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Mark Scheme (Results)

January 2011

GCE

GCE Further Pure Mathematics FP1 (6667) Paper 1

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General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol √ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - ⌊ The second mark is dependent on gaining the first mark

January 2011
Further Pure Mathematics FP1 6667
Mark Scheme

Question Number	Scheme		Marks
1.	$z = 5 - 3\text{i},\ \ w = 2 + 2\text{i}$		
(a)	$z^2 = (5-3\text{i})(5-3\text{i})$		
	$= 25 - 15\text{i} - 15\text{i} + 9\text{i}^2$ $= 25 - 15\text{i} - 15\text{i} - 9$	An attempt to multiply out the brackets to give four terms (or four terms implied). *zw* is M0	M1
	$= 16 - 30\text{i}$	$16 - 30\text{i}$ Answer only 2/2	A1 (2)
(b)	$\dfrac{z}{w} = \dfrac{(5-3\text{i})}{(2+2\text{i})}$		
	$= \dfrac{(5-3\text{i})}{(2+2\text{i})} \times \dfrac{(2-2\text{i})}{(2-2\text{i})}$	Multiplies $\dfrac{z}{w}$ by $\dfrac{(2-2\text{i})}{(2-2\text{i})}$	M1
	$= \dfrac{10 - 10\text{i} - 6\text{i} - 6}{4 + 4}$	Simplifies realising that a real number is needed on the denominator and applies $\text{i}^2 = -1$ on their numerator expression and denominator expression.	M1
	$= \dfrac{4 - 16\text{i}}{8}$		
	$= \dfrac{1}{2} - 2\text{i}$	$\dfrac{1}{2} - 2\text{i}$ or $a = \dfrac{1}{2}$ and $b = -2$ or equivalent Answer as a single fraction A0	A1 (3) **[5]**

Question Number	Scheme		Marks
2.			
(a)	$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$		
	$\mathbf{AB} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$		
	$= \begin{pmatrix} 2(-3)+0(5) & 2(-1)+0(2) \\ 5(-3)+3(5) & 5(-1)+3(2) \end{pmatrix}$	A correct method to multiply out two matrices. Can be implied by two out of four correct elements.	M1
	$= \begin{pmatrix} -6 & -2 \\ 0 & 1 \end{pmatrix}$	Any three elements correct	A1
		Correct answer	A1
		Correct answer only 3/3	(3)
(b)	Reflection; about the y-axis.	Reflection	M1
		y-axis (or $x = 0$.)	A1
			(2)
(c)	$\mathbf{C}^{100} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or $\mathbf{I}$	B1
			(1)
			[6]

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Question Number	Scheme		Marks
3.	$f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, \quad x \geqslant 0$		
(a)	$f(1.6) = -1.29543081...$	awrt -1.30	B1
	$f(1.8) = 0.5401863372...$	awrt 0.54	B1
	$\dfrac{\alpha - 1.6}{"1.29543081..."} = \dfrac{1.8 - \alpha}{"0.5401863372..."}$	Correct linear interpolation method with signs correct. Can be implied by working below.	M1
	$\alpha = 1.6 + \left(\dfrac{"1.29543081..."}{"0.5401863372..." + "1.29543081..."}\right)0.2$		
	$= 1.741143899...$	awrt 1.741 Correct answer seen 4/4	A1 (4)
(b)	$f'(x) = 10x - 6x^{\frac{1}{2}}$	At least one of $\pm ax$ or $\pm bx^{\frac{1}{2}}$ correct. Correct differentiation.	M1 A1 (2)
(c)	$f(1.7) = -0.4161152711...$	$f(1.7) =$ awrt -0.42	B1
	$f'(1.7) = 9.176957114...$	$f'(1.7) =$ awrt 9.18	B1
	$\alpha_2 = 1.7 - \left(\dfrac{"-0.4161152711..."}{"9.176957114..."}\right)$	Correct application of Newton-Raphson formula using their values.	M1
	$= 1.745343491...$		
	$= 1.745 \ (3\text{dp})$	1.745 Correct answer seen 4/4	A1 **cao** (4) **[10]**

Question Number	Scheme		Marks
4.	$z^2 + pz + q = 0$, $z_1 = 2 - 4\text{i}$		
(a)	$z_2 = 2 + 4\text{i}$	$2 + 4\text{i}$	B1 (1)
(b)	$(z - 2 + 4\text{i})(z - 2 - 4\text{i}) = 0$		
	$\Rightarrow z^2 - 2z - 4\text{i}z - 2z + 4 - 8\text{i} + 4\text{i}z - 8\text{i} + 16 = 0$	An attempt to multiply out brackets of two complex factors and no i^2.	M1
	$\Rightarrow z^2 - 4z + 20 = 0$	Any one of $p = -4$, $q = 20$.	A1
		Both $p = -4$, $q = 20$.	A1
		$\Rightarrow z^2 - 4z + 20 = 0$ only 3/3	(3) **[4]**

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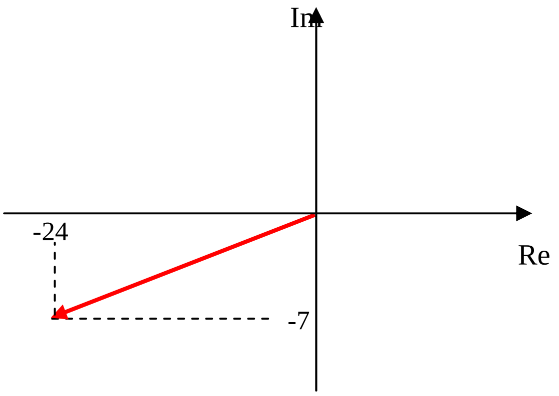
Question Number	Scheme	Marks
5	$\sum_{r=1}^{n} r(r+1)(r+5)$	
(a)	$= \sum_{r=1}^{n} r^3 + 6r^2 + 5r$ Multiplying out brackets and an attempt to use at least one of the standard formulae correctly.	M1
	$= \underline{\frac{1}{4}n^2(n+1)^2 + 6.\frac{1}{6}n(n+1)(2n+1) + 5.\frac{1}{2}n(n+1)}$ <u>Correct expression.</u>	A1
	$= \frac{1}{4}n^2(n+1)^2 + n(n+1)(2n+1) + \frac{5}{2}n(n+1)$	
	$= \frac{1}{4}n(n+1)\left(n(n+1) + 4(2n+1) + 10\right)$ Factorising out at least $n(n+1)$	dM1
	$= \frac{1}{4}n(n+1)\left(n^2 + n + 8n + 4 + 10\right)$	
	$= \frac{1}{4}n(n+1)\left(n^2 + 9n + 14\right)$ Correct 3 term quadratic factor	A1
	$= \frac{1}{4}n(n+1)(n+2)(n+7)$ * Correct proof. No errors seen.	A1 (5)
(b)	$S_n = \sum_{r=20}^{50} r(r+1)(r+5)$	
	$= S_{50} - S_{19}$	
	$= \frac{1}{4}(50)(51)(52)(57) - \frac{1}{4}(19)(20)(21)(26)$ Use of $S_{50} - S_{19}$	M1
	$= 1889550 - 51870$	
	$= 1837680$ 1837680 Correct answer only 2/2	A1
		(2) **[7]**

Question Number	Scheme		Marks
6.	$C: y^2 = 36x \Rightarrow a = \frac{36}{4} = 9$		
(a)	$S(9, 0)$	$(9, 0)$	B1 (1)
(b)	$x + 9 = 0$ or $x = -9$	$x + 9 = 0$ or $x = -9$ or ft using their a from part (a).	B1 $\sqrt{}$ (1)
(c)	$PS = 25 \Rightarrow \underline{QP = 25}$	Either 25 by itself or $PQ = 25$. Do not award if just $PS = 25$ is seen.	B1 (1)
(d)	x-coordinate of $P \Rightarrow x = 25 - 9 = 16$	$x = 16$	B1 $\sqrt{}$
	$y^2 = 36(16)$	Substitutes their x-coordinate into equation of C.	M1
	$\underline{y} = \sqrt{576} = \underline{24}$	$\underline{y = 24}$	A1 (3)
	Therefore $P(16, 24)$		
(e)	Area $OSPQ = \frac{1}{2}(9 + 25)24$	$\frac{1}{2}(\text{their } a + 25)(\text{their } y)$ or rectangle and 2 distinct triangles, correct for their values.	M1
	$= \underline{408}$ (units)^2	408	A1 (2) **[8]**

Question Number	Scheme		Marks
7. **(a)**		Correct quadrant with $(-24, -7)$ indicated.	B1
			(1)
(b)	$\arg z = -\pi + \tan^{-1}\left(\frac{7}{24}\right)$	$\tan^{-1}\left(\frac{7}{24}\right)$ or $\tan^{-1}\left(\frac{24}{7}\right)$	M1
	$= -2.857798544... = -2.86$ (2 dp)	awrt -2.86 or awrt 3.43	A1
			(2)
(c)	$\lvert w\rvert = 4$, $\arg w = \frac{5\pi}{6} \Rightarrow r = 4$, $\theta = \frac{5\pi}{6}$		
	$w = r\cos\theta + \mathrm{i}\, r\sin\theta$		
	$w = 4\cos\left(\frac{5\pi}{6}\right) + 4\mathrm{i}\sin\left(\frac{5\pi}{6}\right)$	Attempt to apply $r\cos\theta + \mathrm{i}\, r\sin\theta$. Correct expression for w.	M1 A1
	$= 4\left(\frac{-\sqrt{3}}{2}\right) + 4\mathrm{i}\left(\frac{1}{2}\right)$		
	$= -2\sqrt{3} + 2\mathrm{i}$	either $-2\sqrt{3} + 2\mathrm{i}$ or awrt $-3.5 + 2\mathrm{i}$	A1
			(3)
	$a = -2\sqrt{3}$, $b = 2$		
(d)	$\lvert z\rvert = \sqrt{(-24)^2 + (-7)^2} = \underline{25}$	$\underline{\lvert z\rvert = 25}$ or $zw = (48\sqrt{3} + 14) + (14\sqrt{3} - 48)\mathrm{i}$ or awrt 97.1-23.8i	B1
	$\lvert zw\rvert = \lvert z\rvert \times \lvert w\rvert = (25)(4)$	Applies $\lvert z\rvert \times \lvert w\rvert$ or $\lvert zw\rvert$	M1
	$= \underline{100}$	$\underline{100}$	A1
			(3) **[9]**

Question Number	Scheme		Marks
8.	$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$		
(a)	$\det \mathbf{A} = 2(3) - (-1)(-2) = 6 - 2 = \underline{4}$	$\underline{4}$	B1 (1)
(b)	$\mathbf{A}^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	$\frac{1}{\det \mathbf{A}}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	M1
		$\frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	A1 (2)
(c)	$\text{Area}(R) = \frac{72}{4} = \underline{18}\text{ (units)}^2$	$\frac{72}{\text{their }\det \mathbf{A}}$ or $72(\text{their }\det \mathbf{A})$	M1
		$\underline{18}$ or ft answer.	A1 $\sqrt{}$ (2)
(d)	$\mathbf{AR} = \mathbf{S} \Rightarrow \mathbf{A}^{-1}\mathbf{AR} = \mathbf{A}^{-1}\mathbf{S} \Rightarrow \mathbf{R} = \mathbf{A}^{-1}\mathbf{S}$		
	$\mathbf{R} = \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 8 & 12 \\ 4 & 16 & 4 \end{pmatrix}$	At least one attempt to apply $\mathbf{A}^{-1}$ by any of the three vertices in $\mathbf{S}$.	M1
	$= \frac{1}{4}\begin{pmatrix} 8 & 56 & 44 \\ 8 & 40 & 20 \end{pmatrix}$		
	$= \begin{pmatrix} 2 & 14 & 11 \\ 2 & 10 & 5 \end{pmatrix}$	At least one correct **column** o.e.	A1 $\sqrt{}$
		At least two correct **columns** o.e.	A1
	Vertices are $(2, 2)$, $(14, 10)$ and $(11, 5)$.	All three **coordinates** correct.	A1 (4) **[9]**

Question Number	Scheme		Marks
9.	$u_{n+1} = 4u_n + 2$, $u_1 = 2$ and $u_n = \frac{2}{3}(4^n - 1)$		
	$n=1$; $u_1 = \frac{2}{3}(4^1 - 1) = \frac{2}{3}(3) = 2$ So u_n is true when $n=1$.	Check that $u_n = \frac{2}{3}(4^n - 1)$ yields 2 when $n=1$.	B1
	Assume that for $n=k$ that, $u_k = \frac{2}{3}(4^k - 1)$ is true for $k \in \mathbb{Z}^+$.		
	Then $u_{k+1} = 4u_k + 2$		
	$= 4\left(\frac{2}{3}(4^k - 1)\right) + 2$	Substituting $u_k = \frac{2}{3}(4^k - 1)$ into $u_{n+1} = 4u_n + 2$.	M1
	$= \frac{8}{3}(4)^k - \frac{8}{3} + 2$	An attempt to multiply out the brackets by 4 or $\frac{8}{3}$	M1
	$= \frac{2}{3}(4)(4)^k - \frac{2}{3}$		
	$= \frac{2}{3}4^{k+1} - \frac{2}{3}$		
	$= \frac{2}{3}(4^{k+1} - 1)$	$\frac{2}{3}(4^{k+1} - 1)$	A1
	Therefore, the general statement, $u_n = \frac{2}{3}(4^n - 1)$ is true when $n = k+1$. (As u_n is true for $n=1$,) then u_n is true for all positive integers by mathematical induction	Require 'True when n=1', 'Assume true when n=k' and 'True when $n = k+1$ ' then true for all n o.e.	A1
			(5) **[5]**

Question Number	Scheme		Marks
10.	$xy = 36$ at $\left(6t, \frac{6}{t}\right)$.		
(a)	$y = \frac{36}{x} = 36x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -36x^{-2} = -\frac{36}{x^2}$	An attempt at $\frac{\mathrm{d}y}{\mathrm{d}x}$. or $\frac{\mathrm{d}y}{\mathrm{d}t}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}$	M1
	At $\left(6t, \frac{6}{t}\right)$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{36}{(6t)^2}$	An attempt at $\frac{\mathrm{d}y}{\mathrm{d}x}$. in terms of t	M1
	So, $m_T = \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{t^2}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{t^2}$ * **Must see working to award here**	A1
	T: $y - \frac{6}{t} = -\frac{1}{t^2}(x - 6t)$	Applies $y - \frac{6}{t} =$ their $m_T(x - 6t)$	M1
	T: $y - \frac{6}{t} = -\frac{1}{t^2}x + \frac{6}{t}$		
	T: $y = -\frac{1}{t^2}x + \frac{6}{t} + \frac{6}{t}$		
	T: $y = -\frac{1}{t^2}x + \frac{12}{t}$ *	Correct solution .	A1 **cso**
			(5)
(b)	Both **T** meet at $(-9, 12)$ gives		
	$12 = -\frac{1}{t^2}(-9) + \frac{12}{t}$	Substituting (-9,12) into **T**.	M1
	$12 = \frac{9}{t^2} + \frac{12}{t}$ $\quad (\times t^2)$		
	$12t^2 = 9 + 12t$		
	$12t^2 - 12t - 9 = 0$	An attempt to form a "3 term quadratic"	M1
	$4t^2 - 4t - 3 = 0$		
	$(2t-3)(2t+1) = 0$	An attempt to factorise.	M1
	$t = \frac{3}{2}, -\frac{1}{2}$	$t = \frac{3}{2}, -\frac{1}{2}$	A1
	$t = \frac{3}{2} \Rightarrow x = 6\left(\frac{3}{2}\right) = 9$, $y = \frac{6}{\left(\frac{3}{2}\right)} = 4 \Rightarrow (9, 4)$	An attempt to substitute either their $t = \frac{3}{2}$ or their $t = -\frac{1}{2}$ into x and y.	M1
	$t = -\frac{1}{2} \Rightarrow x = 6\left(-\frac{1}{2}\right) = -3$,	At least one of $(9, 4)$ or $(-3, -12)$.	A1
	$y = \frac{6}{\left(-\frac{1}{2}\right)} = -12 \Rightarrow (-3, -12)$	Both $(9, 4)$ and $(-3, -12)$.	A1
			(7) **[12]**

Other Possible Solutions

Question Number	Scheme		Marks
4.	$z^2 + pz + q = 0,\ z_1 = 2 - 4\text{i}$		
(a) (i)	$z_2 = 2 + 4\text{i}$	$2 + 4\text{i}$	B1
Aliter (ii) Way 2	Product of roots $= (2 - 4\text{i})(2 + 4\text{i})$	No i^2. Attempt Sum and Product of roots or Sum and discriminant	M1
	$= 4 + 16 = 20$		
	or $b^2 - 4ac = (8\text{i})^2$		
	Sum of roots $= (2 - 4\text{i}) + (2 + 4\text{i}) = 4$		
	$= z^2 - 4z + 20 = 0$	Any one of $p = -4,\ q = 20$.	A1
		Both $p = -4,\ q = 20$.	A1
			(4)
4.	$z^2 + pz + q = 0,\ z_1 = 2 - 4\text{i}$		
(a) (i)	$z_2 = 2 + 4\text{i}$	$2 + 4\text{i}$	B1
Aliter (ii) Way 3	$(2 - 4\text{i})^2 + p(2 - 4\text{i}) + q = 0$	An attempt to substitute either z_1 or z_2 into $z^2 + pz + q = 0$ and no i^2.	M1
	$-12 - 16\text{i} + p(2 - 4\text{i}) + q = 0$		
	Imaginary part: $-16 - 4p = 0$		
	Real part: $-12 + 2p + q = 0$		
	$4p = -16 \Rightarrow p = -4$	Any one of $p = -4,\ q = 20$.	A1
	$q = 12 - 2p \Rightarrow q = 12 - 2(-4) = 20$	Both $p = -4,\ q = 20$.	A1
			(4)

Question Number	Scheme		Marks
Aliter 7. (c) **Way 2**	$\lvert w\rvert = 4$, $\arg w = \frac{5\pi}{6}$ and $w = a + \mathrm{i}b$		
	$\lvert w\rvert = 4 \Rightarrow a^2 + b^2 = 16$	Attempts to write down an equation in terms of a and b for either the modulus or the argument of w.	M1
	$\arg w = \frac{5\pi}{6} \Rightarrow \arctan\left(\frac{b}{a}\right) = \frac{5\pi}{6} \Rightarrow \frac{b}{a} = -\frac{1}{\sqrt{3}}$	Either $a^2 + b^2 = 16$ or $\frac{b}{a} = -\frac{1}{\sqrt{3}}$	A1
	$a = -\sqrt{3}\, b \Rightarrow a^2 = 3b^2$		
	So, $3b^2 + b^2 = 16 \Rightarrow b^2 = 4$		
	$\Rightarrow b = \pm 2$ and $a = \mp 2\sqrt{3}$		
	As w is in the second quadrant		
	$w = -2\sqrt{3} + 2\mathrm{i}$	either $-2\sqrt{3} + 2\mathrm{i}$ or awrt $-3.5 + 2\mathrm{i}$	A1 (3)
	$a = -2\sqrt{3},\ b = 2$		

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