

# Mark Scheme (Results)

## Summer 2008

GCE

### GCE Mathematics (6675/01)

**June 2008**  
**Further Pure Mathematics FP2**  
**Mark Scheme**

Question number	Scheme	Marks
1.	$\frac{d}{dx}(\ln(\tanh x)) = \frac{\operatorname{sech}^2 x}{\tanh x}$ $= \frac{1}{\sinh x \cosh x} = \frac{2}{\sinh 2x} = 2 \operatorname{cosech} 2x$ <p><b>Notes</b></p> <p><b>1M1</b> Any valid differentiation attempt including <math>\ln(e^x - e^{-x}) - \ln(e^x + e^{-x})</math></p> <p><b>1A1</b> c.a.o. (o.e e.g. <math>\frac{\cosh x}{\sinh x} - \frac{\sinh x}{\cosh x}</math> )</p> <p><b>2M1</b> Proceeding to a hyperbolic expression in <math>2x</math></p> <p><b>2A1</b> c.s.o.</p>	<p>M1 A1</p> <p>M1 A1 (*) (4)</p> <p style="text-align: right;"><b>4</b></p>

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2.	$8\left(\frac{e^x + e^{-x}}{2}\right) - 4\left(\frac{e^x - e^{-x}}{2}\right) = 13$ $4e^x + 4e^{-x} - 2e^x + 2e^{-x} = 13$ $2e^{2x} - 13e^x + 6 = 0 \quad (\text{or equiv.})$ $(2e^x - 1)(e^x - 6) = 0$ $e^x = \frac{1}{2}, \quad e^x = 6$ $x = \ln \frac{1}{2} \quad (\text{or } -\ln 2), \quad x = \ln 6$ <p><b>Notes</b></p> <p><b>B1</b> Correctly substituting exponentials for all hyperbolics</p> <p><b>1M1</b> To a three term quadratic in <math>e^x</math></p> <p><b>1A1</b> c.a.o. (o.e.)</p> <p><b>2M1</b> Solving their equation to <math>e^x =</math></p> <p><b>2A1ft</b> f.t. their equation.</p> <p><b>3A1</b> c.a.o.</p>	<p>B1</p> <p>M1 A1</p> <p>M1 A1ft</p> <p>A1 (6)</p> <p style="text-align: right;"><b>6</b></p>

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3.	$\int \frac{3}{\sqrt{x^2-9}} dx + \int \frac{x}{\sqrt{x^2-9}} dx$ $= \left[ 3 \operatorname{arcosh} \frac{x}{3} + \sqrt{x^2-9} \right]$ $= \left[ 3 \ln \left( \frac{x + \sqrt{x^2-9}}{3} \right) + \sqrt{x^2-9} \right]_5^6$ $= \left( 3 \ln \left( \frac{6 + \sqrt{27}}{3} \right) + \sqrt{27} \right) - \left( 3 \ln \left( \frac{5+4}{3} \right) + 4 \right)$ $= 3 \ln \frac{6 + \sqrt{27}}{9} + \sqrt{27} - 4 = 3 \ln \frac{2 + \sqrt{3}}{3} + 3\sqrt{3} - 4$ <p><b>Notes</b></p> <p><b>B1</b> Correctly changing to an integrable form.  <b>1M1</b> Complete attempt to integrate at least one bit.  <b>1A1</b> One term correct  <b>2A1</b> All correct  <b>2DM1</b> Substituting limits in all. <b>Must have got first M1</b>  <b>3A1</b> Correctly (no follow through)  <b>4A1</b> c.s.o.</p>	<p>B1</p> <p>M1 A1 A1</p> <p>M1 A1</p> <p>A1 (7)</p> <p style="text-align: right;">7</p>

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4.	<p>(a) <math>\frac{dy}{dx} = \frac{3x^2}{\sqrt{1+x^6}}</math>,                      At <math>x = \sqrt{2}</math>                      <math>\frac{dy}{dx} = \frac{6}{3} = 2</math></p> <p><math>y - \operatorname{arsinh}(2\sqrt{2}) = 2(x - \sqrt{2})</math></p> <p><math>y = 2x - 2\sqrt{2} + \ln(3 + 2\sqrt{2})</math>                      (*)</p> <p>(b) <math>\frac{3a^2}{\sqrt{1+a^6}} = 2</math>                      <math>9a^4 = 4(1+a^6)</math></p> <p><math>4a^6 - 9a^4 + 4 = 0</math>                      <math>(a^2 - 2)(4a^4 - a^2 - 2) = 0</math></p> <p><math>a^2 = \frac{1 \pm \sqrt{1+32}}{8}</math>                      <math>a = \sqrt{\frac{1+\sqrt{33}}{8}} \approx 0.92</math></p>	<p>M1 A1, A1</p> <p>M1</p> <p>A1                      (5)</p> <p>M1 A1</p> <p>A1</p> <p>M1 A1                      (5)</p> <p style="text-align: right;"><b>10</b></p>
	<p><b>Notes</b></p> <p>(a)<b>1M1</b> Attempt to differentiate need <math>(1+x^6)^{-\frac{1}{2}}</math> at least</p> <p>    <b>1A1</b> correct</p> <p>    <b>2A1</b> c.a.o.</p> <p>    <b>2M1</b> Substituting into straight line equation (linear). Must use <math>x = \sqrt{2}</math></p> <p>    <b>3A1</b> c.s.o.</p> <p>(b)<b>1M1</b> Their derivative = their gradient (condone <math>x</math> throughout)</p> <p>    <b>2M1= A mark cao, any form</b></p> <p>    <b>1A1</b> quartic cao</p> <p>    <b>3M1</b> Solving their quartic to 'a' =</p> <p>    <b>2A1</b> c.a.o. (a.w.r.t. 0.92 to 2dp)</p>	

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5.	<p>(a) <math>I_n = \int_0^\pi e^x \sin^n x dx = [e^x \sin^n x] - \int e^x n \sin^{n-1} x \cos x dx</math></p> <p><math>[e^x \sin^n x - n e^x \sin^{n-1} x \cos x] + n \int e^x (-\sin^n x + (n-1) \cos x \sin^{n-2} x \cos x) dx</math></p> <p><math>[e^x \sin^n x - n e^x \sin^{n-1} x \cos x]_0^\pi = 0</math></p> <p><math>I_n = -n \int e^x \sin^n x dx + n(n-1) \int \sin^{n-2} x (1 - \sin^2 x) dx</math></p> <p><math>I_n = -n I_n + n(n-1) I_{n-2} - n(n-1) I_n \quad I_n = \frac{n(n-1)}{n^2+1} I_{n-2} \quad (*)</math></p>	<p>M1 A1</p> <p>M1 A1</p> <p>B1</p> <p>M1</p> <p>M1 A1 (8)</p>
	<p>(b) <math>I_4 = \frac{4 \times 3}{17} I_2, \quad = \frac{12}{17} \times \frac{2}{5} I_0</math></p> <p><math>I_0 = \int_0^\pi e^x dx = [e^x]_0^\pi = \dots, \quad I_4 = \frac{24}{85} (e^\pi - 1)</math></p>	<p>M1, A1</p> <p>M1, A1 (4)</p>
	<p><b>12</b></p> <p>(a) <b>1M1</b> Complete attempt to use parts once in the right direction need <math>\sin^{n-1} x</math>  <b>1A1</b> cao  <b>2M1</b> Attempt to use parts again with sensible choice of parts, not reversing. Need to be differentiating a product.  <b>2A1</b> cao  <b>1B1</b> both = 0 at some point. (doesn't need to be correct, must =0)  <b>3DM1</b> <math>I_n =</math> expressions in <math>\int e^x \sin^k x dx</math> <b>Depends on 2<sup>nd</sup> M</b>  <b>4DM1</b> Expression in <math>I_n</math> and <math>I_{n-2}</math> to <math>I_n =</math>. <b>Depends on 3<sup>rd</sup> M</b>  <b>3A1</b> c.s.o.</p> <p>(b) <b>1M1</b> <math>I_4</math> in terms of <math>I_2</math>  <b>1A1</b> <math>I_4</math> correctly in terms of <math>I_0</math> [ o.e.]  <b>2M1</b> <math>\int e^x dx</math>  <b>2A1</b> c.a.o for <math>I_4</math> .</p>	

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6.	<p>(a) <math>\int \cosh x \arctan(\sinh x) dx = \sinh x \arctan(\sinh x) - \int \sinh x \frac{\cosh x}{1 + \sinh^2 x} dx</math></p> <p style="margin-left: 40px;"><math>= \sinh x \arctan(\sinh x) - \frac{1}{2} \ln(1 + \sinh^2 x) (+C)</math></p> <p>Or: <span style="margin-left: 100px;">.....</span> <math>- \int \tanh x dx</math></p> <p style="margin-left: 40px;"><math>= \sinh x \arctan(\sinh x) - \ln(\cosh x) (+C)</math> <span style="float: right;">M1 A1</span></p> <p><u>Alternative:</u></p> <p>Let <math>t = \sinh x</math>, <math>\frac{dt}{dx} = \cosh x</math>, <math>\int \arctan t dt = t \arctan t - \int \frac{t}{1+t^2} dt</math> <span style="float: right;">M1 A1 A1</span></p> <p style="margin-left: 150px;"><math>= \dots - \frac{1}{2} \ln(1+t^2)</math> <span style="float: right;">M1</span></p> <p style="margin-left: 40px;"><math>= \sinh x \arctan(\sinh x) - \frac{1}{2} \ln(1 + \sinh^2 x) (+C)</math> (or equiv.) <span style="float: right;">A1</span></p>	<p>M1 A1 A1</p> <p>M1 A1 (5)</p> <p>M1 A1</p> <p>M1 A1 A1</p> <p>M1</p> <p>A1</p>
	<p>(b) <math>\frac{1}{10} [\sinh x \arctan(\sinh x) - \ln(\cosh x)]_0^2 = \dots, \quad 0.34 \quad (*)</math></p>	<p>M1, A1 (2)</p> <p style="text-align: right;"><b>7</b></p>
	<p>(a) <u>Alternative:</u></p> <p>Let <math>\tan t = \sinh x</math>, <math>\sec^2 t \frac{dt}{dx} = \cosh x</math>, <math>\int t \sec^2 t dt = t \tan t - \int \tan t dt</math> <span style="float: right;">M1 A1 A1</span></p> <p style="margin-left: 150px;"><math>= \dots - \ln(\sec t)</math> <span style="float: right;">M1</span></p> <p style="margin-left: 40px;"><math>= \sinh x \arctan(\sinh x) - \ln \sqrt{1 + \sinh^2 x} (+C)</math> (or equiv.) <span style="float: right;">A1</span></p> <p><b>Notes</b></p> <p><b>(a)1M1</b> Complete attempt to use parts</p> <p style="margin-left: 20px;"><b>1A1</b> One term correct.</p> <p style="margin-left: 20px;"><b>2A1</b> All correct.</p> <p style="margin-left: 20px;"><b>2M1</b> All integration completed. Need a ln term.</p> <p style="margin-left: 20px;"><b>3A1</b> c.a.o. ( in x) o.e, any correct form, simplified or not</p> <p><b>(b)1M1</b> Use of limits 0 and 2 and 1/10.</p> <p style="margin-left: 20px;"><b>1A1</b> c.s.o.</p>	

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7.	<p>(a) <math>\frac{2x}{16} - \frac{2y}{9} \frac{dy}{dx} = 0</math> <span style="float: right;"><math>\left[ \frac{dx}{dt} = 4 \sec t \tan t, \frac{dy}{dt} = 3 \sec^2 t \right]</math></span></p> <p><math>\frac{dy}{dx} = \frac{9x}{16y} = \frac{36 \sec t}{48 \tan t} = \frac{3}{4 \sin t}</math></p> <p><math>y - 3 \tan t = \frac{-4 \sin t}{3}(x - 4 \sec t)</math></p> <p><math>4x \sin t + 3y = 25 \tan t</math> <span style="float: right;">(*)</span></p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (6)</p>
	<p>(b) Using <math>b^2 = a^2(e^2 - 1)</math>: <math>ae = \sqrt{a^2 + b^2} = 5</math> or <math>e = \frac{5}{4}</math></p> <p><math>P: 4 \sec t = 5 \quad \cos t = \frac{4}{5}</math></p> <p>Coordinates of <math>P: (4 \sec t, 3 \tan t) = \left(5, \frac{9}{4}\right)</math></p>	<p>M1 A1</p> <p>M1</p> <p>M1 A1 (5)</p>
	<p>(c) <math>R: x = \frac{25 \tan t}{4 \sin t} = \frac{125}{16}</math></p> <p>Area of <math>PRS</math>: <math>\frac{1}{2}(SR \times SP) = \frac{1}{2} \times \left(\frac{125}{16} - 5\right) \times \frac{9}{4} = \frac{405}{128} \left(= 3 \frac{21}{128}\right)</math></p>	<p>M1</p> <p>M1 A1 (3)</p>
	<p><b>Notes</b></p> <p>(a) <b>1M1</b> Differentiating  <b>1A1</b> c.a.o.  <b>2M1</b> <math>\frac{dy}{dx}</math> in terms of <math>t</math>.  <b>2A1</b> c.a.o.  <b>3M1</b> Substituting gradient of <b>normal</b> into straight line equation.  <b>3A1</b> c.s.o.</p> <p>(b) <b>1M1</b> Use of <math>b^2 = a^2(e^2 - 1)</math>  <b>1A1</b> c.a.o. for <math>ae</math> or for <math>e</math>  <b>2M1</b> Using <math>x</math> coordinate of focus = <math>x</math> coordinate of <math>P</math>, to get single term <math>f(t)</math> = constant. (<b>Allow recovery in (c)</b>)  <b>3M1</b> Substituting into <math>P</math> coordinates to a number for <math>x</math> and for <math>y</math>.  <b>2A1</b> c.a.o.</p> <p>(c) <b>1M1</b> Attempt to find <math>x</math> coordinate of <math>R</math>.  <b>2M1</b> Substituting into correct template i.e. <math>\frac{1}{2} \times  \text{their } R_x - \text{their } H_x  \times \text{their } P_y</math>  <b>1A1</b> c.a.o. 3 s.f. or better.</p>	<p><b>14</b></p>



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8.	<p>(a) <math>\dot{x} = 3 + 3\cos t \quad \dot{y} = 3\sin t</math></p> $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{\sin t}{1 + \cos t} = \frac{2\sin \frac{t}{2} \cos \frac{t}{2}}{2\cos^2 \frac{t}{2}} = \tan \frac{t}{2} \quad (*)$ <p>(b) <math>s = \int \sqrt{\dot{x}^2 + \dot{y}^2} dt = 3\sqrt{2} \int \sqrt{1 + \cos t} dt</math></p> $= 6 \int_0^t \cos \frac{t}{2} dt = 12 \sin \frac{t}{2} \quad (\text{Limits or establish } C = 0 \text{ for A1}) \quad (*)$ <p>(c) <math>\tan \psi = \tan \frac{t}{2} \Rightarrow \psi = \frac{t}{2} \Rightarrow s = 12 \sin \psi</math></p> <p>(d) Surface area = <math>\int_0^t 2\pi y \sqrt{\dot{x}^2 + \dot{y}^2} dt = 18\sqrt{2}\pi \int (1 - \cos t) \sqrt{1 + \cos t} dt</math></p> $= 72\pi \int \sin^2 \frac{t}{2} \cos \frac{t}{2} dt$ $= \dots \dots \dots \left( \frac{2}{3} \sin^3 \frac{t}{2} \right)$ <p>But <math>\sin \frac{t}{2} = \frac{s}{12} = \frac{L}{12}</math>, so surface area = <math>\frac{144\pi}{3} \times \frac{L^3}{12^3} = \frac{\pi L^3}{36} \quad (*)</math></p> <p><b>(a) 1B1</b> both  <b>1M1</b> Attempt at <math>y'/x'</math>  <b>1A1</b> cso – on paper need to see half angles</p> <p><b>(b) 1M1</b> Attempt at arc length, integral formula  <b>1A1</b> cao follow through on their <math>x'</math> and <math>y'</math> <b>one variable only</b>  <b>2M1</b> Integrating  <b>2A1</b> cso – on paper</p> <p><b>(c) 1B1</b> cao</p> <p><b>(d) 1M1</b> Attempt at Surface area, integral formula. Condone lack of <math>2\pi</math>.  <b>1A1</b> cao follow through on their <math>x'</math> and <math>y'</math> condone lack of <math>2\pi</math>. <b>one variable only</b>  <b>2DM1</b> Getting to integrable form condone lack of <math>2\pi</math>. <b>Depends on previous M mark.</b>  <b>3DM1</b> integrating condone lack of <math>2\pi</math>. <b>Depends on previous M mark.</b>  <b>2A1</b> cao  <b>4DM1</b> Eliminating <math>t</math> to give expression in <math>L</math> only <b>Depends on previous M mark.</b>  <b>3A1</b> cso – on paper.</p>	<p>B1</p> <p>M1 A1 (3)</p> <p>M1 A1ft</p> <p>M1 A1 (4)</p> <p>B1 (1)</p> <p>M1 A1ft</p> <p>M1</p> <p>M1 A1</p> <p>M1 A1 (7)</p>