

**Question 6**

There were many good solutions to part (a) but a missing minus sign was a common error. Some started off with an energy equation and then differentiated. Some (mostly, but not all, from overseas) appeared not to understand the instruction about finding an equation of motion for the disc. Many candidates followed the instruction to use their answer to part (a) to answer part (b). In fact many had already replaced $\sin \theta$ with $\theta$ in part (a) which could cause problems in part (c). Some however insisted on using the formula they had learned for the period of an approximate SHM and scored nothing in part (b). Many candidates made a reasonable attempt at the final part, using Newton’s second law. Errors arose from signs, using $\theta$ instead of $\sin \theta$ or finding the wrong component.

**Question 7**

Many candidates tried to use conservation of energy (or the equivalent route using Newton’s second law etc) in part (a) in spite of the fact that a collision had occurred. Even those who used conservation of angular momentum often missed out a term and so had to do some interesting fudges to get the printed answer. In part (b) most candidates did not realise that the linear momentum of both $R$ and $Q$ had to be considered and there were very few correct solutions seen. In the third part most realised that an energy method gave the easiest solution but did not always include all of the particles and the pulley. Some tried to write down equations of motion for the particles and the pulley to find an acceleration. Few however successfully reached the end of this long method.
Introduction

This was an accessible paper in which candidates could demonstrate their understanding of statistics at AS level. It was however disappointing to see that a number of candidates had learnt to calculate statistics but had a limited understanding of what they actually meant. Question 3 part (f) demonstrated that they can learn a correct response but Question 1 part (e) and Question 2 part (d) showed they struggle to fully appreciate the meaning of their calculations or diagrams. There was little evidence of candidates running out of time, but occasionally lots of incorrect working crossed out on Question 3 and Question 5 may have left some candidates with insufficient time to complete the paper.

Report on Individual Questions

Question 1

Part (a) often scored full marks although some still mention ‘mean’ instead of ‘median’. Part (d) was very straightforward for the vast majority of candidates. Those candidates who used a scale of 4cm to 10 units were sometimes prone to placing the median inaccurately. Part (e) was also quite well done but some only listed the 5 important values with little or no mention of IQR, range, outliers or skewness. There was occasional confusion thinking the bigger numbers meant school B had done better.

Question 2

In part (a) there were very few correct solutions. It was rare for a candidate to appreciate that the selection was without replacement. The rest of this question was well answered by many, although a surprising number averaged the two means in part (c).

Question 3

In part (a) calculating $\Sigma I$ instead of the required $\Sigma y$ was the most common reason for losing marks. In part (b) premature approximation was frequent and caused a loss of marks in other parts of the question. In part (c) substituting $t=40$ was usually attempted but some then neglected to add on the 2460. Candidates are now very well primed to say that a certain value is out of range and hence the result is not reliable.

Question 4

This often scored full marks. For the variance in part (a) there were a few occasions where the working shown made it clear that the candidate would have forgotten to subtract $(E(X))^2$ if the value of the variance had not been given in the question. As is usually the case, some candidates are not aware of the need for full working when a “show that” question is asked. In part (c) some were using $4^2$ rather than $3^2$.

Question 5

It was unusual if a candidate scored 3 marks for the sketch. The mark for a bell-shaped curve was awarded to most candidates, but a particularly common problem was putting the value 1.65 on the wrong side of the sketch. Putting enough correct probabilities in the spaces was not
always well done. In part (b), very few solutions had the 4 decimal place values for \( z \), hence accuracy was lost. Part (c) was reasonably well answered.

**Question 6**

It was common in the Venn diagram for the value of 41 to be omitted or replaced with a zero. It seems that candidates were assuming that the hundred people in the question all possessed at least one of the attributes, i.e. they didn’t bother to add up the other values in the diagram to see that they did not come to 100. Part (b), part (c) and part (d) were generally well answered and usually followed from the values in the diagram. The conditional probability was better answered than has been the case in the past but this is still a good discriminator.
Introduction

This paper was shown to be accessible to the majority of candidates and there was no evidence of them being unable to complete the paper owing to time constraints. Many of the candidates seemed to be confident with the work they had learnt in statistics at A2 level. It was disappointing to see that some candidates did not relate some of their answers to the context of the question. For example, in Q3, where candidates were asked to give conditions for the choice of a Poisson model, they should have given these conditions by relating the theory to the context posed in the question.

Report on Individual Questions

Question 1

Almost all candidates answered part (a) correctly, a minority failed to mention “census” or “asking all members” when answers referred to long time/expensive/difficult. In part (b) many candidates failed to include the word “all” in their answer. Quite a number did not know or understand the term sampling frame and wrote about sampling methods. Most candidates answered part (c) correctly, but there were occasional references to golfers rather than members or to those selected in the sample.

Question 2

Part (a) was mostly correct although there were some very long-winded solutions seen. Drawing a diagram (as is often the case) was a successful approach to use. Part (b) was generally answered correctly although if integration was used the solution tended to be lengthy. Common wrong answers were 1/8 and ¾ were common wrong answers. Weaker candidates clearly did not understand the use of the word “or” in probability and failed to add the probabilities for the two parts. In part (c) whilst most candidates recognised a binomial situation and found the correct value for \( p \), few candidates were able to cope with a value of \( p > 0.5 \). It was common to see \( P(X > 10) \) given \( X \sim B(20, 0.75) \) interpreted as \( P(Y \leq 10) \), or \( 1 - P(Y \leq 10) \), given \( X \sim B(20, 0.25) \). There was poor understanding of how to use the binomial tables for situations in which \( p \) is greater than 0.5.

Question 3

The majority of candidates knew the conditions for the Poisson distribution but many did not get the marks because they failed to put them into context. As in many previous series, it was very common for candidates to repeat at least some of these conditions parrot-fashion preceded by “events occur” or “it occurs”. Other common errors were listing randomness and independence as separate reasons and citing the fixed time period and lack of an upper limit as reasons. Quite a number failed to mention the parameter for the distribution. The majority of candidates answered part (b) correctly. Most candidates answered part (c) correctly. Where marks were lost it was usually through failing to use a continuity correction rather than applying it wrongly.
Question 4

Most candidates answered Part (a) correctly. A small number of candidates calculated the probability for less than or equal to 3 although a minority thought that dividing by 0! in \( P(X = 0) \) gave zero. In part (b) carrying out the hypothesis test was more challenging though there was clear evidence that candidates had been prepared for this type of question. However, using \( p \) instead of \( \lambda \) or \( \mu \), when stating the hypotheses, was often seen and incorrectly stating \( H_1 : \lambda > 1.25 \) or \( 5 \) also lost marks. Many candidates calculated \( P(X \leq 11) \) instead of looking at \( P(X \geq 11) \). A diagram would have helped them or the use of the phrase “a result as or more extreme than that obtained”. Those who used the critical region approach made more errors. Some candidates correctly calculated the probability and compared it with 0.025 but were then unsure of the implications for the hypotheses. A few candidates used a 2-tailed hypothesis but then used 0.05 rather than 0.025 in their comparison. Most candidates gave their conclusions in context.

Question 5

This was well answered by almost all candidates and many correct solutions were seen. A few candidates tried to use Poisson rather than Binomial for parts (a) and (b). In part (b) a few candidates used \( B(10, 0.6) \) instead of \( B(10, 0.06) \). In part (c)(i) most errors occurred because candidates did not understand what was meant by “between 10 and 13 inclusive” The most common wrong answer was in using \( P(10 \leq X \leq 13) = P(X \leq 13) - P(X \leq 10) \) instead of \( P(X \leq 13) - P(X \leq 9) \) Another fairly common error was using \( P(X \leq 13) - (1 - P(X \leq 10)) \). Some candidates tried to use a continuity correction in this Poisson approximation. Part (c)(ii) was often correct the most common errors being to use 7.5 instead of 7.05 for the variance and to use an incorrect continuity correction.

Question 6

This question was well answered by a high percentage of students who gained full marks or only dropped up to four marks. The vast majority of candidates attempted all parts of this question. Evidence of its challenging nature to a number of candidates was the large amount of crossings out and untidy working. This said, however, it was clear candidates had been well prepared for a question of this type. Part (a) was done well with very few “fudged” solutions seen. Most candidates scored full marks. Part (b) was problematic for a number of candidates who simply wrote an incorrect answer without any working, hence losing up to four marks. Others integrated \( f(x) \) but without a variable upper limit or lower limit of 1. A minority of candidates had difficulty with integration. A significant number of candidates lost many marks on this answer through using \( k \) instead of \( 1/k \) in their working for parts (b), (c), and (d). Candidates who lost marks on part (b) often gained marks later for parts (c) and (d) through working from the original function rather than using their answer to part (b). In part (c) most candidates knew how to find the mean, although a few tried to integrate \( xF(x) \) rather than \( xf(x) \). In part (d) many candidates knew what to do to find the median with the majority of marks lost because the wrong expression for \( F(X) \) was used. A few poor solutions of quadratic solutions were seen but it was good to see many candidates correctly discard the unwanted solution. In part (e) many candidates differentiated to find the mode which was inappropriate in this case. However quite a number drew a good sketch and used this to correctly identify the mode. In part (f) the inequality mean<median<mode was generally known and quoted, often in spite of conflict with their answers to the previous parts!
Question 7

Part (a) was one of the poorest answered questions in the paper. Many candidates quoted the inequalities with little or no understanding of how to apply them and too many merely stated the critical values with no figures to back them up and without going on to give the critical region. It was unclear in some cases whether they knew that the critical region was the two tails rather than the central section. A few candidates used diagrams and this almost always enabled them to give a correct solution. Many misunderstood the wording of the question and thought that one of the tails could be slightly larger than 2.5%. Those that got Part (a) correct usually got part (b) correct, although a minority of weaker candidates did not understand what was meant by significance level. Part (c) was well answered. Those candidates who used the critical region approach did less well, tending to get themselves muddled. A few did not make the correct implication at the end and too many did not state that \(0.2061 > 0.10\) but merely said the result was not significant. The context for accepting/rejecting the null hypothesis was not always given.
Statistics Unit S3
Specification 6691

Introduction

Overall, this was an accessible paper, with good candidates achieving very high marks and showing a high level of competence in this A2 unit. The setting out of work by candidates was usually very clear and precise. However, there was a small number of candidates who were completely unprepared for the examination and scored poorly. Non-numerical answers produced much better responses than previous sessions and lots of completely correct questions were seen, especially question 5 and question 6.

Report on Individual Questions

Question 1

In part (a) they usually scored well, but there were few completely correct answers. Many candidates had little to offer but regurgitated text book definitions.

Question 2

Very well answered on the whole, but many candidates lost a mark by failing to mention the Central Limit Theorem as required.

Question 3

Part (a) was well answered, but hypotheses sometimes appeared in ill thought out words rather than symbols. Part (b) was poorly answered with hardly any candidates going beyond independence and most ignoring the need for context beyond this. Given that two marks were on offer it was surprising that most candidates did not realise the need for a little more thought and a more detailed response.

Question 4

Part (a) was very well answered by the great majority of candidates; part (b) less so but still a very large number of fully correct answers were seen, with the final conclusion well stated in context. A typical error was to conclude that the correlation was positive without any further interpretation.

Question 5

In part (a) a relatively large number of incorrect answers were seen for the variance, but the majority of candidates found it easy to score full marks on this question.

Question 6

There were some excellent responses with a large number of correct answers seen. It was unusual not to see hypotheses well stated and the conclusion given correctly in context.
Question 7

Part (a) was almost always completely correct, but parts (b) and (c) elicited the usual mistakes involving incorrect variances. In part (b) it was not unusual to see $50 \pm 1.96 \frac{0.5}{\sqrt{10}}$.

Question 8

Most candidates answered this question very well and high scores were common. Errors crept in through a failure to pool, flimsy hypotheses, incorrect critical values, and, to a lesser extent, an inability to state a correct conclusion. Only a small minority of candidates failed to read the question properly and used an estimate for the probability. Weaker candidates attempted a Poisson distribution which did not score well.
Introduction

Overall the paper worked well enabling nearly all candidates to demonstrate what they knew but also enabling the stronger candidates to shine. Most students found the first 4 or 5 questions very accessible and many scored highly here. Q6 proved to be a good discriminator and only the better candidates made significant progress through this question.

Candidates should be aware of the notation for the unbiased estimate of population variance as $s^2$. In Q2 and Q4 some candidates treated the given values of $s^2$ (or $s$) as biased estimates and they lost marks through multiplying by $\frac{n}{n-1}$.

Report on Individual Questions

Question 1

This proved to be a good starter and most candidates gave good solutions. Some failed to express the hypotheses in terms of $\mu$ and 1012 and a few did not interpret their conclusion in terms of the mean weight of the squirrels.

Question 2

Most realized that the Chi squared distribution was required to establish the confidence interval in part (a) and there were many correct solutions. The $F$ test was usually used in part (b) but sometimes the degrees of freedom were the wrong way around and some used a 5% significance level.

Question 3

Most candidates identified the need to carry out a paired $t$ test and the method was well known and clearly demonstrated. In part (b) many mentioned that the weekly fuel consumption had to be normally distributed. Whilst this is a sufficient condition the required answer was that the differences in weekly fuel consumption was normally distributed. Only a handful of candidates spotted this.

Question 4

The two tailed $F$ test was usually tackled quite well but the confidence interval in part (b) was not. A pooled estimate of variance $s_p^2$ was required and this was often attempted but the term $\frac{1}{9} + \frac{1}{10}$ was sometimes divided into $s_p$ rather than multiplying it. The interpretation in part (c) was often answered well and the follow through enabled those who could interpret the statistics to gain some credit.
Question 5

The vast majority of candidates realized that Poisson distributions should be used in this question and made some progress, the handful who tried to use binomial distributions did not. The concepts of size and power were generally well understood and all could attempt the graphs in part (f). Those who found a rejection criterion of 6 or more in part (c) were usually able to complete the remainder of the question successfully. The examiners were impressed by the quality of the responses to part (h) where many students showed that they were able to interpret their calculations and make sound decisions based upon them.

Question 6

The structure and given answers helped many candidates here and those who attempted it were often able to pick up marks in at least parts (b), (d) and (g). Perhaps surprisingly part (a) proved to be the most challenging. There were many unconvincing attempts based on integrating $x^n$ and then for some reason dividing by $x$ but those who simply applied their S2 knowledge and wrote $E(X^n) = \int_0^t x^{n-1} \frac{1}{t} \, dt$ were usually able to complete this part and often most of the question. A common error in part (c) was to assume $\text{Var}(XY) = \text{Var}(X)\text{Var}(Y)$ and this was perhaps the most challenging part. In part (f) the reasoning was usually sound, although the values were sometimes incorrect, and candidates who persevered to the end were often able to use the point (2, 3) in their estimator to answer part (g).
Decision Mathematics Unit D1
Specification 6689

Introduction

The paper proved very accessible, and each question had parts accessible to all. Most candidates were able to make a good attempt at all of the questions. The work was generally well-presented and efficient methods of presentation were more common on most questions.

This paper will be marked electronically as of next January and consequently colours will be indistinguishable. It is recommended that candidates use alternative notation, e.g. wavy, dotted, dashed etc. lines to replace colour.

Most candidates reached the end of the paper and attempted all questions.

Q1 proved a good starter but many candidates showed too much working, this resulted in these candidates using additional sheets and, more importantly, the time wasted led to their experiencing time problems towards the end of the paper. This was by far the main cause of additional sheets being used. The only other cause was candidates using photocopies of some of the diagrams and tables to replace incorrect work. Candidates are recommended to complete tables and diagrams in HB or B pencil.

Report on Individual Questions

Question 1

This proved a good starter question for the candidates, with the vast majority scoring full marks. Only a few candidates sorted the list into ascending order, and very few incorrect methods were seen, but a disappointing number of candidates did not seem to be aware that a bubble sort should be performed consistently in one direction. Amongst those candidates using the correct method, more marks were lost by those misreading their own writing and changing one number into another than those lost making errors in applying the algorithm. Some candidates omitted a ‘stop’ statement. Candidates were asked to give the state of the list after each pass, but many showed each exchange and some each comparison, which wasted time, many of these candidates needed to use additional sheets to show all of this working and many got into time difficulties later on in the paper.

Question 2

This proved an accessible question, but some poor definitions were seen in part (a), with candidates using technical language inaccurately, however, many were able to score some credit. Part (b) was generally very well done. The great majority of the candidates found at least one correct alternating path, and most both. Some candidates did not indicate the ‘change status’ step and others did not take into account their first alternating path when seeking their second. Candidates are reminded that colour will be indistinguishable as this paper will be marked electronically from next January.

Question 3

Parts (a) and (b) proved very successful for most candidates. A substantial minority failed to identify the correct four odd vertices, either by miscounting or some, having listed the all the valencies, selected an even vertex. Similarly some did not select their least pairing, but selecting the pairing using the least number of edges. Candidates were required to list the arc to
be repeated but many, for example, listed AE as an arc, a few listed AB + BE rather than AD + DE. Part (c) was a good discriminator but some very good, clear explanations were seen. The most common mistake was to focus on the longest arc to be eliminated, rather than the smallest one to be repeated.

**Question 4**

Once again the use of technical terms was very confused and very few candidates were able to give a full, clear definition in part (a), but most were able to gain some credit. The rest of the question proved accessible to most candidates, with some weaker candidates gaining a lot of their marks here. There were, of course, the usual problems with candidates listing the working values in any order in their working for part (b). Candidates must list the working values in the order in which they occur if they are to demonstrate that they are using the algorithm properly. This carelessness was a major source of mark loss for some candidates. Mistakes often appeared at D and E, 62 often being seen as an extra working value at E with the orders of these 2 vertices reversed. In part (c) candidates who listed the subtractions they had used, together with listing the arcs this indicated, were usually much more successful than those who attempted a more general explanation, but it was noted that the general quality of the response to this part of the question has improved.

**Question 5**

This was a good source of marks for many candidates, but it also discriminated well. In part (a) the most frequent error occurred in calculating the late event at the end of activity B. In part (b) many listed J as a critical activity and some only listed one path’s worth of critical activities. Part (c) was very revealing. Candidates were required to show their working and although most used numbers from the correct parts of their diagram many did not. Some subtracted two numbers at one event, others used the two late times or the two early times. Some, predictably, found the sum of their total floats. A surprisingly good number of candidates completed part (d), but many showed no floats, probably trying to draw a scheduling diagram, and others made errors when indicating activity lengths. A significant number overlapped B and D, and L and N so that activity length and floats could not be clearly identified. Again part (e) was surprisingly well-answered by very many candidates. Some listed a few extra activities with C, with D, E and F being popular companions, and a few only listed G and H for day 25.

**Question 6**

Part (a) was well answered in general with most candidates stating the correct equations. The profit equation certainly proved the most challenging for the candidates to obtain. Some candidates omitted the slack variables, or used inequality signs (often retaining slack variables) in the constraints and there were some sign difficulties, double equal signs, and a few inequalities seen in the profit equation as well as P itself being omitted. Most candidates clearly had an awareness of the Simplex algorithm, but there were many arithmetic slips. The pivot was correctly identified in the majority of cases. Some candidates did not change the basic variables. When slips are made and negative numbers appear where they shouldn’t, there is no awareness that this is the case. Some rather sloppy notation was sometimes used to describe the row operations, which is disappointing at this level. In part (c), a significant number of candidates read down the columns rather than across the rows. Some candidates chose to miss out part (b), and therefore part (c), entirely, others scored very well on both parts.

**Question 7**

A surprisingly few candidates gained all three marks in part (a), with 177 rarely seen and 103 seen only slightly more frequently. Most candidates completed the labelling correctly; the only commonly seen error was that of swapping the labels on DE. Most candidates were able to find
one alternating path but only the better candidates found all, many over-saturated DT. Many
who attempted diagram 2 in part (d) didn’t check their flows into and out of nodes. Candidates
had to be looking at a flow of 98 to gain any credit in part (e), although some clearly felt they
managed to prove their flow of 82 was maximal. A disappointing number of good candidates
found the correct maximum flow and correct minimum cut but did not use the theorem to link
them together.
Introduction

The paper proved to be accessible to the majority of candidates, with most candidates being able to make good attempts at most of the questions. Q1, Q6 and Q7 proved challenging for many candidates, but good attempts were usually seen to questions 2, 3, 4 and 5.

Examiners were a little disappointed by the confused language and poor use of technical terms by some candidates.

The questions requiring candidates to formulate a linear programming problem (questions 2 and 7a) were of a much improved standard in general.

A number of candidates did not complete Q7 but from comments seen on the scripts this seemed due to their not expecting to see simplex, rather than having time difficulties.

Report on Individual Questions

Question 1

This question was often poorly answered. Many candidates did not seem to know Bellman’s principle and even those that did were often unable to give an accurate statement. Many candidates gave a confused definition of a minimax route, often including statements such as “minimise the maximum route”. In the final part, candidates often had some idea of a practical problem, such as “walkers in mountains” or “a plane making a multistage journey” but then sometimes failed to give a full statement.

Question 2

Good attempts were often seen to this question, which is very pleasing since this has traditionally been an area that candidates find challenging. A few candidates treated it as a game theory problem, but most were able to set up the constraints correctly, although a small number did not use coefficients of 1. Some candidates failed to correctly define \( x_{ij} \) as being a task, \( i \), allocated to a worker, \( j \), and/or failed to explain the values these variables could take and why. A small number of candidates used poor notation such as \( p1 \) etc. Most candidates were able to state the objective but some failed to state that they needed to minimise the cost.

Question 3

This question proved accessible to all candidates. Most candidates were able to gain some credit in part (a), but few gave complete answers, most were able to relate this practical problem to the TSP, but few made it clear that they must complete each activity once, in a minimum total time and return to the start. The remainder of the question was often very well done. Most candidates obtained the initial upper bound, but a small number omitted the return to B. In part (c) candidates either used Nearest Neighbour or 2 x MST and shortcuts to arrive at a better upper bound. The most common errors were failing to state a route or omitting the final arc to return to B. In part (d) most candidates found the correct lower bound, however the most common error was to select DT (102) in the RMST rather than CT (60).
Question 4

The vast majority of candidates did not seem know how to apply the algorithm when one worker could not be assigned to a particular job and they found a series of creative ways to deal with this situation. The two most common were to ignore this altogether, or to place a zero in the blank cell, (although the latter approach led to a solution in which a worker was allocated a task which he could not perform). The concept of assigning a ‘large’ value (usually at least twice the value of largest element) does not seem to be well understood. Having arrived at a solution most candidates were able state an allocation and give the cost in thousands of pounds. Part (b) was usually answered well, examiners following through from the candidate’s final table, but a disappointing number merely made vague reference to the number of zeros in each row and column rather than presenting an argument.

Question 5

This question was well done by the vast majority of candidates, although some lost marks to minor arithmetical errors or failing to indicate their maximum values. Some candidates lost marks through the omission of one column such as “State” or more seriously by confusing the order of actions within one stage. A few candidates chose to do something other than maximise and some worked forwards rather than backwards. Other minor errors occurred in stating the route and sometimes omitting the units of the profit.

Question 6

This question caused problems for most candidates. Many candidates failed to give an adequate definition for degeneracy, with some referring to the “magic number” (m + n – 1), but did not define m and n. Others tried to describe the physical set of circumstances leading to degeneracy, but omitted part of the definition. Most candidates failed to correctly give the reason for a dummy, with the most common error being the statement that supply ≠ demand, rather than supply > demand. A significant minority stated that the number of suppliers did not equal the number of demand points. Most candidates gave the correct initial table, but many candidates then made errors. Common initial errors included miscalculating shadow costs or improvement indices, stating an incorrect stepping stone route or not giving the correct next solution. Those candidates who successfully completed the first iteration then ran into further problems, as many were confused by the zero in the dummy column. Some candidates did not calculate further shadow costs, believing that they had reached an optimum solution, others did not realise that their stepping stone route could have a value of 0. A number of candidates just chose to move the 0 with no justification. Further errors then occurred with candidates not calculating their final improvement indices, or failing to draw a conclusion and calculating the cost incorrectly.

Question 7

Many candidates struggled to answer this question and those that did make a decent attempt encountered difficulties and made a succession of errors. For those candidates who made a reasonable attempt at the question, by far the most popular approach was to divide the probabilities by the value of the game. Unfortunately this then created additional difficulties for the candidate, as for player A it was then necessary to minimise, which a large number of candidates failed to state. Candidates also either failed to turn their inequalities into equations or added slack variables when, in this instance, they should have been subtracted. Candidates also failed to define their probabilities etc. Most candidates who tried to answer the question in this way then used the equations for player A in their simplex tableau, not realizing that they needed to change these to player B’s perspective to allow them to maximise. Other candidates who started in the same manner, incorrectly set up their equations (inequalities) from B’s
perspective, but were then able to use these in their tableau. A minority of candidates adopted one of the other approaches and these candidates were generally more successful. Many candidates failed to mention that simplex was necessary because it was a 3 x 3 problem and it could not be reduced by dominance arguments. Those candidates who reached a correct initial tableau were generally able to manipulate this correctly, although minor errors, either arithmetic or omitting the change of base variable, did occur. Some candidates failed to state the row operations that they had used.
Grade Boundaries
June 2006 GCE Mathematics Examinations

The table below gives the lowest raw marks for the award of the stated uniform marks (UMS).

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