Mark Scheme (Results)
Summer 2008

GCE

GCE Mathematics (6665/01)

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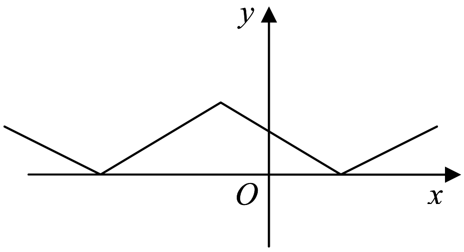
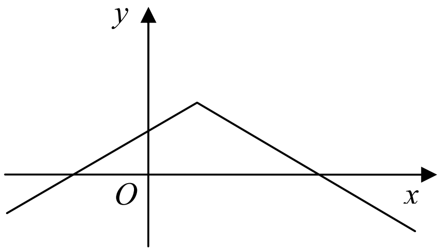
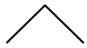
June 2008
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $e^{2x+1} = 2$	
	$2x + 1 = \ln 2$	M1
	$x = \frac{1}{2}(\ln 2 - 1)$	A1 (2)
	(b) $\frac{dy}{dx} = 8e^{2x+1}$	B1
	$x = \frac{1}{2}(\ln 2 - 1) \Rightarrow \frac{dy}{dx} = 16$	B1
	$y - 8 = 16\left(x - \frac{1}{2}(\ln 2 - 1)\right)$	M1
	$y = 16x + 16 - 8\ln 2$	A1 (4)
		[6]

Question Number	Scheme		Marks	
2.	(a) $R^2 = 5^2 + 12^2$		M1	
	$R = 13$		A1	
	$\tan\alpha = \frac{12}{5}$		M1	
	$\alpha \approx 1.176$	cao	A1	(4)
	(b) $\cos(x-\alpha) = \frac{6}{13}$		M1	
	$x-\alpha = \arccos\frac{6}{13} = 1.091\ \ldots$		A1	
	$x = 1.091\ \ldots + 1.176\ \ldots \approx 2.267\ldots$	awrt 2.3	A1	
	$x-\alpha = -1.091\ \ldots$	accept … = 5.19 … for M	M1	
	$x = -1.091\ \ldots + 1.176\ \ldots \approx 0.0849\ \ldots$	awrt 0.084 or 0.085	A1	(5)
	(c)(i) $R_{\max} = 13$	ft their R	B1 ft	
	(ii) At the maximum, $\cos(x-\alpha) = 1$ *or* $x-\alpha = 0$		M1	
	$x = \alpha = 1.176\ \ldots$	awrt 1.2, ft their α	A1ft	(3)
				[12]

Question Number	Scheme	Marks
3.	(a) 	
	 shape	B1
	Vertices correctly placed	B1 (2)
	(b) 	
	 shape	B1
	Vertex and intersections with axes correctly placed	B1 (2)
	(c) $P:(-1,2)$	B1
	$Q:(0,1)$	B1
	$R:(1,0)$	B1 (3)
	(d) $x > -1;\quad 2-x-1=\frac{1}{2}x$	M1 A1
	Leading to $x=\frac{2}{3}$	A1
	$x < -1;\quad 2+x+1=\frac{1}{2}x$	M1
	Leading to $x=-6$	A1 (5)
		[12]

Question Number	Scheme	Marks
4.	(a) $x^2-2x-3=(x-3)(x+1)$	B1
	$f(x)=\dfrac{2(x-1)-(x+1)}{(x-3)(x+1)}$ $\left(or\ \dfrac{2(x-1)}{(x-3)(x+1)}-\dfrac{x+1}{(x-3)(x+1)} \right)$	M1 A1
	$=\dfrac{x-3}{(x-3)(x+1)}=\dfrac{1}{x+1}$ * cso	A1 (4)
	(b) $\left(0,\dfrac{1}{4}\right)$ Accept $0<y<\dfrac{1}{4},\ 0<f(x)<\dfrac{1}{4}$ etc.	B1 B1 (2)
	(c) Let $y=f(x)$ $y=\dfrac{1}{x+1}$	
	$x=\dfrac{1}{y+1}$	
	$yx+x=1$	
	$y=\dfrac{1-x}{x}$ or $\dfrac{1}{x}-1$	M1 A1
	$f^{-1}(x)=\dfrac{1-x}{x}$	
	Domain of f^{-1} is $\left(0,\dfrac{1}{4}\right)$ ft their part (b)	B1 ft (3)
	(d) $fg(x)=\dfrac{1}{2x^2-3+1}$	
	$\dfrac{1}{2x^2-2}=\dfrac{1}{8}$	M1
	$x^2=5$	A1
	$x=\pm\sqrt{5}$ both	A1 (3)
		[12]

Question Number	Scheme	Marks
5.	(a) $\sin^2\theta+\cos^2\theta=1$	
	$\div\sin^2\theta$ $\quad \dfrac{\sin^2\theta}{\sin^2\theta}+\dfrac{\cos^2\theta}{\sin^2\theta}=\dfrac{1}{\sin^2\theta}$	M1
	$1+\cot^2\theta=\operatorname{cosec}^2\theta$ ∗ cso	A1 (2)
	Alternative for (a)	
	$1+\cot^2\theta=1+\dfrac{\cos^2\theta}{\sin^2\theta}=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}=\dfrac{1}{\sin^2\theta}$	M1
	$=\operatorname{cosec}^2\theta$ ∗ cso	A1
	(b) $2\left(\operatorname{cosec}^2\theta-1\right)-9\operatorname{cosec}\theta=3$	M1
	$2\operatorname{cosec}^2\theta-9\operatorname{cosec}\theta-5=0$ *or* $5\sin^2\theta+9\sin\theta-2=0$	M1
	$(2\operatorname{cosec}\theta+1)(\operatorname{cosec}\theta-5)=0$ *or* $(5\sin\theta-1)(\sin\theta+2)=0$	M1
	$\operatorname{cosec}\theta=5$ *or* $\sin\theta=\dfrac{1}{5}$	A1
	$\theta=11.5°, 168.5°$	A1 A1 (6)
		[8]

Question Number	Scheme	Marks
6.	(a)(i) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3x}\left(\sin x+2\cos x\right)\right)=3\mathrm{e}^{3x}\left(\sin x+2\cos x\right)+\mathrm{e}^{3x}\left(\cos x-2\sin x\right)$	M1 A1 A1 (3)
	$\left(=\mathrm{e}^{3x}\left(\sin x+7\cos x\right)\right)$	
	(ii) $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3\ln\left(5x+2\right)\right)=3x^2\ln\left(5x+2\right)+\frac{5x^3}{5x+2}$	M1 A1 A1 (3)
	(b) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(x+1\right)^2\left(6x+6\right)-2\left(x+1\right)\left(3x^2+6x-7\right)}{\left(x+1\right)^4}$	M1 $\frac{\text{A1}}{\text{A1}}$
	$=\frac{\left(x+1\right)\left(6x^2+12x+6-6x^2-12x+14\right)}{\left(x+1\right)^4}$	M1
	$=\frac{20}{\left(x+1\right)^3}$ * cso	A1 (5)
	(c) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{60}{\left(x+1\right)^4}=-\frac{15}{4}$	M1
	$\left(x+1\right)^4=16$	M1
	$x=1,-3$ both	A1 (3) **[14]**
	Note: The simplification in part (b) can be carried out as follows	
	$\frac{\left(x+1\right)^2\left(6x+6\right)-2\left(x+1\right)\left(3x^2+6x-7\right)}{\left(x+1\right)^4}$	
	$=\frac{\left(6x^3+18x^2+18x+6\right)-\left(6x^3+18x^2-2x-14\right)}{\left(x+1\right)^4}$	
	$=\frac{20x+20}{\left(x+1\right)^4}=\frac{20\left(x+1\right)}{\left(x+1\right)^4}=\frac{20}{\left(x+1\right)^3}$	M1 A1

Question Number	Scheme		Marks	
7.	(a) $f(1.4) = -0.568\ \ldots < 0$			
	$f(1.45) = 0.245\ \ldots > 0$		M1	
	Change of sign (and continuity) $\Rightarrow\ \alpha \in (1.4, 1.45)$		A1	(2)
	(b) $3x^3 = 2x + 6$			
	$x^3 = \frac{2x}{3} + 2$			
	$x^2 = \frac{2}{3} + \frac{2}{x}$		M1 A1	
	$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}$ ∗	cso	A1	(3)
	(c) $x_1 = 1.4371$		B1	
	$x_2 = 1.4347$		B1	
	$x_3 = 1.4355$		B1	(3)
	(d) Choosing the interval $(1.4345, 1.4355)$ or appropriate tighter interval.		M1	
	$f(1.4345) = -0.01$ ...			
	$f(1.4355) = 0.003\ \ldots$		M1	
	Change of sign (and continuity) $\Rightarrow\ \alpha \in (1.4345, 1.4355)$			
	$\Rightarrow \alpha = 1.435$, correct to 3 decimal places ∗	cso	A1	(3) **[11]**
	Note: $\alpha = 1.435\,304\,553\ \ldots$			