

# Mark Scheme (Results) January 2010

GCE

## Further Pure Mathematics FP2 (6675)

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Question Number	Scheme	Marks
Q1	<p>(a) <math>\frac{x^2}{4a^2} - \frac{y^2}{a^2} = 1</math></p> <p>Using “<math>b^2 = a^2(e^2 - 1)</math>”</p> $a^2 = 4a^2(e^2 - 1)$ <p>Leading to <math>e = \frac{\sqrt{5}}{2}</math></p> <p>(b) “<math>x = \frac{a}{e}</math>” <math>\Rightarrow x = \frac{2a}{e}</math></p> $a = \frac{ex}{2} = \frac{5\sqrt{5}}{2}$ <p style="text-align: right;">ft their <math>e</math></p>	<p>M1 A1</p> <p>A1 (3)</p> <p>M1 A1ft (2)</p> <p>[5]</p>
Q2	<p><math>\rho = \frac{ds}{d\psi} = \tan \psi</math></p> $s = \int \tan \psi \, d\psi = \ln \sec \psi \quad (+A)$ <p>At <math>s = 0, \psi = \frac{\pi}{4}</math> <math>0 = \ln \sqrt{2} + A \Rightarrow A = -\ln \sqrt{2}</math></p> $\left( s = \ln \left( \frac{\sec \psi}{\sqrt{2}} \right) \right)$	<p>B1</p> <p>M1 A1</p> <p>M1 A1 (5)</p> <p>[5]</p>

Question Number	Scheme	Marks
Q3	<p>(a) <math display="block">2 \sinh x \cosh x = 2 \left( \frac{e^x - e^{-x}}{2} \right) \left( \frac{e^x + e^{-x}}{2} \right)</math> <math display="block">= \frac{e^{2x} - e^{-2x}}{2}</math> <math display="block">= \sinh 2x \quad *</math></p> <p>(b) <math display="block">x = 0</math> <math display="block">2 \sinh x \cosh x = 6 \sinh^2 x + 7 \sinh x</math> <math display="block">2 \cosh x = 6 \sinh x + 7</math> <math display="block">e^x + e^{-x} = 3e^x - 3e^{-x} + 7</math> <math display="block">2e^{2x} + 7e^x - 4 = 0</math> <math display="block">(2e^x - 1)(e^x + 4) = 0</math> <math display="block">e^x = \frac{1}{2}</math> <math display="block">x = -\ln 2</math></p>	<p>M1 A1 cso (2)</p> <p>B1 M1 M1 A1 M1 A1 A1 (7) [9]</p>

Question Number	Scheme	Marks
Q4	<p>(a) <math>I_n = \int 1 \cdot (\ln x)^n dx</math>  <math>= x(\ln x)^n - \int x \cdot n(\ln x)^{n-1} \times \frac{1}{x} dx</math>  <math>= x(\ln x)^n - n \int (\ln x)^{n-1} dx</math>  <math>= n(\ln x)^n - nI_{n-1}</math> *</p> <p>(b) <math>I_3 = x(\ln x)^3 - 3I_2</math>  <math>= x(\ln x)^3 - 3(x(\ln x)^2 - 2I_1)</math>  <math>= x(\ln x)^3 - 3x(\ln x)^2 + 6I_1</math>  <math>= x(\ln x)^3 - 3x(\ln x)^2 + 6(x \ln x - I_0)</math>  <math>= x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x - 6x</math>            With the limits 1 and e  <math>I_3 = e - 3e + 6e - 6e + 6 = 6 - 2e</math></p>	<p>M1 A1 A1            cso A1 (4)            M1            M1            M1 A1            M1 A1 (6)            [10]</p>
	<p><i>If <math>I_1</math> is integrated directly</i></p> <p>(b) <math>I_3 = x(\ln x)^3 - 3I_2</math>  <math>= x(\ln x)^3 - 3(x(\ln x)^2 - 2I_1)</math>  <math>= x(\ln x)^3 - 3x(\ln x)^2 + 6I_1</math>  <math>I_1 = \int 1 \cdot \ln x dx = x \ln x - \int x \times \frac{1}{x} dx = x \ln x - x</math> Allow if quoted  <math>(I_3 = x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x - 6x)</math>            With the limits 1 and e  <math>I_3 = e - 3e + 6e - 6e + 6 = 6 - 2e</math></p>	<p>M1            M1            M1 A1            M1 A1 (6)</p>

Question Number	Scheme	Marks
Q5	<p>(a)</p> $\frac{dy}{dx} = 3 \sinh 3x - 4$ $3 \sinh 3x - 4 = 0$ $\sinh 3x = \frac{4}{3}$ $3x = \operatorname{arsinh} \frac{4}{3}$ $x = \frac{1}{3} \operatorname{arsinh} \frac{4}{3}$ $= \frac{1}{3} \ln \left[ \frac{4}{3} + \sqrt{\left(\frac{4}{3}\right)^2 + 1} \right] = \frac{1}{3} \ln 3$ <p>(b)</p> $\int (\cosh 3x - 4x) dx = \frac{\sinh 3x}{3} - 2x^2$ $A = \left[ \frac{\sinh 3x}{3} - 2x^2 \right]_0^{\frac{1}{3} \ln 3} = \frac{\sinh(\ln 3)}{3} - \frac{2(\ln 3)^2}{9}$ $= \frac{e^{\ln 3} - e^{-\ln 3}}{6} - \dots = \frac{3 - \frac{1}{3}}{6} - \dots$ $= \frac{4}{9} - \frac{2(\ln 3)^2}{9}$ $= \frac{2}{9} [2 - (\ln 3)^2] \quad *$	<p>M1 A1</p> <p>M1</p> <p>M1</p> <p>A1 (5)</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>cso A1 (6)</p> <p>[11]</p>

Question Number	Scheme	Marks
Q6	<p>(a) <math display="block">x = \frac{a}{u} \Rightarrow \frac{dx}{du} = -\frac{a}{u^2}</math></p> $\int \frac{1}{x\sqrt{(a^2-x^2)}} dx = \int \frac{1}{\frac{a}{u}\sqrt{\left(a^2-\frac{a^2}{u^2}\right)}} \left(-\frac{a}{u^2}\right) du$ $= -\frac{1}{a} \int \frac{1}{\sqrt{(u^2-1)}} du$ $= -\frac{1}{a} \operatorname{arcosh} u \quad (+A)$ $= -\frac{1}{a} \operatorname{arcosh}\left(\frac{a}{x}\right) \quad (+A)$ <p>(b) <math display="block">\int \frac{1}{x\sqrt{(25-x^2)}} dx = -\frac{1}{5} \operatorname{arcosh}\left(\frac{5}{x}\right)</math></p> $\left[-\frac{1}{5} \operatorname{arcosh}\left(\frac{5}{x}\right)\right]_3^4 = \frac{1}{5} \left(\operatorname{arcosh}\left(\frac{5}{3}\right) - \operatorname{arcosh}\left(\frac{5}{4}\right)\right)$ $= \frac{1}{5} [\ln 3 - \ln 2]$ $= \frac{1}{5} \ln \frac{3}{2}$	<p>B1</p> <p>M1 A1</p> <p>M1</p> <p>M1</p> <p>A1 (6)</p> <p>M1</p> <p>M1 A1</p> <p>M1 A1 (5)</p> <p>[11]</p>
	<p><i>Alternative for part (a)</i></p> $x = \frac{a}{\cosh u} \Rightarrow \frac{dx}{du} = -\frac{\sinh u}{\cosh^2 u}$ $\int \frac{1}{x\sqrt{(a^2-x^2)}} dx = \int \frac{1}{\frac{a}{\cosh u}\sqrt{\left(a^2-\left(\frac{a}{\cosh u}\right)^2\right)}} \left(-\frac{a \sinh u}{\cosh^2 u}\right) du$ $= -\frac{1}{a} \int 1 du = -\frac{1}{a} u \quad (+A)$ $= -\frac{1}{a} \operatorname{arcosh}\left(\frac{a}{x}\right) \quad (+A)$	<p>B1</p> <p>M1 A1</p> <p>M1 M1</p> <p>A1 (6)</p>

Question Number	Scheme	Marks
Q7	<p>(a)</p> $u = k \arcsin 2x, \frac{du}{dx} = \frac{2k}{\sqrt{(1-4x^2)}}$ $\frac{dy}{du} = \cos u$ $\frac{dy}{dx} = \cos u \times \frac{2k}{\sqrt{(1-4x^2)}}$ $\cos u = \sqrt{(1 - \sin^2 u)} = \sqrt{(1 - y^2)}$ $\frac{dy}{dx} = \sqrt{(1 - y^2)} \times \frac{2k}{\sqrt{(1-4x^2)}}$ $\sqrt{(1-4x^2)} \frac{dy}{dx} = 2k \sqrt{(1 - y^2)}$ $(1-4x^2) \left( \frac{dy}{dx} \right)^2 = 4k^2 (1 - y^2) *$ <p>Using <math>u = \arcsin 2x</math> is marked similarly.</p> <p>(b)</p> $-8x \left( \frac{dy}{dx} \right)^2 + (1-4x^2) 2 \frac{dy}{dx} \frac{d^2y}{dx^2} = -8k^2 y \frac{dy}{dx}$ $-4x \frac{dy}{dx} + (1-4x^2) \frac{d^2y}{dx^2} = -4k^2 y$ $(1-4x^2) \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 4k^2 y = 0 *$	<p>M1 A1</p> <p>A1</p> <p>cs0 M1 A1 (5)</p> <p>M1 A1 A1</p> <p>M1</p> <p>cs0 A1 (5)</p> <p>[10]</p>
	<p><i>Alternative for part (a)</i></p> $\arcsin y = k \arcsin 2x$ $\frac{1}{\sqrt{(1-y^2)}} \frac{dy}{dx} = \frac{2k}{\sqrt{(1-4x^2)}}$ $\sqrt{(1-4x^2)} \frac{dy}{dx} = 2k \sqrt{(1 - y^2)}$ $(1-4x^2) \left( \frac{dy}{dx} \right)^2 = 4k^2 (1 - y^2) *$	<p>M1 A1 A1</p> <p>cs0 M1 A1 (5)</p>



Question Number	Scheme	Marks
Q8	<p>(a) Eliminating <math>y</math> between <math>y = mx + c</math> and <math>y^2 = 4ax</math></p> $(mx + c)^2 = 4ax$ $m^2x^2 + 2(mc - 2a)x + c^2 = 0$ <p>For a tangent <math>“b^2 - 4ac = 0”</math></p> $4(mc - 2a)^2 - 4m^2c^2 = 0$ <p>Leading to <math>c = \frac{a}{m} *</math></p> <p>(b) <math>y = mx + \frac{a}{m}</math> passes through <math>(4a, 5a)</math></p> $5a = 4ma + \frac{a}{m}$ $4m^2 - 5m + 1 = 0$ $(4m - 1)(m - 1) = 0$ $m = \frac{1}{4}, 1$ $y = \frac{1}{4}x + 4a, \quad y = x + a$ <p>(c) Obtaining the points of contact between the tangents and the curve e.g. Substituting <math>m = \frac{1}{4}, c = 4a</math> into the quadratic in part (a)</p> $\frac{1}{16}x^2 - 2ax + 16a^2 = \left(\frac{x}{4} - 4a\right)^2 = 0 \Rightarrow x = 16a$ <p>Point has coordinates <math>(16a, 8a)</math></p> <p>Substituting <math>m = 1, c = a</math> into the quadratic in part (a)</p> $x^2 - 2ax + a^2 = (x - a)^2 = 0 \Rightarrow x = a$ <p>Point has coordinates <math>(a, 2a)</math></p> <p>With all methods, M1 is for finding the <math>x</math>- or <math>y</math>-coordinate of one point of contact. A1 is for any two coordinates correct. The second A1 is for all four coordinates correct.</p> $RQ^2 = (16a - a)^2 + (8a - 2a)^2 = 261a^2$ $RQ = a\sqrt{261} \quad (= 3a\sqrt{29})$	<p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1 (5)</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1 (5)</p> <p>[14]</p>

Question Number	Scheme	Marks
Q8	<p><i>Alternatives for part (a)</i></p> <p>① Let the point of contact of <math>l</math> and <math>C</math> be <math>(2at, at^2)</math></p> $m = \frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{2a}{2at} = \frac{1}{t}$ <p><math>l</math> has the form <math>y = \frac{1}{t}x + c</math></p> <p><math>(2at, at^2)</math> lies on <math>l</math> <math>2at = \frac{1}{t}at^2 + c \Rightarrow c = at</math></p> $c = at = \frac{a}{m} \quad *$ <p>② Eliminate <math>y</math> between <math>y = mx + \frac{a}{m}</math> and <math>y^2 = 4ax</math></p> $\left(mx + \frac{a}{m}\right)^2 = 4ax$ $m^2x^2 + 2ax + \frac{a^2}{m^2} = 4ax$ $m^2x^2 - 2ax + \frac{a^2}{m^2} = 0$ $\left(mx - \frac{a}{m}\right)^2 = 0$ <p>As this is a complete square, <math>l</math> is a tangent to <math>C</math> when <math>c = \frac{a}{m} \quad *</math></p> <p><i>Note:</i> Alternative ② allows the coordinates of <math>R</math> and <math>Q</math> in part (c) to be written down using <math>x = \frac{a}{m^2}</math>.</p> <p><i>Alternative for part (b)</i></p> $y = mx + \frac{a}{m} = \frac{x}{t} + at \text{ passes through } (4a, 5a)$ $5a = \frac{4a}{t} + at$ $t^2 - 5t + 4 = 0$ $(t-1)(t-4) = 0$ $t = 1, 4$ $y = \frac{1}{4}x + 4a, \quad y = x + a$	<p>M1</p> <p>A1</p> <p>M1</p> <p>cs0 A1 (4)</p> <p>M1 A1</p> <p>M1</p> <p>cs0 A1 (4)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>both A1 (5)</p>



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