

# Mark Scheme (Results) Summer 2010

GCE

## Core Mathematics C3 (6665)

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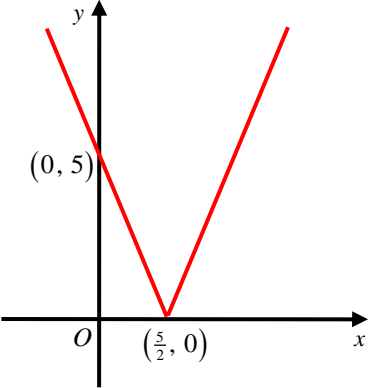


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June 2010  
6665 Core Mathematics C3  
Mark Scheme

Question Number	Scheme	Marks
1.	<p>(a) <math>\frac{2 \sin \theta \cos \theta}{1 + 2 \cos^2 \theta - 1}</math>  <del><math>\frac{\cancel{2} \sin \theta \cancel{\cos \theta}}{\cancel{2} \cos \theta \cancel{\cos \theta}}</math></del> = <math>\tan \theta</math> (as required) <b>AG</b></p> <p>(b) <math>2 \tan \theta = 1 \Rightarrow \tan \theta = \frac{1}{2}</math>  <math>\theta_1 = \text{awrt } 26.6^\circ</math>  <math>\theta_2 = \text{awrt } -153.4^\circ</math></p>	<p>M1</p> <p>A1 cso</p> <p style="text-align: right;">(2)</p> <p>M1</p> <p>A1</p> <p>A1 <math>\sqrt{\quad}</math></p> <p style="text-align: right;">(3) [5]</p>
	<p>(a) M1: Uses <b>both</b> a correct identity for <math>\sin 2\theta</math> <b>and</b> a correct identity for <math>\cos 2\theta</math>.            Also allow a candidate writing <math>1 + \cos 2\theta = 2 \cos^2 \theta</math> on the denominator.            Also note that angles <b>must be consistent</b> in when candidates apply these identities.            A1: Correct proof. No errors seen.</p> <p>(b) 1<sup>st</sup> M1 for either <math>2 \tan \theta = 1</math> or <math>\tan \theta = \frac{1}{2}</math>, seen or implied.            A1: awrt 26.6            A1 <math>\sqrt{\quad}</math>: awrt <math>-153.4^\circ</math> or <math>\theta_2 = -180^\circ + \theta_1</math></p> <p><b>Special Case:</b> For candidate solving, <math>\tan \theta = k</math>, where <math>k \neq \frac{1}{2}</math>, to give <math>\theta_1</math> and <math>\theta_2 = -180^\circ + \theta_1</math>, then award M0A0B1 in part (b).  <b>Special Case:</b> Note that those candidates who writes <math>\tan \theta = 1</math>, and gives ONLY two answers of <math>45^\circ</math> and <math>-135^\circ</math> that are inside the range will be awarded SC M0A0B1.</p>	

Question Number	Scheme	Marks
2.	<p>At <math>P</math>, <math>y = 3</math></p> $\frac{dy}{dx} = \frac{3(-2)(5-3x)^{-3}(-3)}{(5-3x)^3} \left\{ \text{or } \frac{18}{(5-3x)^3} \right\}$ $\frac{dy}{dx} = \frac{18}{(5-3(2))^3} \{ = -18 \}$ $m(\mathbf{N}) = \frac{-1}{-18} \text{ or } \frac{1}{18}$ $\mathbf{N}: y - 3 = \frac{1}{18}(x - 2)$ $\mathbf{N}: \underline{x - 18y + 52 = 0}$	<p>B1</p> <p>M1A1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;"><b>[7]</b></p>
	<p>1<sup>st</sup> M1: <math>\pm k(5-3x)^{-3}</math> can be implied. See appendix for application of the quotient rule.</p> <p>2<sup>nd</sup> M1: Substituting <math>x = 2</math> into an equation involving their <math>\frac{dy}{dx}</math>;</p> <p>3<sup>rd</sup> M1: Uses <math>m(\mathbf{N}) = -\frac{1}{\text{their } m(\mathbf{T})}</math>.</p> <p>4<sup>th</sup> M1: <math>y - y_1 = m(x - 2)</math> with 'their NORMAL gradient' or a "changed" tangent gradient and their <math>y_1</math>. Or uses a complete method to express the equation of the tangent in the form <math>y = mx + c</math> with 'their NORMAL ("changed" <b>numerical</b>) gradient', their <math>y_1</math> and <math>x = 2</math>.</p> <p><b>Note:</b> To gain the final A1 mark all the previous 6 marks in this question need to be earned. Also there must be a completely correct solution given.</p>	

Question Number	Scheme	Marks
3.	<p>(a) <math>f(1.2) = 0.49166551\dots</math>, <math>f(1.3) = -0.048719817\dots</math>            Sign change (and as <math>f(x)</math> is continuous) therefore a root <math>\alpha</math> is such that <math>\alpha \in [1.2, 1.3]</math></p> <p>(b) <math>4\operatorname{cosec}x - 4x + 1 = 0 \Rightarrow 4x = 4\operatorname{cosec}x + 1</math>  <math>\Rightarrow x = \operatorname{cosec}x + \frac{1}{4} \Rightarrow x = \frac{1}{\sin x} + \frac{1}{4}</math></p> <p>(c) <math>x_1 = \frac{1}{\sin(1.25)} + \frac{1}{4}</math>  <math>x_1 = 1.303757858\dots</math>, <math>x_2 = 1.286745793\dots</math>  <math>x_3 = 1.291744613\dots</math></p> <p>(d) <math>f(1.2905) = 0.00044566695\dots</math>, <math>f(1.2915) = -0.00475017278\dots</math>            Sign change (and as <math>f(x)</math> is continuous) therefore a root <math>\alpha</math> is such that  <math>\alpha \in (1.2905, 1.2915) \Rightarrow \alpha = 1.291</math> (3 dp)</p>	<p>M1A1 (2)</p> <p>M1 A1 * (2)</p> <p>M1 A1 A1 (3)</p> <p>M1 A1 (2)</p> <p>[9]</p>
	<p>(a) M1: Attempts to evaluate both <math>f(1.2)</math> and <math>f(1.3)</math> and evaluates at least one of them correctly to awrt (or truncated) 1 sf.            A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion.</p> <p>(b) M1: Attempt to make <math>4x</math> or <math>x</math> the subject of the equation.            A1: Candidate must then rearrange the equation to give the required result. It must be clear that candidate has made their initial <math>f(x) = 0</math>.</p> <p>(c) M1: An attempt to substitute <math>x_0 = 1.25</math> into the iterative formula            Eg <math>= \frac{1}{\sin(1.25)} + \frac{1}{4}</math>.            Can be implied by <math>x_1 = \text{awrt } 1.3</math> or <math>x_1 = \text{awrt } 46^\circ</math>.            A1: Both <math>x_1 = \text{awrt } 1.3038</math> and <math>x_2 = \text{awrt } 1.2867</math>            A1: <math>x_3 = \text{awrt } 1.2917</math></p> <p>(d) M1: Choose suitable interval for <math>x</math>, e.g. <math>[1.2905, 1.2915]</math> or tighter and at least one attempt to evaluate <math>f(x)</math>.            A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion.</p>	

Question Number	Scheme	Marks
<p>4. (a)</p>  <p>(b) <math>x = 20</math>  <math>2x - 5 = -(15 + x) ; \Rightarrow x = -\frac{10}{3}</math></p> <p>(c) <math>fg(2) = f(-3) =  2(-3) - 5  ; =  -11  = 11</math></p> <p>(d) <math>g(x) = x^2 - 4x + 1 = (x - 2)^2 - 4 + 1 = (x - 2)^2 - 3</math>. Hence <math>g_{\min} = -3</math>  Either <math>g_{\min} = -3</math> or <math>g(x) \geq -3</math>  or <math>g(5) = 25 - 20 + 1 = 6</math>  <u><math>-3 \leq g(x) \leq 6</math></u> or <u><math>-3 \leq y \leq 6</math></u></p>	<p>M1A1</p> <p>(2)</p> <p>B1 M1;A1 oe.</p> <p>(3)</p> <p>M1;A1</p> <p>(2)</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>(3)</p> <p>[10]</p>	
	<p>(a) M1: V or  or  graph with vertex on the <math>x</math>-axis.</p> <p>A1: <math>(\frac{5}{2}, \{0\})</math> and <math>(\{0\}, 5)</math> seen and the graph appears in both the first and second quadrants.</p> <p>(b) M1: Either <math>2x - 5 = -(15 + x)</math> or <math>-(2x - 5) = 15 + x</math></p> <p>(c) M1: <b>Full method</b> of inserting <math>g(2)</math> into <math>f(x) =  2x - 5 </math> or for inserting <math>x = 2</math> into <math> 2(x^2 - 4x + 1) - 5 </math>. There must be evidence of the modulus being applied.</p> <p>(d) M1: <b>Full method</b> to establish the minimum of <math>g</math>. Eg: <math>(x \pm \alpha)^2 + \beta</math> leading to <math>g_{\min} = \beta</math>. Or for candidate to differentiate the quadratic, set the result equal to zero, find <math>x</math> and insert this value of <math>x</math> back into <math>f(x)</math> in order to find the minimum.</p> <p>B1: For either finding the correct minimum value of <math>g</math> (can be implied by <math>g(x) \geq -3</math> or <math>g(x) &gt; -3</math>) or for stating that <math>g(5) = 6</math>.</p> <p>A1: <u><math>-3 \leq g(x) \leq 6</math></u> or <u><math>-3 \leq y \leq 6</math></u> or <u><math>-3 \leq g \leq 6</math></u>. <b>Note that:</b> <math>-3 \leq x \leq 6</math> is A0.</p> <p><b>Note that:</b> <math>-3 \leq f(x) \leq 6</math> is A0. <b>Note that:</b> <math>-3 \geq g(x) \geq 6</math> is A0.</p> <p><b>Note that:</b> <math>g(x) \geq -3</math> or <math>g(x) &gt; -3</math> or <math>x \geq -3</math> or <math>x &gt; -3</math> with no working gains M1B1A0.</p> <p><b>Note that for the final Accuracy Mark:</b>  If a candidate writes down <math>-3 &lt; g(x) &lt; 6</math> or <math>-3 &lt; y &lt; 6</math>, then award M1B1A0.  If, however, a candidate writes down <math>g(x) \geq -3</math>, <math>g(x) \leq 6</math>, then award A0.  If a candidate writes down <math>g(x) \geq -3</math> or <math>g(x) \leq 6</math>, then award A0.</p>	

Question Number	Scheme	Marks
5.	<p>(a) Either <math>y = 2</math> or <math>(0, 2)</math></p> <p>(b) When <math>x = 2</math>, <math>y = (8 - 10 + 2)e^{-2} = 0e^{-2} = 0</math>  <math>(2x^2 - 5x + 2) = 0 \Rightarrow (x - 2)(2x - 1) = 0</math>            Either <math>x = 2</math> (for possibly B1 above) or <math>x = \frac{1}{2}</math>.</p> <p>(c) <math>\frac{dy}{dx} = (4x - 5)e^{-x} - (2x^2 - 5x + 2)e^{-x}</math></p> <p>(d) <math>(4x - 5)e^{-x} - (2x^2 - 5x + 2)e^{-x} = 0</math>  <math>2x^2 - 9x + 7 = 0 \Rightarrow (2x - 7)(x - 1) = 0</math>  <math>x = \frac{7}{2}, 1</math>            When <math>x = \frac{7}{2}</math>, <math>y = 9e^{-\frac{7}{2}}</math>, when <math>x = 1</math>, <math>y = -e^{-1}</math></p>	<p>B1 (1)</p> <p>B1 M1 A1 (3)</p> <p>M1A1A1 (3)</p> <p>M1 M1 A1 ddM1A1 (5)</p> <p>[12]</p>
	<p>(b) If the candidate believes that <math>e^{-x} = 0</math> solves to <math>x = 0</math> or gives an extra solution of <math>x = 0</math>, then withhold the final accuracy mark.</p> <p>(c) M1: (their <math>u'</math>)<math>e^{-x} + (2x^2 - 5x + 2)</math>(their <math>v'</math>)            A1: Any one term correct.            A1: Both terms correct.</p> <p>(d) 1<sup>st</sup> M1: For setting their <math>\frac{dy}{dx}</math> found in part (c) equal to 0.            2<sup>nd</sup> M1: Factorise or eliminate out <math>e^{-x}</math> correctly and an attempt to factorise a 3-term quadratic or apply the formula to candidate's <math>ax^2 + bx + c</math>.            See rules for solving a three term quadratic equation on page 1 of this Appendix.            3<sup>rd</sup> ddM1: An attempt to use at least one <math>x</math>-coordinate on <math>y = (2x^2 - 5x + 2)e^{-x}</math>.            Note that this method mark is dependent on the award of the two previous method marks in this part.            Some candidates write down corresponding <math>y</math>-coordinates without any working. It may be necessary on some occasions to use your calculator to check that at least one of the two <math>y</math>-coordinates found is correct to awrt 2 sf.            Final A1: Both <math>\{x = 1\}</math>, <math>y = -e^{-1}</math> and <math>\{x = \frac{7}{2}\}</math>, <math>y = 9e^{-\frac{7}{2}}</math>. <b>cao</b>            Note that both exact values of <math>y</math> are required.</p>	

Question Number	Scheme	Marks
<p>6. (a) (i) (3, 4) (ii) (6, -8)</p> <p>(b)</p> <p>(c) <math>f(x) = (x - 3)^2 - 4</math> or <math>f(x) = x^2 - 6x + 5</math></p> <p>(d) Either: The function <math>f</math> is a many-one {mapping}. Or: The function <math>f</math> is not a one-one {mapping}.</p>	<p>B1 B1 B1 B1</p> <p>(4)</p> <p>B1 B1 B1</p> <p>(3)</p> <p>M1A1</p> <p>(2)</p> <p>B1</p> <p>(1)</p> <p>[10]</p>	
	<p>(b) B1: Correct shape for <math>x \geq 0</math>, with the curve meeting the positive <math>y</math>-axis and the turning point is found below the <math>x</math>-axis. (providing candidate does not copy the whole of the original curve and adds nothing else to their sketch.). B1: Curve is symmetrical about the <math>y</math>-axis or correct shape of curve for <math>x &lt; 0</math>. <b>Note:</b> The first two B1B1 can only be awarded if the curve has the correct shape, with a cusp on the positive <math>y</math>-axis and with both turning points located in the correct quadrants. Otherwise award B1B0. B1: Correct turning points of <math>(-3, -4)</math> and <math>(3, -4)</math>. Also, <math>(\{0\}, 5)</math> is marked where the graph cuts through the <math>y</math>-axis. Allow <math>(5, 0)</math> rather than <math>(0, 5)</math> if marked in the "correct" place on the <math>y</math>-axis.</p> <p>(c) M1: Either states <math>f(x)</math> in the form <math>(x \pm \alpha)^2 \pm \beta</math>; <math>\alpha, \beta \neq 0</math> Or uses a complete method on <math>f(x) = x^2 + ax + b</math>, with <math>f(0) = 5</math> and <math>f(3) = -4</math> to find both <math>a</math> and <math>b</math>. A1: Either <math>(x - 3)^2 - 4</math> or <math>x^2 - 6x + 5</math></p> <p>(d) B1: Or: The inverse is a one-many {mapping and not a function}. Or: Because <math>f(0) = 5</math> and also <math>f(6) = 5</math>. Or: One <math>y</math>-coordinate has 2 corresponding <math>x</math>-coordinates {and therefore cannot have an inverse}.</p>	



Question Number	Scheme	Marks
7.	<p>(a) <math>R = \sqrt{6.25}</math> or 2.5  <math>\tan \alpha = \frac{1.5}{2} = \frac{3}{4} \Rightarrow \alpha = \text{awrt } 0.6435</math></p> <p>(b) (i) Max Value = 2.5  (ii) <math>\sin(\theta - 0.6435) = 1</math> or <math>\theta - \text{their } \alpha = \frac{\pi}{2}; \Rightarrow \theta = \text{awrt } 2.21</math></p> <p>(c) <math>H_{\text{Max}} = 8.5</math> (m)  <math>\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1</math> or <math>\frac{4\pi t}{25} = \text{their (b) answer}; \Rightarrow t = \text{awrt } 4.41</math></p> <p>(d) <math>\Rightarrow 6 + 2.5 \sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7; \Rightarrow \sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5} = 0.4</math>  <math>\left\{\frac{4\pi t}{25} - 0.6435\right\} = \sin^{-1}(0.4)</math> or awrt 0.41  Either <math>t = \text{awrt } 2.1</math> or awrt 6.7  So, <math>\left\{\frac{4\pi t}{25} - 0.6435\right\} = \{\pi - 0.411517... \text{ or } 2.730076...^c\}</math>  Times = <math>\{14:06, 18:43\}</math></p>	<p>B1  M1A1  (3)</p> <p>B1 <math>\sqrt{\quad}</math>  M1;A1 <math>\sqrt{\quad}</math>  (3)</p> <p>B1 <math>\sqrt{\quad}</math>  M1;A1  (3)</p> <p>M1;M1  A1  A1  ddM1  A1 (6)  [15]</p>
	<p>(a) B1: <math>R = 2.5</math> or <math>R = \sqrt{6.25}</math>. For <math>R = \pm 2.5</math>, award B0.  M1: <math>\tan \alpha = \pm \frac{1.5}{2}</math> or <math>\tan \alpha = \pm \frac{2}{1.5}</math>  A1: <math>\alpha = \text{awrt } 0.6435</math></p> <p>(b) B1 <math>\sqrt{\quad}</math>: 2.5 or follow through the value of <math>R</math> in part (a).  M1: For <math>\sin(\theta - \text{their } \alpha) = 1</math>  A1 <math>\sqrt{\quad}</math>: awrt 2.21 or <math>\frac{\pi}{2} + \text{their } \alpha</math> rounding correctly to 3 sf.</p> <p>(c) B1 <math>\sqrt{\quad}</math>: 8.5 or <math>6 + \text{their } R</math> found in part (a) as long as the answer is greater than 6.  M1: <math>\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 1</math> or <math>\frac{4\pi t}{25} = \text{their (b) answer}</math>  A1: For <math>\sin^{-1}(0.4)</math> This can be implied by awrt 4.41 or awrt 4.40.</p> <p>(d) M1: <math>6 + (\text{their } R) \sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 7</math>, M1:  <math>\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = \frac{1}{\text{their } R}</math>  A1: For <math>\sin^{-1}(0.4)</math>. This can be implied by awrt 0.41 or awrt 2.73 or other values for different <math>\alpha</math>'s. Note this mark can be implied by seeing 1.055.  A1: Either <math>t = \text{awrt } 2.1</math> or <math>t = \text{awrt } 6.7</math>  ddM1: either <math>\pi - \text{their PV}^c</math>. Note that this mark is dependent upon the two M marks. This mark will usually be awarded for seeing either 2.730... or 3.373...  A1: Both <math>t = 14:06</math> and <math>t = 18:43</math> or both 126 (min) and 403 (min) or both 2 hr 6 min and 6 hr 43 min.</p>	

Question Number	Scheme	Marks
8.	<p>(a) <math>\frac{(x+5)(2x-1)}{(x+5)(x-3)} = \frac{(2x-1)}{(x-3)}</math></p> <p>(b) <math>\ln\left(\frac{2x^2+9x-5}{x^2+2x-15}\right) = 1</math></p> $\frac{2x^2+9x-5}{x^2+2x-15} = e$ $\frac{2x-1}{x-3} = e \Rightarrow 3e-1 = x(e-2)$ $\Rightarrow x = \frac{3e-1}{e-2}$	<p>M1 B1 A1 aef (3)</p> <p>M1</p> <p>dM1</p> <p>M1</p> <p>A1 aef cso (4) [7]</p>
	<p>(a) M1: An attempt to factorise the numerator. B1: Correct factorisation of denominator to give <math>(x+5)(x-3)</math>. Can be seen anywhere.</p> <p>(b) M1: Uses a correct law of logarithms to combine at least two terms. This usually is achieved by the subtraction law of logarithms to give</p> $\ln\left(\frac{2x^2+9x-5}{x^2+2x-15}\right) = 1.$ <p>The product law of logarithms can be used to achieve</p> $\ln(2x^2+9x-5) = \ln(e(x^2+2x-15)).$ <p>The product and quotient law could also be used to achieve</p> $\ln\left(\frac{2x^2+9x-5}{e(x^2+2x-15)}\right) = 0.$ <p>dM1: Removing ln's correctly by the realisation that the anti-ln of 1 is e. Note that this mark is dependent on the previous method mark being awarded. M1: Collect <math>x</math> terms together and factorise. Note that this is not a dependent method mark.</p> <p>A1: <math>\frac{3e-1}{e-2}</math> or <math>\frac{3e^1-1}{e^1-2}</math> or <math>\frac{1-3e}{2-e}</math>. aef</p> <p>Note that the answer needs to be in terms of <math>e</math>. The decimal answer is 9.9610559... Note that the solution must be correct in order for you to award this final accuracy mark.</p> <p><b>Note: See Appendix for an alternative method of long division.</b></p>	



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