Mark Scheme (Results)
June 2008

GCE

GCE Mathematics (6690/01)

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June 2008
6690 Decision Mathematics D2
Mark Scheme

Question Number	Scheme	Marks
Q1		
(a)	A walk is a **finite sequence of arcs** such that the **end vertex of one arc is the start vertex of the next.**	B2,1,0
(b)	A tour is a walk that visits **every vertex**, **returning to its stating vertex.**	B2,1,0 (4)
		Total 4

Notes:

(a) 1B1: Probably one of the two below but accept correct relevant statement– bod gets B1, generous.

2B1: A good clear complete answer: End vertex=start vertex + finite.

(b) 1B1: Probably one of the two below but accept correct relevant statement– bod gets B1, generous.

2B1: A good clear complete answer: Every vertex + return to start.

From the D1 and D2 glossaries

D1

A **path** is a finite sequence of edges, such that the end vertex of one edge in the sequence is the start vertex of the next, and in which no vertex appears more than once.

A **cycle (circuit)** is a closed path, ie the end vertex of the last edge is the start vertex of the first edge.

D2

A **walk** in a network is a finite sequence of edges such that the end vertex of one edge is the start vertex of the next.

A walk which visits every vertex, returning to its starting vertex, is called a **tour**.

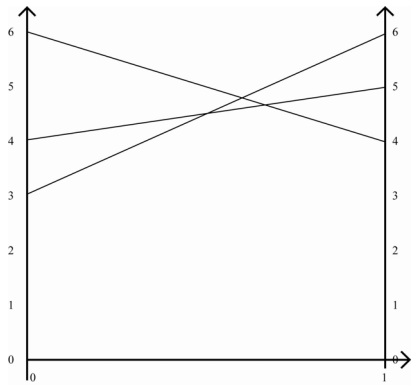
Question Number	Scheme	Marks
Q2		
(a)	Total supply > total demand	B2,1,0 (2)
(b)	Adds 0, 0 and 5 to the dummy column	B2,1,0 (2)
(c)	<table><tr><td></td><td>L</td><td>E</td><td>D</td></tr><tr><td>A</td><td>35</td><td>20</td><td></td></tr><tr><td>B</td><td></td><td>40</td><td>5</td></tr></table>	B1 (1)
(d)	<table><tr><td></td><td></td><td>80</td><td>70</td><td>20</td></tr><tr><td></td><td></td><td>L</td><td>E</td><td>D</td></tr><tr><td>0</td><td>A</td><td>35</td><td>20</td><td></td></tr><tr><td>-20</td><td>B</td><td></td><td>40</td><td>5</td></tr></table>	M1 A1
	$I_{AD} = 0 - 0 - 20 = -20$ $I_{BL} = 60 + 20 - 80 = 0$	A1 (3)
	<table><tr><td></td><td>L</td><td>E</td><td>D</td></tr><tr><td>A</td><td>35</td><td>20-θ</td><td>θ</td></tr><tr><td>B</td><td></td><td>40+θ</td><td>5-θ</td></tr></table>	M1
	θ = 5 ; entering square is AD; exiting square is BD	
	<table><tr><td></td><td></td><td>80</td><td>70</td><td>0</td></tr><tr><td></td><td></td><td>L</td><td>E</td><td>D</td></tr><tr><td>0</td><td>A</td><td>35</td><td>15</td><td>5</td></tr><tr><td>-20</td><td>B</td><td></td><td>45</td><td></td></tr></table>	A1ft (2) B1ft
	$I_{BL} = 60 + 20 - 80 = 0$ $I_{BD} = 0 + 20 - 0 = 20$	B1ft (2)
(e)	Cost is (£) 6100	B1 (1)
		Total 13

Question Number	Scheme	Marks
Q3 (a)	Maximin : we seek a route where the shortest arc used is a great as possible. Minimax : we seek a route where the longest arc used is a small as possible.	B2,1,0 (2)
(b)		

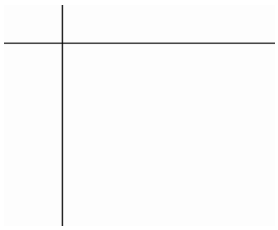
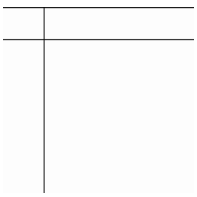
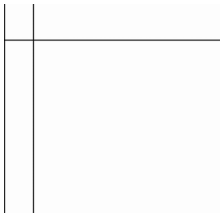
Stage	State	Action	Dest.	Value	
	G	GR	R	132*	
1	H	HR	R	175*	M1A1 (2)
	I	IR	R	139*	
	D	DG	G	min (175,132) = 132	
		DH	H	min (160,175) = 160*	M1A1
2	E	EG	G	min (162,132) = 132	
		EH	H	min (144,175) = 144*	
		EI	I	min (102,139) = 102	A1 (3)
	F	FH	H	min (145,175) = 145*	
		FI	I	min (210,139) = 139	
	A	AD	D	min (185,160) = 160*	
		AE	E	min (279,144) = 144	M1A1ft
3	B	BD	D	min (119,160) = 119	
		BE	E	min (250,144) = 144*	A1ft
		BF	F	min (123,145) = 123	
	C	CE	E	min (240,144) = 144	
		CF	F	min (170,145) = 145*	
	L	LA	A	min (155,160) = 155*	A1ft
4		LB	B	min (190,144) = 144	
		LC	C	min (148,145) = 145	

Maximin route: LADHR — A1ft (5)

Total 12

Question Number	Scheme	Marks
Q4		
(a)	For each row the element in column x must be less than the element in column y.	B2,1,0 (2)
(b)	Row minimum {2,4,3} row maximin = 4 Column maximum {6,5,6} column minimax = 5 $4 \neq 5$ so not stable	M1 A1 A1 (3)
(c)	Row 3 dominates row 1, so matrix reduces to	
		B1
	Let Liz play 2 with probability p and 3 with probability (1- p) If Mark plays 1: Liz's gain is $4p + 6(1- p) = 6 - 2p$ If Mark plays 2: Liz's gain is $5p + 4(1- p) = 4 + p$ If Mark plays 3: Liz's gain is $6p + 3(1- p) = 3 + 3p$	M1 A1 (3)
		B2, 1ft, 0 (2)
	$4 + p = 6 - 2p$ $p = \frac{2}{3}$	M1 A1
	Liz should play row 1 – never, row 2 - $\frac{2}{3}$ of the time, row 3 - $\frac{1}{3}$ of the time and the value of the game is $4\frac{2}{3}$ to her.	A1ft A1 (4)
(d)	Row 3 no longer dominates row 1 and so row 1 can not be deleted. Use Simplex (linear programming).	B1 B1 (2) **Total 16**

	M1	M2	M3
L2	4	5	6
L3	6	4	3

Question Number	Scheme	Marks
Q5 (a)	Since maximising, subtract all elements from some $n \geq 53$	
	$\begin{bmatrix} 5 & 4 & 11 & 11 \\ 0 & 4 & 2 & 3 \\ 2 & 0 & 5 & 5 \\ 6 & 3 & 7 & 10 \end{bmatrix}$	M1 A1 (2)
	Reduce rows $\begin{bmatrix} 1 & 0 & 7 & 7 \\ 0 & 4 & 2 & 3 \\ 2 & 0 & 5 & 5 \\ 3 & 0 & 4 & 7 \end{bmatrix}$ then columns $\begin{bmatrix} 1 & 0 & 5 & 4 \\ 0 & 4 & 0 & 0 \\ 2 & 0 & 3 & 2 \\ 3 & 0 & 2 & 4 \end{bmatrix}$	M1 A1ft (2)
	Minimum element 1	M1
	 $\begin{bmatrix} 0 & 0 & 4 & 3 \\ 0 & 5 & 0 & 0 \\ 1 & 0 & 2 & 1 \\ 2 & 0 & 1 & 3 \end{bmatrix}$	A1ft A1ft (3)
	 	M1
	$\begin{bmatrix} 0 & 1 & 4 & 3 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 3 & 2 \\ 1 & 6 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix}$	A1ft A1ft (3)
(b)		

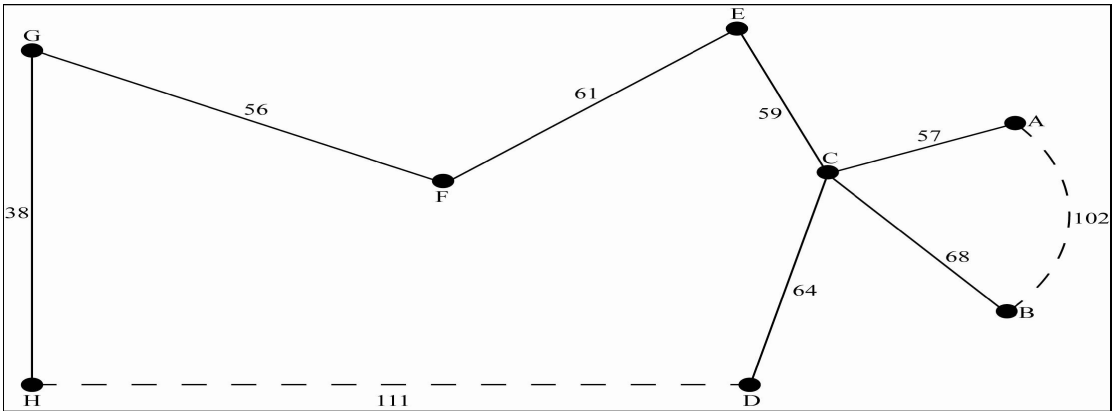
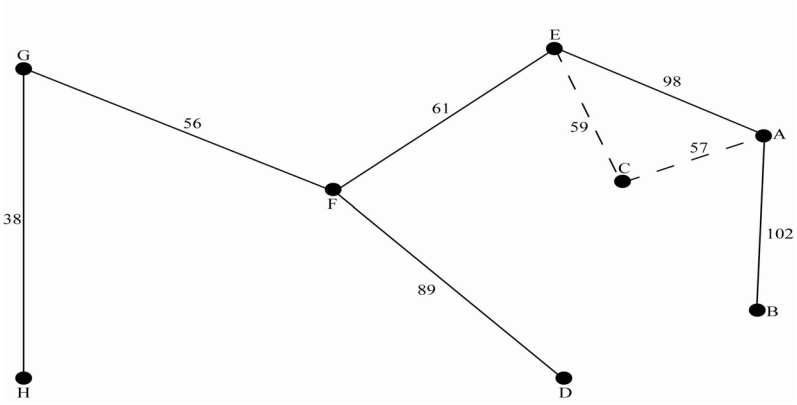
Joe	A	A
Min-Seong	C	D
Olivia	D	B
Robert	B	C

M1 A1ft (2)

Value £197 000

M1A1 (2)

Total 14

Question Number	Scheme	Marks
Q6		
(a)	GH(38) GF(56) CA(57) EC(59) FE(61) CD(64) CB(68)	M1A1 (2)
(b)	2 x 403 = 806 (km)	B1 (1)
(c)	e.g. DH saves 167 AB saves 23 806 – 190 = 616 (km)	M1 A1 A1
		
	eg A B C E F G H D C A	A1 (4)
(d)	B C A E F G H D B 68 +57 + 98 + 61 + 56 + 38 + 111 + 108 = 597 (km)	M1 A1 A1 (3)
(e)	Delete C	
		M1 A1
	RMST weight = 444 Lower bound = 444 + 59 + 57 = 560 (km)	M1 A1ft (4)
(f)	560 < length ≤ 597	B2,1,0 (2) **Total 16**

6690 JUNE 2008 Question 2 notes

(a) 1B1: Close, accept supply ≠ demand
2B1: CAO
(b) 1B1: One error
2B1: CAO
(c) 1B1: CAO
(d) 1M1: 5 shadow costs and precisely 2 improvement indices stated (no extra zeros)
1A1: 5 shadow costs correct.
2A1: 2 improvement indices correct.
2M1: A valid route, negative II chosen, only one empty square used, θ's balance.
3A1ft: optimal solution (no extra zeros)
1B1ft: ft 5 correct shadow costs
2B1ft: ft precisely 2 Improvement indices, both correct (no extra zeros)
(e) 1B1: CAO condone lack of £s

Note There is a second correct solution. It is unlikely to be found except as a result of an earlier error, or in continuing to develop the solution on mark scheme.

		80	70	0
		L	E	D
0	A		50	5
-20	B	35	10	

$I_{AL} = 80 - 0 - 80 = 0$ $I_{BD} = 0 + 20 - 0 = 20$ Cost (£) 1600

(d) Accept

		0	-10	-60
		L	E	D
80	A	35	20	
60	B		40	5

		0	-10	-80
		L	E	D
80	A	35	15	5
60	B		45	

Do not accept

		35	20	-15
		L	E	D
0	A	35	20	
20	B		40	5

		35	15	5
		L	E	D
0	A	35	15	5
30	B		45	

		0	-15	-60
		L	E	D
35	A	35	20	
65	B		40	5

		0	-20	-30
		L	E	D
35	A	35	15	5
65	B		45	

6690 JUNE 2008 Question 3 notes

(a) 1B1: Close. Condone swapped definitions here. bod gets B1.
2B1: Good, clear answer.

Throughout section (b):

- **Condone lack of destination column and/or reversed stage numbers throughout.**
- **Only penalise incorrect result in Value – ie ignore working values.**
- **Penalise absence of state or action column with first two A marks earned only**
- **Penalise empty/errors in stage column with first A mark earned only.**

(b) 1M1: First stage complete and working backwards.
1A1: CAO (condone lack of *)
2M1: Second stage completed. Penalise reversed states here and at end. Bod if something in each column.
2A1: Any 2 states correct. Penalise * errors with the first A mark earned here and only once.
3A1: All 3 states correct. (Penalise * errors only once in the question).
3M1: 3rd and 4th stages completed. Bod if something in each column.
4A1ft: Any 2 states correct. (Penalise * errors only once in the question). A, B or C
5A1ft: All 3 states correct. (Penalise * errors only once in the question). A, B and C.
6A1ft: Final, L, state correct. (Penalise * errors only once in the question).
7A1ft: CAO penalise reversed states again here.

Special cases (and misreads)

SC1 Minimax: treat as misread – see sheet. **MAX 8/10**

SC2 Maximum: 1M1,1A1; 2M0; 3M1,4A1ft,5A0,6A1ft,7A1ft **MAX 6/10**

SC3 Minimum: Marks awarded as above SC2

SC4 Maximax: 1M1,1A1; 2M0; 3M1,4A0,5A0,6A0,7A1ft **MAX 4/10**

SC5 Minimin: Marks awarded as above SC4

SC6 Working forwards: see sheet. **MAX 4/10**

Anything else annotate and send to review.

Question Number	Scheme	Marks
Q3(b)	**SC1 (Minimax – MISREAD)** **Misread: Award marks as usual.** **Remove the last 2 A (or B) marks earned** **ANNOTATE: MR**	

Stage	State	Action	Dest	Value	
	G	GR	R	132*	M1 A1 (2)
1	H	HR	R	175*	
	I	IR	R	139*	
	D	DG	G	max (175, 132) = 175*	
		DH	H	max (160, 175) = 175*	M1 A1
2	E	EG	G	max (162, 132) = 162	
		EH	H	max (144, 175) = 175	
		EI	I	max (102, 139) = 139*	A1 (3)
	F	FH	H	max (145, 175) = 175*	
		FI	I	max (210, 139) = 210	
	A	AD	D	max (185, 175) = 185*	M1 A1ft
		AE	E	max (279, 139) = 279	
3	B	BD	D	max (119, 175) = 175*	A1ft
		BE	E	max (250, 139) = 250	
		BF	F	max (123, 175) = 175*	
	C	CE	E	max (240, 139) = 240	
		CF	F	max (170, 175) = 175*	
	L	LA	A	max (155, 185) = 185	
4		LB	B	max (190, 175) = 190	A1ft
		LC	C	max (148, 175) = 175*	

LCFHR

A1ft (5)

- last 2 A/B for MR

Question Number	Scheme	Marks
Q3(b)	**SC2 (Maximum)** RHEBL	M1 A1 M0 M1 A1ft (for ALL correct) A0 A1ft A1ft **6 max**

Stage	State	Action	Dest	Value
	G	GR	R	132*
1	H	HR	R	175*
	I	IR	R	139*
	D	DG	G	175 + 132 = 307
		DH	H	160 + 175 = 335*
2	E	EG	G	162 + 132 = 294
		EH	H	144 + 175 = 319*
		EI	I	102 + 139 = 241
	F	FH	H	145 + 175 = 320
		FI	I	210 + 139 = 349*
	A	AD	D	185 + 335 = 520
		AE	E	279 + 319 = 598*
3	B	BD	D	119 + 335 = 454
		BE	E	250 + 319 = 569*
		BF	F	123 + 349 = 472
	C	CE	E	240 + 319 = 559*
		CF	F	170 + 349 = 519
	L	LA	A	155 + 598 = 753
4		LB	B	190 + 569 = 759*
		LC	C	148 + 559 = 707

Question Number	Scheme	Marks
Q3(b)	**SC3 (Minimum)** **Stage** | **State** | **Action** | **Dest** | **Value** | | G | GR | R | 132* 1 | H | HR | R | 175* | I | IR | R | 139* | D | DG | G | 175 + 132 = 307* | | DH | H | 160 + 175 = 335 2 | E | EG | G | 162 + 132 = 294 | | EH | H | 144 + 175 = 319 | | EI | I | 102 + 139 = 241* | F | FH | H | 145 + 175 = 320* | | FI | I | 210 + 139 = 349 | A | AD | D | 185 + 307 = 492* | | AE | E | 279 + 241 = 520 3 | B | BD | D | 119 + 307 = 426* | | BE | E | 250 + 241 = 491 | | BF | F | 123 + 320 = 443 | C | CE | E | 240 + 241 = 481* | | CF | F | 170 + 320 = 490 | L | LA | A | 155 + 492 = 647 4 | | LB | B | 190 + 426 = 616* | | LC | C | 148 + 481 = 629 RGDBL	M1 A1 M0 M1 A1ft (for ALL correct) A0 A1ft A1ft **6 max**

Question Number	Scheme	Marks
Q3(b)	**SC4 (Maximax)**	

Stage	State	Action	Dest	Value	Marks
	G	GR	R	132*	M1
1	H	HR	R	175*	A1
	I	IR	R	139*	
	D	DG	G	max(175, 132) = 175*	
		DH	H	max(160, 175) = 175*	
2	E	EG	G	max(162, 132)= 162	M0
		EH	H	max(144, 175) = 175*	
		EI	I	max(102, 139) = 139	
	F	FH	H	max(145, 175) = 175	
		FI	I	max(210, 139) = 210*	
	A	AD	D	max(185, 175) = 185	M1
		AE	E	max(279, 175) = 279*	A0
3	B	BD	D	max(119, 175) = 175	A0
		BE	E	max(250, 175) = 250*	
		BF	F	max(123, 210) = 210	
	C	CE	E	max(240, 175) = 240*	
		CF	F	max(170, 210) = 210	
	L	LA	A	max(155, 279) = 279*	
4		LB	B	max(190, 250) = 250	A0
		LC	C	max(148, 240) = 240	

RHEAL — A1ft

4 max

Question Number	Scheme	Marks
Q3(b)	**SC5 (Minimin)**	

Stage	State	Action	Dest	Value	Marks
	G	GR	R	132*	M1
1	H	HR	R	175*	A1
	I	IR	R	139*	
	D	DG	G	min(175, 132) = 132*	
		DH	H	min(160, 175) = 160	
2	E	EG	G	min(162, 132) = 132	M0
		EH	H	min(144, 175) = 144	
		EI	I	min(102, 139) = 102*	
	F	FH	H	min(145, 175) = 145	
		FI	I	min(210, 139) = 139*	
	A	AD	D	min(185, 132) = 132	M1
		AE	E	min(279, 102) = 102*	A0
3	B	BD	D	min(119, 132) = 119	A0
		BE	E	min(250, 102) = 102*	
		BF	F	min(123, 139) = 123	
	C	CE	E	min(240, 102) = 102*	
		CF	F	min(170, 139) = 139	
	L	LA	A	min(155, 102) = 102*	A0
4		LB	B	min(190, 102) = 102*	
		LC	C	min (148, 102) = 102*	A1ft

RIEAL
RIEBL
RIECL

4 max

Question Number	Scheme	Marks
Q3(b)	**<u>SC6 (Working forwards + Maximin)</u>** **Stage** | **State** | **Action** | **Dest** | **Value** | | A | AL | L | 155* 1 | B | BL | L | 190* | C | CL | L | 148* | D | DA | A | min (185, 155) = 155* | | DB | B | min (119, 190) = 119 2 | E | EA | A | min (279, 155) = 155 | | EB | B | min (250, 190) = 190* | | EC | C | min (240, 148) = 148 | F | FB | B | min (123, 190) = 123 | | FC | C | min (170, 148) = 148* | G | GD | D | min (175, 155) = 155 | | GE | E | min (162, 190) = 162* 3 | H | HD | D | min (160, 155) = 155* | | HE | E | min (144, 190) = 144 | | HF | F | min (145, 148) = 145 | I | IE | E | min (102, 190) = 102 | | IF | F | min (210, 148) = 148* | R | RG | G | min (132, 162) = 132 4 | | RH | H | min (175, 155) = 155* | | RI | I | min (139, 148) = 139 RHDAL	M0 M1 A1 (for ALL correct) A0 M1 A1ft (for ALL correct) A0 A0 A0 **4 max**

6690 JUNE 2008 Question 4 notes

(a) 1B1: Generous, but need idea of $x < y$. Bod gets B1
2B1: Good clear answer, idea of 'per row'.

(b) 1M1: Finds row maximin and column minimax. All values enough.
1A1: Row maximin = 4 col minimax = 5 identified in some way.
2A1: Row maximin (4) ≠ column minimax (5) stated and a clear link to statement.

(c) 1B1: Matrix reduced correctly. Could be implicit from equations.
1M1: Setting up three probability equations, implicit definition of p.
1A1: CAO
2B1ft: At least two lines correct, accept p>1 or p<0 here.
3B1: 3 lines cao, $0 \le p \le 1$, scale clear (or 1 line = 1), condone lack of labels.
2M1: Finding their correct optimal point, must have three lines, and setting up an equation to find $0 \le p \le 1$.
1A1: CAO
2A1ft: All three options listed.
3A1: CAO

(d) 1B1: CAO (generous)
2B1: CAO (generous)

6690 JUNE 2008 Question 5 notes

(a) 1M1: Subtracting from some $n \geq 53$
1A1: CAO
2M1: Reducing rows then columns
2A1ft: ft
3M1: Double covered +e; one uncovered – e; and one single covered unchanged.
3A1ft: ft correct accept one error
4A1ft: ft correct - no errors
4M1: Double covered +e; one uncovered – e; and one single covered unchanged.
5A1ft: ft correct accept one error
6A1ft: ft correct - no errors

(b) 1M1: One complete solution.
1A1ft: ft all possible solutions for their diagram
2M1: ft their result – should be 197
2A1: cao (£) 197 000

MISREAD – minimises

Reduce rows $\begin{bmatrix} 6 & 7 & 0 & 0 \\ 4 & 0 & 2 & 1 \\ 3 & 5 & 0 & 0 \\ 4 & 7 & 3 & 0 \end{bmatrix}$ then reduce columns $\begin{bmatrix} 3 & 7 & 0 & 0 \\ 1 & 0 & 2 & 1 \\ 0 & 5 & 0 & 0 \\ 1 & 7 & 3 & 0 \end{bmatrix}$

This is optimal. J – C
M – B
O – A
R – D
Cost (£) 185 000

Marks:
(a) 1M0 2M1 and 2A1 available. 3M0 4M0
(b) 1M1 1A1 2M0 2AO **Max of 4/ 14**

6690 JUNE 2008 Question 6 notes

(a) 1M1: First three arcs correct
1A1: CAO

(b) 1B1: CAO 806
(c)1M1: Finding at least one shortcut, **must be shortcut method so shortcuts need to be clear**, stated or drawn.
1A1: At least two short cuts clear, stated or drawn, valid tour remains.
2DA1: depends on 1st A. Bound stated, below 630, valid tour remains.Consistent.
3DA1: depends on 2nd A. A correct, consistent tour stated for a value below 630.Accept a diagram with letters.

(d) 1M1: Nearest Neighbour each vertex visited at least once (condone lack of return to start)
1A1: Correct route CAO – must return to start.
2A1: 597 CAO (do not ignore subsequent doubling)

(e) 1M1: Finding correct RMST (maybe implicit) 444 sufficient,or correct numbers. 6 arcs.
1A1: CAO tree or 444.
2M1: Adding 2 least arcs to C, 57 and 59 or 116 only
2A1: CAO 560

(f) 1B1: CSO 560 + all marks in (e). Accept better, correct lower bound
2B: CSO all marks in (c) and (d) 597 or 592

(c) Some shortcuts

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**
A		23			18			
B	23			24				
C						45		
D		24				95		167
E	18						30	
F			45	95				19
G					30			
H				167		19		

Some routes and lengths

ABCDFHGEA	**607**	ABDHFGECA	655
ABCDHFGEA	661	ABDHGEFCA	639
ABCDHGFEA	**598**	ABDHGFCEA	647
ABCDHGFECA	**616**	**ABDHGFECA**	**592**
ABCEFGHDCA	**616**	**ACBDFHGEA**	**620**
ABCFDHGEA	668	ACBDHFGEA	660
ABDCFHGEA	647	**ACBDHGFEA**	**597**
ABDFHGECA	**615**		