

Mark Scheme (Results)

Summer 2013

GCE Statistics 4 (6686/01R)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

# EDEXCEL GCE MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.

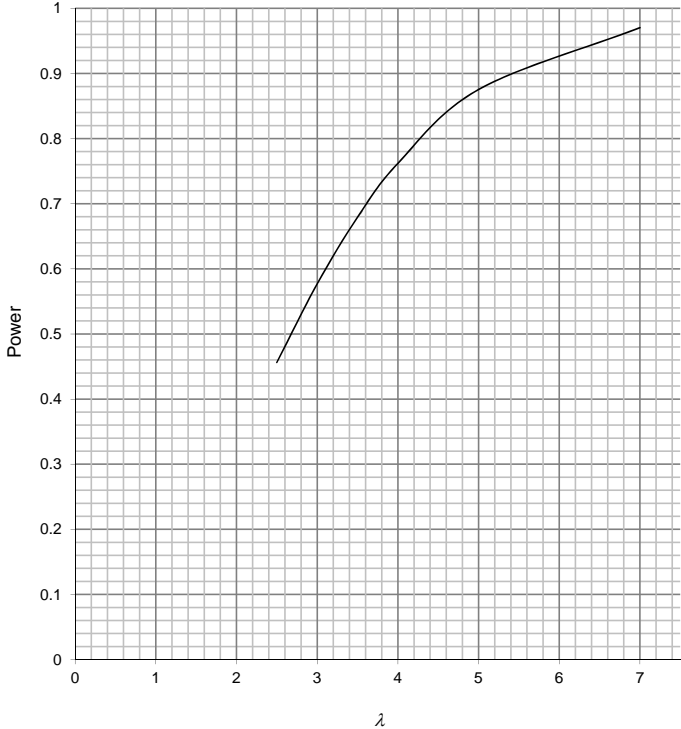
### 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\surd$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
  5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
  6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
  7. Ignore wrong working or incorrect statements following a correct answer.
  8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

Question Number	Scheme	Marks
<p><b>1.(a)</b></p> <p><b>(b)</b></p>	<p><math>P(X &gt; 1.690) = 0.975</math>  <math>P(X &gt; a) = 0.025</math>  <math>a = 16.013</math></p> <p>Upper critical value of <math>F_{6,4} = 15.21</math>  Lower critical value of <math>F_{6,4} = \frac{1}{9.15} = 0.109</math></p>	<p>M1  A1  <b>(2)</b></p> <p>B1  B1  <b>(2)</b>  <b>[4]</b></p>
<b>Notes</b>		
<p><b>(a)</b></p> <p><b>(b)</b></p>	<p>M1 for using 0.025</p> <p>2<sup>nd</sup> B1 either <math>\frac{1}{9.15}</math> or awrt 0.109</p>	

Question Number	Scheme	Marks
<p><b>2.</b></p> <p><b>(a)</b></p> <p><b>(b)</b></p>	$\frac{29 \times 0.36}{45.722} < \sigma^2 < \frac{29 \times 0.36}{16.047}$ $0.228 < \sigma^2 < 0.651$ <p>Since 0.495 lies in the interval or <math>0.228 &lt; 0.495 &lt; 0.651</math> yes</p>	<p>M1B1,B1</p> <p>M1 A1</p> <p><b>(5)</b></p> <p>B1ft</p> <p>B1ftd</p> <p><b>(2)</b></p> <p><b>[7]</b></p>
	<b>Notes</b>	
<p><b>(a)</b></p> <p><b>(b)</b></p>	<p>1<sup>st</sup> M1 use of <math>\frac{29 \times s^2}{\chi^2}</math> ( May use <math>\frac{s^2}{F_{29,\infty}}</math> or <math>s^2 \times F_{29,\infty}</math> )</p> <p>( Based on <math>\frac{s^2}{\sigma^2} = F_{29,\infty}</math> )</p> <p>1<sup>st</sup> B1 45.722 ( using <math>\frac{s^2}{F_{29,\infty}}</math> and <math>s^2 \times F_{29,\infty}</math> )</p> <p>2<sup>nd</sup> B1 16.047 ( may use <math>F_{29,\infty} = 1.4686</math> )</p> <p>2<sup>nd</sup> M1 correct answer using their <math>\chi^2</math> value ( correct using their <math>F_{29,\infty}</math> )</p> <p>A1 awrt 0.228 and awrt 0.651 ( awrt 0.245 and awrt 0.529 )</p> <p>ft their interval</p>	

Question Number	Scheme	Marks
<p><b>3. (a)</b></p> <p><b>(b)</b></p> <p><b>(c)</b></p> <p><b>(d)</b></p> <p><b>(e)</b></p>	<p><math>X \sim \text{Po}(2)</math></p> <p>Size = <math>P(X \geq 3 / \lambda = 2)</math></p> <p><math>= 1 - 0.6767</math></p> <p><math>= 0.3233</math></p> <p>0.323</p> <p>Power = <math>1 - P(0) - P(1) - P(2)</math></p> <p><math>= 1 - e^{-\lambda} - \lambda e^{-\lambda} - \frac{\lambda^2 e^{-\lambda}}{2!}</math></p> <p><math>= 1 - \frac{1}{2} e^{-\lambda} (2 + 2\lambda + \lambda^2)</math></p> <p><math>r = 0.58 \quad s = 0.76</math></p>  <p><math>\lambda &gt; 3.1</math></p>	<p>awrt</p> <p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>A1</p> <p>A1 cso (3)</p> <p>B1, B1 (2)</p> <p>B1ft points</p> <p>B1ft curve (2)</p> <p>allow numbers in range 3.1-3.2</p> <p>B1 (1)</p> <p>[10]</p>
<b>Notes</b>		
<p><b>(a)</b></p> <p><b>(b)</b></p> <p><b>(c)</b></p> <p><b>(d)</b></p>	<p>M1 for correct expression for size using Po(2)</p> <p>1<sup>st</sup> M1 for a correct expression in terms of probabilities. Allow <math>1 - P(X \leq 2)</math> or <math>1 - P(X &lt; 3)</math></p> <p>1<sup>st</sup> A1 for correct equation in <math>\lambda</math></p> <p>2<sup>nd</sup> A1 cso</p> <p>SC if both correct but not to 2dp award B1B0</p> <p>1<sup>st</sup> B1ft points</p> <p>2<sup>nd</sup> B1ft curve (or straight lines) through points</p>	

Question Number	Scheme	Marks
<p><b>4. (a)(i)</b></p> <p><b>(ii)</b></p> <p><b>(b)</b></p> <p><b>(c)</b></p>	<p>Ardo <math>s^2 = \frac{1}{6}(1257.78 - 7(13.4)^2)</math>  <math>= 0.143\dots</math></p> <p>Bards <math>0.261 = \frac{6 \times 0.143\dots + 8 \times s^2}{7 + 9 - 2}</math>  <math>s^2 = 0.349\dots</math></p> <p><math>H_0 : \sigma_1^2 = \sigma_2^2, H_1 : \sigma_1^2 \neq \sigma_2^2</math></p> <p>critical values <math>F_{8,6} = 4.15</math> <math>\left( \frac{1}{F_{8,6}} = 0.241 \right)</math></p> <p><math>\frac{s_2^2}{s_1^2} = \frac{0.349}{0.143} = \text{awrt } 2.44</math> <math>\left( \frac{s_1^2}{s_2^2} = \frac{0.143}{.349} = 0.41 \right)</math></p> <p>Since 2.44... (0.424) is not in the critical region we accept <math>H_0</math> and conclude there is no evidence that the two variances are different</p> <p><math>H_0 : \mu_B - \mu_A = 0.9; H_1 : \mu_B - \mu_A &gt; 0.9</math></p> <p>CR: <math>t_{14}(0.05) &gt; 1.761</math></p> <p><math>t = \pm \frac{14.8 - 13.4 - 0.9}{\sqrt{0.261(\frac{1}{7} + \frac{1}{9})}} = \pm 1.94\dots</math></p> <p>awrt <math>\pm 1.94</math></p> <p>Since 1.94... is in the critical region we reject <math>H_0</math> and conclude that the <b>mean strength</b> of rods from <b>Bards is more than 0.9 kN</b> than that from <b>Ardo</b>.</p>	<p>M1</p> <p>awrt 0.143 A1</p> <p>M1</p> <p>A1 (4)</p> <p>B1</p> <p>B1</p> <p>M1; A1</p> <p>A1cso (5)</p> <p>both B1</p> <p>1.761 B1</p> <p>M1 A1</p> <p>awrt <math>\pm 1.94</math> A1</p> <p>A1 ft (6)</p>
	<b>Notes</b>	<b>[15]</b>
<p><b>(a)(i)</b></p> <p><b>(ii)</b></p> <p><b>(b)</b></p> <p><b>(c)</b></p>	<p>M1 for attempt to calculate <math>s^2</math></p> <p>M1 use of correct formula for <math>s_p^2</math>  A1 awrt 0.349 / 0.3495</p> <p>1<sup>st</sup> B1 allow <math>H_0 : \sigma_1 = \sigma_2, H_1 : \sigma_1 \neq \sigma_2</math></p> <p>M1 For use of a correct formula</p> <p>B1 must use <math>\mu</math>. If not use <math>A</math> and <math>B</math> it must be clear which is which</p> <p>M1 for attempt at correct test statistic – matching their hypotheses</p> <p>1<sup>st</sup> A1 correct test statistic for their hypotheses</p>	



Question Number	Scheme	Marks
5.	<p>D = Paper I score – paper II score</p> <p><math>H_0: \mu_D = 1</math>   <math>H_1: \mu_D &gt; 1</math></p> <p>d: 4, 1, 7, 3, -1, 1, 9, 2</p> $\bar{d} = 3.25; \quad s^2 = \frac{162 - 8 \times 3.25^2}{7} = 11.07.. \quad (s = 3.32)$ $t_7 = \frac{3.25 - 1}{\frac{3.32}{\sqrt{8}}} = 1.9126...$ <p style="text-align: right;">awrt 1.91</p> <p><math>t_7(5\%) = 1.895</math></p> <p>There is evidence to support the <b>teacher's</b> belief or the score on <b>paper I</b> is more than one mark <b>higher</b> than on <b>paper II</b></p>	<p>B1</p> <p>M1</p> <p>M1;M1</p> <p>M1A1</p> <p>B1</p> <p>A1 ft</p> <p><b>(8)</b></p> <p><b>[8]</b></p>
	<b>Notes</b>	
(a)	<p>1<sup>st</sup> M1 for attempting differences</p> <p>2<sup>nd</sup> M1 for attempting <math>\bar{d}</math></p> <p>3<sup>rd</sup> M1 for attempting <math>s_d</math> or <math>s_d^2</math>, correct expression with their <math>\sum d^2</math> and <math>\bar{d}</math> or correct calculation (to 2 sf or better)</p> <p>4<sup>th</sup> M1 for use of <math>\frac{\bar{d} - 1}{\frac{s}{\sqrt{8}}}</math>, ft their values.</p> <p>1<sup>st</sup> A1 awrt 1.91</p> <p>2<sup>nd</sup> B1 for 1.895</p> <p>2<sup>nd</sup> A1 contextual conclusion ft their values.</p> <p>SC if they use a 2 sample test they may get the first B1 for <math>H_0: \mu_I - \mu_{II} = 1</math> and <math>H_1: \mu_I - \mu_{II} &gt; 1</math></p>	

Question Number	Scheme	Marks
<p><b>6.</b></p> <p><b>(a)</b></p>	<p><math>H_0: \mu = 500</math> [accept <math>\leq 500</math>], <math>H_1: \mu &gt; 500</math></p> $t = \frac{502 - 500}{\sqrt{5.6} / \sqrt{12}} = 2.93$ <p>critical value <math>t_{11}(1\%) = 2.718</math></p> <p>sufficient evidence that the mean <b>amount</b> of <b>water</b> is more than <b>500</b> ml</p>	<p>B1</p> <p>M1A1</p> <p>B1</p> <p>A1 ft (5)</p>
<p><b>(b)</b></p>	<p><math>H_0: \sigma^2 = 9</math> or <math>(\sigma = 3)</math>, <math>H_1: \sigma^2 &lt; 9</math> or <math>(\sigma &lt; 3)</math></p> <p>test statistic <math>\frac{11s^2}{\sigma^2} =, \frac{61.6}{9} = 6.84</math></p> <p>critical values <math>\chi_{11}^2(1\%)</math> lower tail = 3.053</p> <p>Insufficient evidence to suggest that the <b>standard deviation</b> of the amount of <b>water</b> is less than 3</p>	<p>B1</p> <p>M1 A1</p> <p>B1</p> <p>A1cso (5)</p> <p>[10]</p>
<b>Notes</b>		
<p><b>(a)</b></p>	<p>M1 attempt at correct statistic</p>	
	<p>1<sup>st</sup> A1 awrt 2.93</p>	
	<p>2<sup>nd</sup> A1ft correct contextual comment including amount , water and 500</p>	
<p><b>(b)</b></p>	<p>1<sup>st</sup> B1 Both hypotheses, must use <math>\sigma</math></p>	
	<p>2<sup>nd</sup> B1 for critical value, this should be compatible with their alternative hypothesis</p>	
	<p>3<sup>rd</sup> A1cso cso. contextual comment, include standard deviation/ variance and water</p>	

Question Number	Scheme	Marks
<p><b>7.</b></p> <p><b>(a)</b></p> <p><b>(b)</b></p>	$\frac{CV - 202}{2/\sqrt{n}} = -2.3263$ $CR \leq 202 - \frac{4.6526}{\sqrt{n}} \quad \text{or} \quad 202 - 2.3263\sqrt{\frac{4}{n}}$ $\frac{CV - 200}{2/\sqrt{n}} = 1.6449 \quad \text{or} \quad \frac{2 - \frac{4.6526}{\sqrt{n}}}{2/\sqrt{n}} > 1.6449$ $CV = 200 + \frac{3.2898}{\sqrt{n}}$ <p>Solving simultaneously</p> $2 = \frac{7.9424}{\sqrt{n}} \quad \text{or} \quad \sqrt{n} - \frac{4.6526}{2} > 1.6449$ $\sqrt{n} = 3.9712$ $n = 15.77$ $n = 16$	<p>M1 B1</p> <p>A1</p> <p><b>(3)</b></p> <p>M1 B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p><b>(6)</b></p> <p><b>[9]</b></p>
	<b>Notes</b>	
<p><b>(a)</b></p> <p><b>(b)</b></p>	<p>Note only lose one B1 for not reading from points table. This should be deducted the first time it is done</p> <p>1<sup>st</sup> M1 use correct formula equal a z value</p> <p>A1 allow use of &lt;</p> <p>1<sup>st</sup> M1 use correct formula equal a z value</p> <p>B1 – if B mark lost in part (a) allow 1.64 or 1.65</p> <p>1<sup>st</sup> A1 awrt 3.97 may be implied by an answer of 15.77 <b>or</b> an answer of 16 and using 1.6449</p> <p>2<sup>nd</sup> A1 awrt 15.8 may be implied by an answer of 16</p>	

Question Number	Scheme	Marks
8.		
(a)	$E\left(\sum_{i=1}^n W_i\right) = n\mu$ $E(W_i^2) = \text{Var}(W_i) + (E(W_i))^2$ $= \sigma^2 + \mu^2$ $E\left(\sum_{i=1}^n W_i^2\right) = E(W_1^2 + W_2^2 + \dots + W_n^2)$ $= n(\sigma^2 + \mu^2)$	B1 M1 A1  A1 cso <b>(4)</b>
(b)	$E\left(\frac{1}{n} \sum_{i=1}^n W_i\right) = \frac{1}{n} E\left(\sum_{i=1}^n W_i\right)$ $= \mu$ $\text{Var}\left(\frac{1}{n} \sum_{i=1}^n W_i\right) = \frac{1}{n^2} \text{Var}(W_1 + W_2 + \dots + W_n)$ $= \frac{1}{n^2} n\sigma^2$ $= \frac{\sigma^2}{n}, \rightarrow 0 \text{ as } n \rightarrow \infty$	B1  B1, B1d <b>(3)</b>
(c)	$E\left[\frac{1}{n} \left(\sum w_i^2\right) - (\bar{w})^2\right] = \frac{1}{n} \times n(\sigma^2 + \mu^2) - E(\bar{w}^2)$ $\text{Var}(\bar{w}) = E(\bar{w}^2) - [E(\bar{w})]^2 \Rightarrow E(\bar{w}^2) - \mu^2 = \frac{\sigma^2}{n}$ <p>Hence expected value is <math>(\sigma^2 + \mu^2) - \frac{\sigma^2}{n} - \mu^2 = \frac{(n-1)\sigma^2}{n}</math></p> $\text{Bias} = (-) \frac{\sigma^2}{n}$	M1 M1 A1 A1 <b>(4)</b>
(d)	$\frac{n}{(n-1)} U$	B1  <b>(1)</b> <b>[12]</b>

	Notes	
<b>(a)</b>	1 <sup>st</sup> M1 using $E(W_i^2) = \text{Var}(W_i) + (E(W_i))^2$	
<b>(b)</b>	2 <sup>nd</sup> B1 stating $\text{Var}\left(\frac{1}{n} \sum_{i=1}^n W_i\right) = \frac{\sigma^2}{n}$	
	3 <sup>rd</sup> B1 dependent on 2 <sup>nd</sup> B1, stating $\frac{\sigma^2}{n} \rightarrow 0$ as $n \rightarrow \infty$	
<b>(c)</b>	1 <sup>st</sup> M1 attempting correct method with their answer to part (a) – award for $(\sigma^2 + \mu^2) - E\left(\frac{1}{n} \sum_{i=1}^n w_i\right)^2$	
<b>(d)</b>	2 <sup>nd</sup> M1 using $\text{Var}(\bar{w}) = E(\bar{w}^2) - [E(\bar{w})]^2$ Allow $\frac{n}{(n-1)}\sigma^2$	

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