

1. $z = 5 - 3i, \quad w = 2 + 2i$

Express in the form $a + bi$, where a and b are real constants,

(a) z^2 , **(2)**

(b) $\frac{z}{w}$. **(3)**

(Total 5 marks)

Q1



2.

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$$

(a) Find \mathbf{AB} .

(3)

Given that

$$\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(b) describe fully the geometrical transformation represented by \mathbf{C} ,

(2)

(c) write down \mathbf{C}^{100} .

(1)

(Total 6 marks)

Q2



3.

$$f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, \quad x \geq 0$$

The root α of the equation $f(x) = 0$ lies in the interval $[1.6, 1.8]$.

(a) Use linear interpolation once on the interval $[1.6, 1.8]$ to find an approximation to α . Give your answer to 3 decimal places.

(4)

(b) Differentiate $f(x)$ to find $f'(x)$.

(2)

(c) Taking 1.7 as a first approximation to α , apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α . Give your answer to 3 decimal places.

(4)



5. (a) Use the results for $\sum_{r=1}^n r$, $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r^3$, to prove that

$$\sum_{r=1}^n r(r+1)(r+5) = \frac{1}{4}n(n+1)(n+2)(n+7)$$

for all positive integers n .

(5)

- (b) Hence, or otherwise, find the value of

$$\sum_{r=20}^{50} r(r+1)(r+5)$$

(2)



Question 5 continued

Lined writing area for the answer to Question 5.



6.

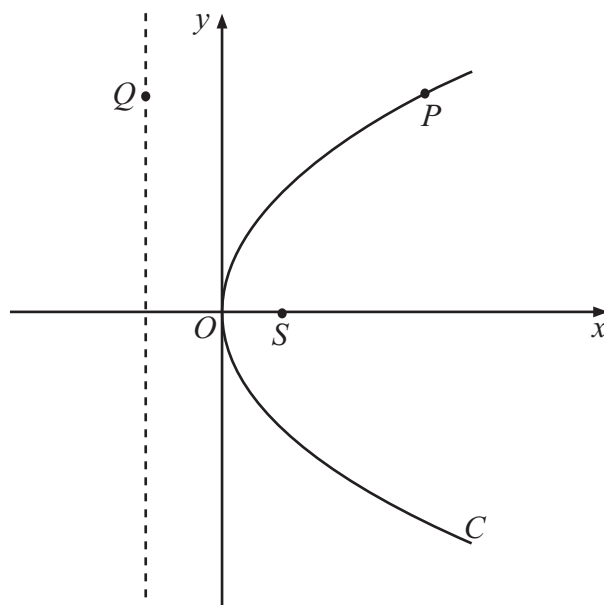


Figure 1

Figure 1 shows a sketch of the parabola C with equation $y^2 = 36x$. The point S is the focus of C .

- (a) Find the coordinates of S . (1)
- (b) Write down the equation of the directrix of C . (1)

Figure 1 shows the point P which lies on C , where $y > 0$, and the point Q which lies on the directrix of C . The line segment QP is parallel to the x -axis.

Given that the distance PS is 25,

- (c) write down the distance QP , (1)
- (d) find the coordinates of P , (3)
- (e) find the area of the trapezium $OSPQ$. (2)



7. $z = -24 - 7i$

(a) Show z on an Argand diagram. **(1)**

(b) Calculate $\arg z$, giving your answer in radians to 2 decimal places. **(2)**

It is given that

$$w = a + bi, \quad a \in \mathbb{R}, b \in \mathbb{R}$$

Given also that $|w| = 4$ and $\arg w = \frac{5\pi}{6}$,

(c) find the values of a and b , **(3)**

(d) find the value of $|zw|$. **(3)**



8.

$$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$$

(a) Find $\det \mathbf{A}$. **(1)**

(b) Find \mathbf{A}^{-1} . **(2)**

The triangle R is transformed to the triangle S by the matrix \mathbf{A} .
Given that the area of triangle S is 72 square units,

(c) find the area of triangle R . **(2)**

The triangle S has vertices at the points $(0, 4)$, $(8, 16)$ and $(12, 4)$.

(d) Find the coordinates of the vertices of R . **(4)**



10. The point $P\left(6t, \frac{6}{t}\right)$, $t \neq 0$, lies on the rectangular hyperbola H with equation $xy = 36$.

(a) Show that an equation for the tangent to H at P is

$$y = -\frac{1}{t^2}x + \frac{12}{t} \tag{5}$$

The tangent to H at the point A and the tangent to H at the point B meet at the point $(-9, 12)$.

(b) Find the coordinates of A and B . (7)



