

1. Find the set of values of x for which

$$\frac{3}{x+3} > \frac{x-4}{x} \tag{7}$$



2.

$$\frac{d^2 y}{dx^2} = e^x \left(2y \frac{dy}{dx} + y^2 + 1 \right)$$

(a) Show that

$$\frac{d^3 y}{dx^3} = e^x \left[2y \frac{d^2 y}{dx^2} + 2 \left(\frac{dy}{dx} \right)^2 + ky \frac{dy}{dx} + y^2 + 1 \right],$$

where k is a constant to be found.

(3)

Given that, at $x = 0$, $y = 1$ and $\frac{dy}{dx} = 2$,

(b) find a series solution for y in ascending powers of x , up to and including the term in x^3 .

(4)



3. Find the general solution of the differential equation

$$x \frac{dy}{dx} + 5y = \frac{\ln x}{x}, \quad x > 0$$

giving your answer in the form $y = f(x)$.

(8)



5. The point P represents the complex number z on an Argand diagram, where

$$|z - i| = 2$$

The locus of P as z varies is the curve C .

(a) Find a cartesian equation of C .

(2)

(b) Sketch the curve C .

(2)

A transformation T from the z -plane to the w -plane is given by

$$w = \frac{z+i}{3+iz}, \quad z \neq 3i$$

The point Q is mapped by T onto the point R . Given that R lies on the real axis,

(c) show that Q lies on C .

(5)



Question 5 continued

Blank area for writing the answer to Question 5, consisting of numerous horizontal lines.



6.

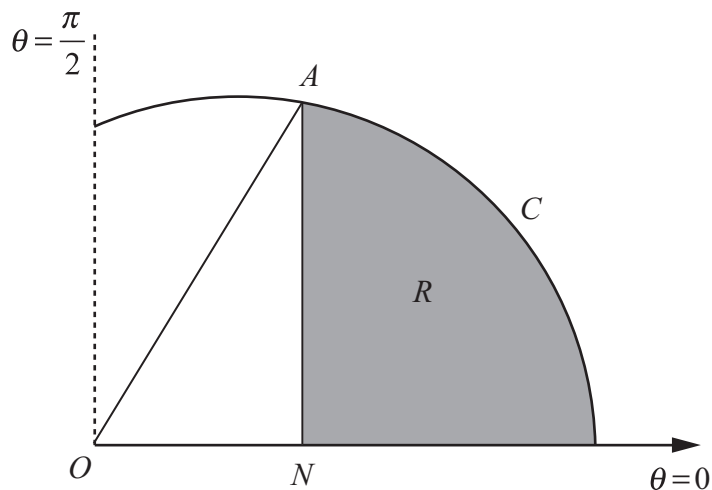


Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 2 + \cos \theta, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

At the point A on C , the value of r is $\frac{5}{2}$.

The point N lies on the initial line and AN is perpendicular to the initial line.

The finite region R , shown shaded in Figure 1, is bounded by the curve C , the initial line and the line AN .

Find the exact area of the shaded region R .

(9)



8. The differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = \cos 3t, \quad t \geq 0$$

describes the motion of a particle along the x -axis.

(a) Find the general solution of this differential equation. **(8)**

(b) Find the particular solution of this differential equation for which, at $t = 0$,

$$x = \frac{1}{2} \text{ and } \frac{dx}{dt} = 0. \quad \text{(5)}$$

On the graph of the particular solution defined in part (b), the first turning point for $t > 30$ is the point A .

(c) Find approximate values for the coordinates of A . **(2)**



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