Edexcel GCE
Statistics S4
Advanced/Advanced Subsidiary
Friday 19 June 2009 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Orange or Green)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions. You must write your answer for each question in the space following the question.
Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 6 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Total

Turn over
1. A company manufactures bolts with a mean diameter of 5 mm. The company wishes to check that the diameter of the bolts has not decreased. A random sample of 10 bolts is taken and the diameters, $x$ mm, of the bolts are measured. The results are summarised below.

\[
\sum x = 49.1 \quad \sum x^2 = 241.2
\]

Using a 1\% level of significance, test whether or not the mean diameter of the bolts is less than 5 mm.

(You may assume that the diameter of the bolts follows a normal distribution.)
2. An emission-control device is tested to see if it reduces CO\textsubscript{2} emissions from cars. The emissions from 6 randomly selected cars are measured with and without the device. The results are as follows.

<table>
<thead>
<tr>
<th>Car</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissions without device</td>
<td>151.4</td>
<td>164.3</td>
<td>168.5</td>
<td>148.2</td>
<td>139.4</td>
<td>151.2</td>
</tr>
<tr>
<td>Emissions with device</td>
<td>148.9</td>
<td>162.7</td>
<td>166.9</td>
<td>150.1</td>
<td>140.0</td>
<td>146.7</td>
</tr>
</tbody>
</table>

(a) State an assumption that needs to be made in order to carry out a \( t \)-test in this case. (1)

(b) State why a paired \( t \)-test is suitable for use with these data. (1)

(c) Using a 5% level of significance, test whether or not there is evidence that the device reduces CO\textsubscript{2} emissions from cars. (8)

(d) Explain, in context, what a type II error would be in this case. (2)
Question 2 continued

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Question 2 continued
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3. Define, in terms of $H_0$ and/or $H_1$,

(a) the size of a hypothesis test,  

(b) the power of a hypothesis test.  

The probability of getting a head when a coin is tossed is denoted by $p$.

This coin is tossed 12 times in order to test the hypotheses $H_0: p = 0.5$ against $H_1: p \neq 0.5$, using a 5% level of significance.

(c) Find the largest critical region for this test, such that the probability in each tail is less than 2.5%.

(d) Given that $p = 0.4$

   (i) find the probability of a type II error when using this test,

   (ii) find the power of this test.

(e) Suggest two ways in which the power of the test can be increased.
Question 3 continued
4. A farmer set up a trial to assess whether adding water to dry feed increases the milk yield of his cows. He randomly selected 22 cows. Thirteen of the cows were given dry feed and the other 9 cows were given the feed with water added. The milk yields, in litres per day, were recorded with the following results.

<table>
<thead>
<tr>
<th>Sample size</th>
<th>Mean</th>
<th>$s^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry feed</td>
<td>13</td>
<td>25.54</td>
</tr>
<tr>
<td>Feed with water added</td>
<td>9</td>
<td>27.94</td>
</tr>
</tbody>
</table>

You may assume that the milk yield from cows given the dry feed and the milk yield from cows given the feed with water added are from independent normal distributions.

(a) Test, at the 10% level of significance, whether or not the variances of the populations from which the samples are drawn are the same. State your hypotheses clearly.

(b) Calculate a 95% confidence interval for the difference between the two mean milk yields.

(c) Explain the importance of the test in part (a) to the calculation in part (b).
Question 4 continued
Question 4 continued
5. A machine fills jars with jam. The weight of jam in each jar is normally distributed. To check the machine is working properly the contents of a random sample of 15 jars are weighed in grams. Unbiased estimates of the mean and variance are obtained as 

$$\hat{\mu} = 560 \quad s^2 = 25.2$$

Calculate a 95% confidence interval for,

(a) the mean weight of jam,  

(b) the variance of the weight of jam.

A weight of more than 565 g is regarded as too high and suggests the machine is not working properly.

(c) Use appropriate confidence limits from parts (a) and (b) to find the highest estimate of the proportion of jars that weigh too much.
Question 5 continued
6. A continuous uniform distribution on the interval \([0, k]\) has mean \(\frac{k}{2}\) and variance \(\frac{k^2}{12}\).

A random sample of three independent variables \(X_1, X_2\) and \(X_3\) is taken from this distribution.

(a) Show that \(\frac{2}{3}X_1 + \frac{1}{2}X_2 + \frac{5}{6}X_3\) is an unbiased estimator for \(k\).

(b) Show that \(\text{Var}(\hat{k}) = (a^2 - 2a + 2) \frac{k^2}{6}\)

(c) Hence determine the value of \(a\) and the value of \(b\) for which \(\hat{k}\) has minimum variance, and calculate this minimum variance.
Question 6 continued
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Question 6 continued