

3.

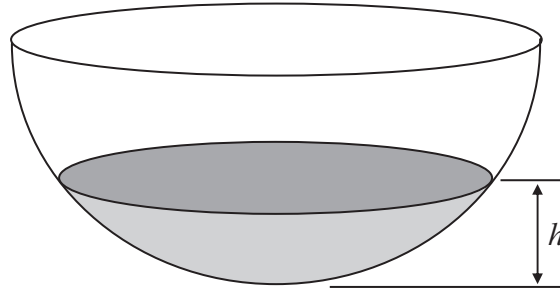


Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl. When the depth of the water is h m, the volume $V\text{m}^3$ is given by

$$V = \frac{1}{12} \pi h^2 (3 - 4h), \quad 0 \leq h \leq 0.25$$

(a) Find, in terms of π , $\frac{dV}{dh}$ when $h = 0.1$ (4)

Water flows into the bowl at a rate of $\frac{\pi}{800} \text{m}^3\text{s}^{-1}$.

(b) Find the rate of change of h , in ms^{-1} , when $h = 0.1$ (2)



4.

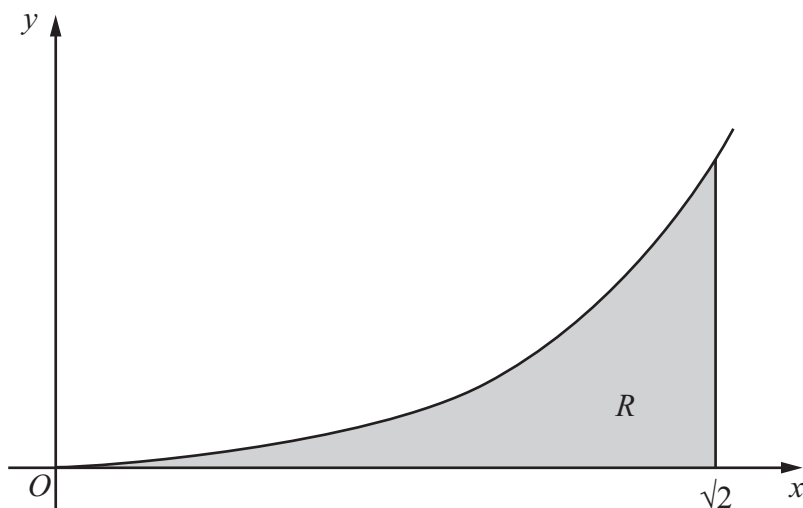


Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2)$, $x \geq 0$. The finite region R , shown shaded in Figure 2, is bounded by the curve, the x -axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln(x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

(a) Complete the table above giving the missing values of y to 4 decimal places. (2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 2 decimal places. (3)

(c) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2} \int_2^4 (u - 2) \ln u \, du \quad (4)$$

(d) Hence, or otherwise, find the exact area of R . (6)



7.

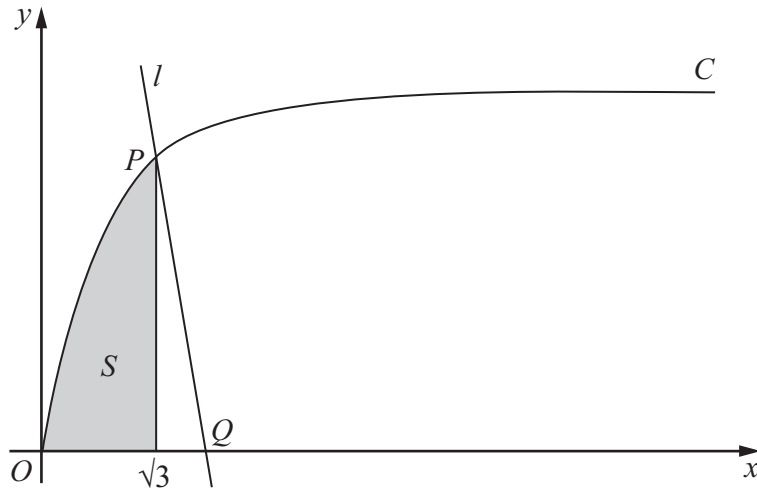


Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan \theta, \quad y = \sin \theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$.

(a) Find the value of θ at the point P .

(2)

The line l is a normal to C at P . The normal cuts the x -axis at the point Q .

(b) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k .

(6)

The finite shaded region S shown in Figure 3 is bounded by the curve C , the line $x = \sqrt{3}$ and the x -axis. This shaded region is rotated through 2π radians about the x -axis to form a solid of revolution.

(c) Find the volume of the solid of revolution, giving your answer in the form $p\pi\sqrt{3} + q\pi^2$, where p and q are constants.

(7)



