







2.

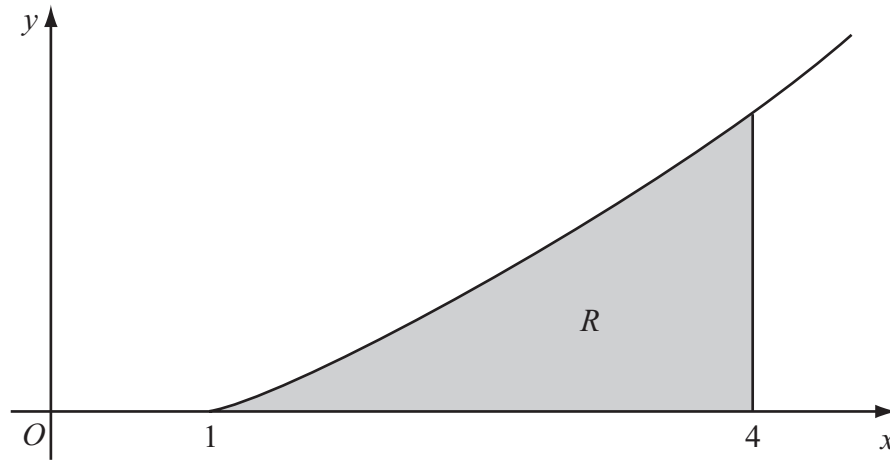


Figure 1

Figure 1 shows a sketch of the curve with equation  $y = x \ln x$ ,  $x \geq 1$ . The finite region  $R$ , shown shaded in Figure 1, is bounded by the curve, the  $x$ -axis and the line  $x = 4$ .

The table shows corresponding values of  $x$  and  $y$  for  $y = x \ln x$ .

$x$	1	1.5	2	2.5	3	3.5	4
$y$	0	0.608			3.296	4.385	5.545

- (a) Complete the table with the values of  $y$  corresponding to  $x = 2$  and  $x = 2.5$ , giving your answers to 3 decimal places. (2)
- (b) Use the trapezium rule, with all the values of  $y$  in the completed table, to obtain an estimate for the area of  $R$ , giving your answer to 2 decimal places. (4)
- (c) (i) Use integration by parts to find  $\int x \ln x \, dx$ .
- (ii) Hence find the exact area of  $R$ , giving your answer in the form  $\frac{1}{4}(a \ln 2 + b)$ , where  $a$  and  $b$  are integers. (7)

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Question 2 continued

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**Question 3 continued**

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4. The line  $l_1$  has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

and the line  $l_2$  has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where  $\lambda$  and  $\mu$  are parameters.

The lines  $l_1$  and  $l_2$  intersect at the point  $A$  and the acute angle between  $l_1$  and  $l_2$  is  $\theta$ .

(a) Write down the coordinates of  $A$ . (1)

(b) Find the value of  $\cos \theta$ . (3)

The point  $X$  lies on  $l_1$  where  $\lambda = 4$ .

(c) Find the coordinates of  $X$ . (1)

(d) Find the vector  $\overrightarrow{AX}$ . (2)

(e) Hence, or otherwise, show that  $|\overrightarrow{AX}| = 4\sqrt{26}$ . (2)

The point  $Y$  lies on  $l_2$ . Given that the vector  $\overrightarrow{YX}$  is perpendicular to  $l_1$ ,

(f) find the length of  $AY$ , giving your answer to 3 significant figures. (3)

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5. (a) Find  $\int \frac{9x+6}{x} dx$ ,  $x > 0$ .

(2)

(b) Given that  $y = 8$  at  $x = 1$ , solve the differential equation

$$\frac{dy}{dx} = \frac{(9x+6)y^{\frac{1}{3}}}{x}$$

giving your answer in the form  $y^2 = g(x)$ .

(6)

Lined area for writing answers.

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**Question 5 continued**

A series of 30 horizontal lines for writing.



N 3 5 3 8 2 A 0 1 7 2 8





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6. The area  $A$  of a circle is increasing at a constant rate of  $1.5 \text{ cm}^2 \text{ s}^{-1}$ . Find, to 3 significant figures, the rate at which the radius  $r$  of the circle is increasing when the area of the circle is  $2 \text{ cm}^2$ . (5)

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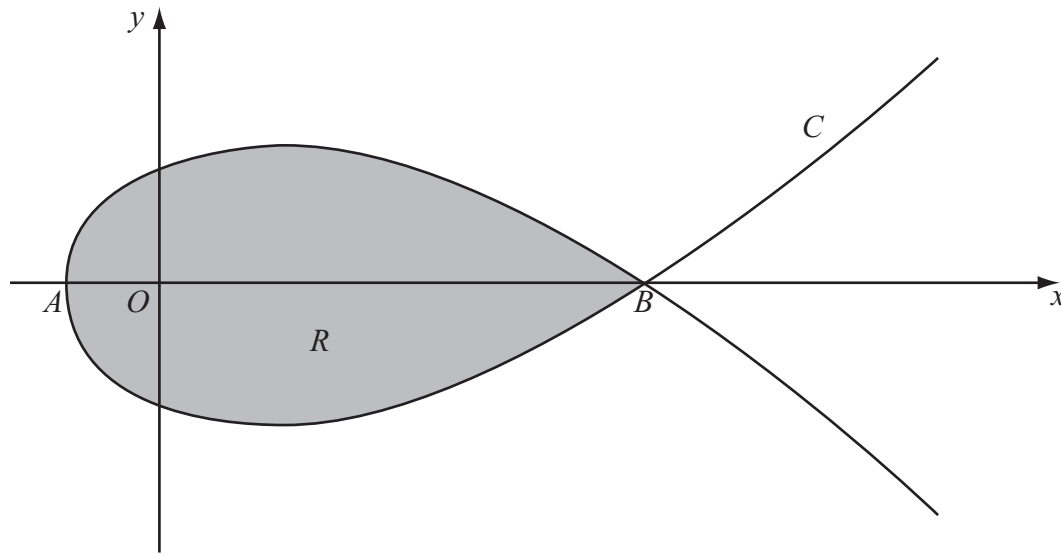
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7.



**Figure 2**

Figure 2 shows a sketch of the curve  $C$  with parametric equations

$$x = 5t^2 - 4, \quad y = t(9 - t^2)$$

The curve  $C$  cuts the  $x$ -axis at the points  $A$  and  $B$ .

- (a) Find the  $x$ -coordinate at the point  $A$  and the  $x$ -coordinate at the point  $B$ . **(3)**

The region  $R$ , as shown shaded in Figure 2, is enclosed by the loop of the curve.

- (b) Use integration to find the area of  $R$ . **(6)**

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8. (a) Using the substitution  $x = 2 \cos u$ , or otherwise, find the exact value of

$$\int_1^{\sqrt{2}} \frac{1}{x^2 \sqrt{4-x^2}} dx \quad (7)$$

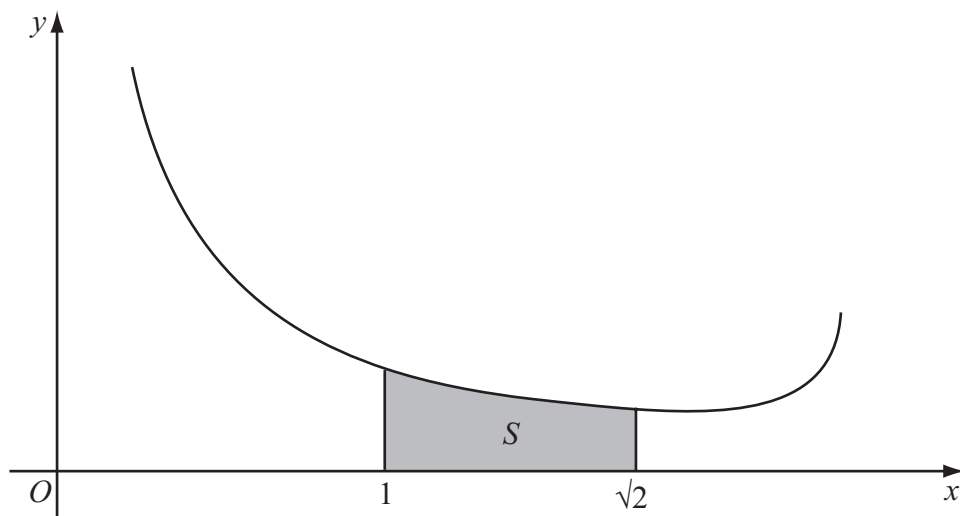


Figure 3

Figure 3 shows a sketch of part of the curve with equation  $y = \frac{4}{x(4-x^2)^{\frac{1}{2}}}$ ,  $0 < x < 2$ .

The shaded region  $S$ , shown in Figure 3, is bounded by the curve, the  $x$ -axis and the lines with equations  $x = 1$  and  $x = \sqrt{2}$ . The shaded region  $S$  is rotated through  $2\pi$  radians about the  $x$ -axis to form a solid of revolution.

(b) Using your answer to part (a), find the exact volume of the solid of revolution formed.

(3)

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**Question 8 continued**

Lined area for student answers, consisting of approximately 28 horizontal lines.

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**Q8**

**(Total 10 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

