Mark Scheme (Results)

Summer 2012

GCE Core Mathematics C2
(6664) Paper 1
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General Marking Guidance

• All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

• Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

• Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

• There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

• All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.

• Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

• When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.

• Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.
   - bod – benefit of doubt
   - ft – follow through
   - the symbol \( \checkmark \) will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - □ The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:
1. Factorisation
   \[(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \ldots\]
   \[(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \ldots\]

2. Formula
   Attempt to use correct formula (with values for \(a, b \) and \(c\)), leading to \(x = \ldots\)

3. Completing the square
   Solving \(x^2 + bx + c = 0\):
   \[(x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0, \quad \text{leading to } x = \ldots\]

Method marks for differentiation and integration:
1. Differentiation
   Power of at least one term decreased by 1. \((x^n \rightarrow x^{n-1})\)

2. Integration
   Power of at least one term increased by 1. \((x^n \rightarrow x^{n+1})\)

Use of a formula
Where a method involves using a formula that has been learnt, the advice given in recent examiners’ reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers
Examiners’ reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working
The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done “in your head”, detailed working would not be required.
<table>
<thead>
<tr>
<th>Question number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[ (2 - 3x)^5 ] = \ldots + \binom{5}{1} 2^4 (-3x) + \binom{5}{2} 2^3 (-3x)^2 + \ldots ]</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>= 32, -240x, +720x^2</td>
<td>B1, A1, A1</td>
</tr>
</tbody>
</table>

**Notes**

**M1:** The method mark is awarded for an attempt at Binomial to get the second and/or third term – need correct binomial coefficient combined with correct power of x. Ignore errors (or omissions) in powers of 2 or 3 or sign or bracket errors. Accept any notation for \( \binom{5}{1} \) and \( \binom{5}{2} \), e.g. \( \binom{5}{1} \) and \( \binom{5}{2} \) (unsimplified) or 5 and 10 from Pascal’s triangle. This mark may be given if no working is shown, but either or both of the terms including x is correct.

**B1:** must be simplified to 32 (writing just \( \binom{5}{2} \) is B0). 32 must be the only constant term in the final answer– so 32 +80 - 3x+80 + 9x^3 is B0 but may be eligible for M1A0A0.

**A1:** is cao and is for -240x. (not +240x) The x is required for this mark

**A1:** is c.a.o and is for 720x^2 (can follow omission of negative sign in working)

A list of correct terms may be given credit i.e. series appearing on different lines

Ignore extra terms in x^3 and/or x^4 (isw)

**Special Case**

Special Case: Descending powers of x would be

\((-3x)^5 + 2 \times 5 \times (-3x)^4 + 2^2 \times \binom{5}{3} \times (-3x)^3 + \ldots \), i.e. \(-243x^5 + 810x^4 - 1080x^3 + \ldots \). This is a misread but award as s.c. M1B1A0A0 if completely “correct” or M1 B0A0A0 for correct binomial coefficient in any form with the correct power of x

**Alternative Method**

**Method 1:** \[ (2 - 3x)^5 ] = 2^5 (1 + \binom{5}{1} (-\frac{3x}{2}) + \binom{5}{2} (\frac{-3x}{2})^2 + \ldots \) is M1B0A0A0 { The M1 is for the expression in the bracket and as in first method– need correct binomial coefficient combined with correct power of x. Ignore bracket errors or errors (or omissions) in powers of 2 or 3 or sign or bracket errors} – answers must be simplified to = 32, -240x, +720x^2 for full marks (awarded as before)

\[ (2 - 3x)^5 ] = 2(1 + \binom{5}{1} (-\frac{3x}{2}) + \binom{5}{2} (\frac{-3x}{2})^2 + \ldots \) would also be awarded M1B0A0A0

**Method 2:** Multiplying out: B1 for 32 and M1A1A1 for other terms with M1 awarded if x or x^2 term is correct. Completely correct is 4/4
<table>
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</tr>
</thead>
</table>
| 2               | $2 \log x = \log x^2$<br>$\log_3 x^2 - \log_3 (x-2) = \log_3 \frac{x^2}{x-2}$<br>$\frac{x^2}{x-2} = 9$<br>Solves $x^2 - 9x + 18 = 0$ to give $x = ...$
|                 | $x = 3, x = 6$ |       |
|                 | B1      |       |
|                 | M1      |       |
|                 | A1 o.e. |       |
|                 | M1      |       |
|                 | A1      |       |
|                 | **Total 5** |     |

**Notes**<br>B1 for this correct use of power rule (may be implied)<br>M1: for correct use of subtraction rule (or addition rule) for logs<br>N.B. $2 \log_3 x - \log_3 (x-2) = 2 \log_3 \frac{x}{x-2}$ is M0<br>A1: for correct equation without logs (Allow any correct equivalent including $3^2$ instead of 9.)<br>M1 for attempting to solve $x^2 - 9x + 18 = 0$ to give $x =$ (see notes on marking quadratics)<br>A1 for these two correct answers<br>

**Alternative Method**<br>$\log_3 x^2 = 2 + \log_3 (x-2)$ is B1,<br>so $x^2 = 3^{2+\log_3 (x-2)}$ needs to be followed by $(x^2) = 9(x-2)$ for M1 A1<br>Here M1 is for complete method i.e. correct use of powers after logs are used correctly<br>

**Common Slips**<br>$2 \log x - \log x + \log 2 = 2$ may obtain B1 if $\log x^2$ appears but the statement is M0 and so leads to no further marks<br>$2 \log_3 x - \log_3 (x-2) = 2$ so $\log_3 x = \log_3 (x-2) = 1$ and $\log_3 \frac{x}{x-2} = 1$ can earn M1 for correct subtraction rule following error, but no other marks<br>

**Special Case**<br>$\frac{\log x^2}{\log (x-2)} = 2$ leading to $\frac{x^2}{x-2} = 9$ and then to $x = 3, x = 6$, usually earns B1M0A0, but may then earn M1A1 (special case) so 3/5 [This recovery after uncorrected error is very common]<br>Trial and error, Use of a table or just stating answer with both $x = 3$ and $x = 6$ should be awarded B0M0A0 then final M1A1 i.e. 2/5
<table>
<thead>
<tr>
<th>Question number</th>
<th>Scheme</th>
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<tbody>
<tr>
<td>3 (a)</td>
<td>Obtain $(x \pm 10)^2$ and $(y \pm 8)^2$</td>
<td></td>
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<tr>
<td></td>
<td>Obtain $(x - 10)^2$ and $(y - 8)^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Centre is (10, 8). N.B. This may be indicated on diagram only as (10, 8)</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>See $(x \pm 10)^2 + (y \pm 8)^2 = 25 (= r^2)$ or $(r^2 = &quot;100&quot; + &quot;64&quot; - 139)$</td>
<td></td>
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<tr>
<td></td>
<td>$r = 5$ * (this is a printed answer so need one of the above two reasons)</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>Use $x = 13$ in either form of equation of circle and solve resulting quadratic to give $y =$</td>
<td></td>
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<tr>
<td></td>
<td>e.g $x = 13 \Rightarrow (13 - 10)^2 + (y - 8)^2 = 25 \Rightarrow (y - 8)^2 = 16$ so $y =$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>or $13^2 + y^2 - 20 \times 13 - 16y + 139 = 0 \Rightarrow y^2 - 16y + 48 = 0$ so $y =$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y = 4$ or $12$ (on EPEN mark one correct value as A1A0 and both correct as A1A1)</td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>Use of $r\theta$ with $r = 5$ and $\theta = 1.855$ (may be implied by 9.275)</td>
<td></td>
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<tr>
<td></td>
<td>Perimeter $PTQ = 2r + \text{their arc } PQ$ (Finding perimeter of triangle is M0 here)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 19.275$ or $19.28$ or $19.3$</td>
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</table>

**Alternatives**

- Method 2: From $x^2 + y^2 + 2gx + 2fy + c = 0$ centre is $(\pm g, \pm f)$ |
- Centre is $(-g, -f)$, and so centre is $(10, 8)$. |
- Method 3: Use any value of $y$ to give two points (L and M) on circle. $x$ co-ordinate of mid point of LM is “10” and Use any value of $x$ to give two points (P and Q) on circle. $y$ co-ordinate of mid point of PQ is “8” (Centre – chord theorem). (10,8) is M1A1A1 |

- Method 2: Using $\sqrt{g^2 + f^2 - c}$ or $(r^2 = "100" + "64" - 139)$ |
- $r = 5$ * |
- Method 3: Use point on circle with centre to find radius. Eg $\sqrt{(13-10)^2 + (12-8)^2}$ |
- $r = 5$ * |

- Divide triangle PTQ and use Pythagoras with $r^2 - (13 - "10")^2 = h^2$, then evaluate "8 ± h" - (N.B. Could use 3,4,5 Triangle and 8 ± 4). |
- Accuracy as before |

**Notes**

- **Mark (a) and (b) together** |
- M1 as in scheme and can be implied by $(\pm 10, \pm 8)$. **Correct centre (10, 8) implies M1A1A1** |
- M1 for a correct method leading to $r = \ldots$, or $r^2 = "100" + "64" - 139$ (not 139 – “100” – “64”) |
- Special case: if centre is given as (-10, -8) or (10, -8) or (-10, 8) allow M1A1 for $r = 5$ worked correctly |
- Full marks available for calculation using major sector so Use of $r\theta$ with $r = 5$ and $\theta = 4.428$ leading to perimeter of 32.14 for major sector
<table>
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</table>
| **4 (a)** | \( f(-2) = 2(-2)^3 - 7(-2)^2 - 10(-2) + 24 \)  
\( = 0 \) so \((x+2)\) is a factor | M1  
A1 | (2) |
| **(b)** | \( f(x) = (x+2)(2x^2 - 11x + 12) \)  
\( f(x) = (x+2)(2x-3)(x-4) \) | M1  
A1  
dM1  
A1 | (4) |

**Notes (a)**

- M1: Attempts \( f(\pm 2) \) (Long division is M0)  
- A1: is for \( = 0 \) and conclusion  
- Note: Stating “hence factor” or “it is a factor” or a “√“ (tick) or “QED” is fine for the conclusion.  
- Note also that a conclusion can be implied from a preamble, eg: “If \( f(-2) = 0, (x + 2) \) is a factor…” (Not just \( f(-2)=0 \))

**Notes (b)**

1st M1: Attempts long division by correct factor or other method leading to obtaining \( (2x^2 \pm ax \pm b), a \neq 0, b \neq 0 \), even with a remainder. Working need not be seen as could be done “by inspection.”

Or Alternative Method: 1st M1: Use \( (x+2)(ax^2 + bx + c) = 2x^3 - 7x^2 - 10x + 24 \) with expansion and comparison of coefficients to obtain \( a = 2 \) and to obtain values for \( b \) and \( c \)  

1st A1: For seeing \( (2x^2 - 11x + 12) \). [Can be seen here in (b) after work done in (a)]

2nd M1: Factorises quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded and needs factors  

2nd A1: is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.)

Note: Some candidates will go from \( \{(x + 2)\}{(2x^2 - 11x + 12) \) to \( \{x = -2\}, x = 1.5 , 4 \), and not list all three factors. Award these responses M1A1M0A0.

Finds \( x = 4 \) and \( x = 1.5 \) by factor theorem, formula or calculator and produces factors M1  

\( f(x) = (x+2)(2x-3)(x-4) \) or \( f(x) = 2(x+2)(x-1.5)(x-4) \) o.e. is full marks  

\( f(x) = (x+2)(x-1.5)(x-4) \) loses last A1
Question number | Scheme | Marks
--- | --- | ---
Method 1 5 (a) | Puts $10 - x = 10x - x^2 - 8$ and rearranges to give three term quadratic | M1
Solves their $x^2 - 11x + 18 = 0$ using acceptable method as in general principles to give $x =$ | M1
Obtains $x = 2, x = 9$ (may be on diagram or in part (b) in limits) | A1
Substitutes their $x$ into a given equation to give $y =$ (may be on diagram) | M1
Or puts $y = 10(10 - y) - (10 - y)^2 - 8$ and rearranges to give three term quadratic | M1
Solves their $y^2 - 9y + 8 = 0$ using acceptable method as in general principles to give $y =$ | A1
Obtains $y = 8, y = 1$ (may be on diagram) | M1
Substitutes their $y$ into a given equation to give $x =$ (may be on diagram or in part (b)) | A1 (5)

(b) | $\int (10x - x^2 - 8)dx = \frac{10x^2}{2} - \frac{x^3}{3} - 8x \{+ c\}$ | M1 A1
$\left[\frac{10x^2}{2} - \frac{x^3}{3} - 8x\right]^9_2 = (......) - (......)$ | A1
d$M1$ | $= 90 - \frac{4}{3} = 88\frac{2}{3}$ or $26\frac{2}{3}$ | B1
Area of trapezium $= \frac{1}{2}(8+1)(9-2) = 31.5$ | M1A1 cao
So area of $R$ is $88\frac{2}{3} - 31.5 = 57\frac{1}{6}$ or $\frac{343}{6}$ | (7)

Notes (a) | First $M1$: See scheme | 12 marks
Second $M1$: See notes relating to solving quadratics | 
Third $M1$: This may be awarded if one substitution is made | 
Two correct Answers following tables of values, or from Graphical calculator are $5/5$ | 
Just one pair of correct coordinates – no working or from table is M0M0A0M1A0 | 
$M1$: $x^n \rightarrow x^{n+1}$ for any one term. | 
$1^{st} A1$: at least two out of three terms correct | 
$2^{nd} A1$: All three correct | 
d$M1$: Substitutes 9 and 2 (or limits from part(a)) into an “integrated function” and subtracts, | 
either way round | 
(NB: If candidate changes all signs to get $\int (-10x + x^2 + 8)dx = -\frac{10x^2}{2} + \frac{x^3}{3} + 8x \{+ c\}$) This is $M1$ $A1$ $A1$ | 
Then uses limits d$M1$ and trapezium is $B1$ | 
Needs to change sign of value obtained from integration for final $M1A1$ so $-88\frac{2}{3} - 31.5$ is M0A0 | 
$B1$: Obtains 31.5 for area under line using any correct method (could be integration) or triangle minus | 
triangle $\frac{1}{2} \times 8 \times 8 - \frac{1}{2}$ or rectangle plus triangle [may be implied by correct $57\ 1/6$] | 
$M1$: Their Area under curve – Their Area under line (if integrate both need same limits) | 
$A1$: Accept 57.16 recurring but not 57.16 | 
PTO for Alternative method |
| **Method 2 for (b)** | Area of $R$  
$$\int_2^9 (10x - x^2 - 8) - (10 - x) \, dx$$  
$$\int_2^9 x^2 + 11x - 18 \, dx$$  
$$= \frac{x^3}{3} + \frac{11x^2}{2} - 18x \{+ C\}$$  
$$\left[ \frac{x^3}{3} + \frac{11x^2}{2} - 18x \right]_2^9 = (\ldots) - (\ldots)$$ |  
$3^{rd}$ M1 (in (b)): Uses difference between two functions in integral.  
$M$: $x^n \to x^{n+1}$ for any one term.  
$A1$ at least two out of these three simplified terms.  
Correct integration. (Ignore $+ C$).  
Substitutes 9 and 2 (or limits from part (a)) into an “integrated function” and subtracts, either way round.  
This mark is implied by final answer which rounds to 57.2  
*See above working (allow bracketing errors) to decide to award $3^{rd}$ M1 mark for (b) here:*  
$$40.5 - (-16\frac{1}{2}) = 57\frac{1}{2} \text{ cao}$$ |
|---|---|---|
| **Special case of above method** | $\int_2^9 x^2 + 11x + 18 \, dx = \frac{x^3}{3} - \frac{11x^2}{2} + 18x \{+ C\}$  
$$\left[ \frac{x^3}{3} - \frac{11x^2}{2} + 18x \right]_2^9 = (\ldots) - (\ldots)$$ |  
$M1A1A1$  
$DM1$  
This mark is implied by final answer which rounds to 57.2 (not -57.2)  
Difference of functions implied (see above expression)  
$$40.5 - (-16\frac{1}{2}) = 57\frac{1}{2} \text{ cao}$$ |
| **Special Case 2** | Integrates expression in $y$ e.g. "$y^2 - 9y + 8 = 0"$: This can have first $M1$ in part (b) and no other marks. (It is not a method for finding this area) |  
| **Notes** | Take away trapezium again having used Method 2 loses last two marks  
Common Error:  
Integrates $-x^2 + 9x - 18$ is likely to be $M1A1A0dM1B0M1A0$  
Integrates $2 - 11x - x^2$ is likely to be $M1A0A0dM1B0M1A0$  
Writing $\int_2^9 (10x - x^2 - 8) - (10 - x) \, dx$ only earns final M mark |
<table>
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</table>
| 6(a)            | States or uses $\tan 2x = \frac{\sin 2x}{\cos 2x}$
|                 | $\sin 2x = 5 \sin 2x \Rightarrow \sin 2x - 5 \sin 2x \cos 2x = 0 \Rightarrow \sin 2x(1 - 5 \cos 2x) = 0 \,*$ | M1 |
|                 | $\cos 2x$ | A1 |

(b) $\sin 2x = 0$ gives $2x = 0, 180, 360 \quad \text{so } x = 0, 90, 180$

|                  | $\cos 2x = \frac{1}{2}$ gives $2x = 78.46 \text{ (or 78.5 or 78.4)} \quad \text{or } 2x = 281.54 \text{ (or 281.6)}$ | B1, B1 |
|                  | $x = 39.2 \text{ (or 39.3)}, \quad 140.8 \text{ (or 141)}$ | B1, B1 |

Notes

(a) **M1**: Statement that $\tan \theta = \frac{\sin \theta}{\cos \theta}$ or Replacement of $\tan$ (wherever it appears). Must be a correct statement but may involve $\theta$ instead of $2x$.

- **A1**: the answer is given so all steps should be given.

N.B. $\sin 2x - 5 \sin 2x \cos 2x = 0$ or $-5 \sin 2x \cos 2x + \sin 2x = 0$ or $\sin 2x(\frac{1}{\cos 2x} - 5) = 0 \quad \text{o.e.}$

- **must be seen** and be followed by printed answer for A1 mark
- $\sin 2x = 5 \sin 2x \cos 2x$ is not sufficient.

(b) Statement of 0 and 180 with no working gets B1 B0 (bod) as it is two solutions

**M1**: This mark for one of the two statements given (must relate to $2x$ not just to $x$)

**A1, A1**: first A1 for 39.2, second for 140.8

Special case solving $\cos 2x = -\frac{1}{5}$ giving $2x = 101.5$ or 258.5 is awarded M1A0A0

140.8 omitted would give M1A1A0

Allow answers which round to 39.2 or 39.3 and which round to 140.8 and allow 141

Answers in radians lose last A1 awarded (These are 0, 0.68, 1.57, 2.46 and 3.14)

Excess answers in range lose last A1 Ignore excess answers outside range.

All 5 correct answers with no extras and no working gets **full marks** in part (b). The answers imply the method here
### Question

#### 7 (a)

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>1.251</td>
<td>1.494</td>
<td>1.741</td>
<td>2</td>
</tr>
</tbody>
</table>

#### (b)

\[
\frac{1}{2} \times 0.25, \quad \{(1 + 2) + 2(1.251 + 1.494 + 1.741)\} \text{ o.e.}
\]

\[= 1.4965\]

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### Scheme

#### Marks

<table>
<thead>
<tr>
<th>Question number</th>
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<th>Marks</th>
</tr>
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<tbody>
<tr>
<td>7 (a)</td>
<td></td>
<td>B1, B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2)</td>
</tr>
<tr>
<td>7 (b)</td>
<td></td>
<td>B1, M1, A1 ft</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
</tr>
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<td></td>
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<td>(4)</td>
</tr>
</tbody>
</table>

### Notes

(a) first B1 for 1.494 and second B1 for 1.741 (1.740 is B0)

Wrong accuracy e.g. 1.49, 1.74 is B1B0

(b) B1: Need \(\frac{1}{2}\) of 0.25 or 0.125 o.e.

**M1:** requires first bracket to contain first plus last values and second bracket to include no additional values from the three in the table. If the only mistake is to omit one value from second bracket this may be regarded as a slip and M mark can be allowed (An extra repeated term forfeits the M mark however)

x values: M0 if values used in brackets are x values instead of y values

**A1 ft** follows their answers to part (a) and is for \{correct expression\}

Final A1: Accept 1.4965, 1.497 or 1.50 only after correct work. (No follow through except one special case below following 1.740 in table)

Separate trapezia may be used: B1 for 0.125, M1 for \(\frac{1}{2}h(a+b)\) used 3 or 4 times (and A1 ft if it is all correct)

e.g., 0.125(1 + 1.251) + 0.125(1.251 + 1.494) + 0.125(1.741 + 2) is M1 A0 equivalent to missing one term in \{\} in main scheme

Special Case: Bracketing mistake: i.e. 0.125(1+2) + 2(1.251+1.494+1.741) scores B1 M1 A0 A0 for 9.347 If the final answer implies that the calculation has been done correctly i.e. 1.4965 (then full marks can be given).

**Need to see trapezium rule – answer only (with no working) is 0/4 any doubts send to review**

Special Case: Uses 1.740 to give 1.49625 or 1.4963 or 1.496 or 1.50 gets B1 B0 B1 M1 A1 ft then A1 (lose 1 mark)

**NB Bracket is 11.972**
<table>
<thead>
<tr>
<th>Question number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 (a)</td>
<td>( h = \frac{60}{\pi x^2} ) or equivalent exact (not decimal) expression e.g. ( h = 60 \div \pi x^2 )</td>
<td>B1 (1)</td>
</tr>
<tr>
<td>(b)</td>
<td>((A) = 2\pi x^2 + 2\pi xh) or ((A) = 2\pi r^2 + 2\pi rh) or ((A) = 2\pi r^2 + \pi dh) may not be simplified and may appear on separate lines</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Either ((A) = 2\pi x^2 + 2\pi x\left(\frac{60}{\pi x^2}\right)) or As ( \pi xh = \frac{60}{x} ) then ((A) = 2\pi x^2 + 2\left(\frac{60}{x}\right))</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>( A = 2\pi x^2 + \left(\frac{120}{x}\right) ) *</td>
<td>A1 cso</td>
</tr>
<tr>
<td>(c)</td>
<td>( \frac{dA}{dx} = 4\pi x - \frac{120}{x^2} ) or ( = 4\pi x - 120x^{-2} )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( 4\pi x - \frac{120}{x^2} = 0 ) implies ( x^3 = \sqrt[3]{\frac{120}{4\pi}} ) or answers which round to 2.12 ( -2.12 is A0)</td>
<td>dM1 A1</td>
</tr>
<tr>
<td>(d)</td>
<td>( A = 2\pi(2.12)^2 + \frac{120}{2.12} ), = 85 (only ft ( x = 2 ) or 2.1 – both give 85)</td>
<td>M1, A1 (2)</td>
</tr>
<tr>
<td>(e)</td>
<td>Either ( \frac{d^2A}{dx^2} = 4\pi + \frac{240}{x^3} ) and sign considered (May appear in (c))</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( \text{Or (method 2) considers gradient to left and right of their 2.12 (e.g at 2 and 2.5)} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \text{Or (method 3) considers value of } A \text{ either side} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \text{Find numerical values for gradients and observes which is } &gt; 0 \text{ and therefore minimum (most substitute 2.12 but it is not essential to see a substitution ) (may appear in (c))} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \text{concludes minimum OR finds numerical values of } A , \text{ observing greater than minimum value and draws conclusion} )</td>
<td></td>
</tr>
</tbody>
</table>

Notes

(a) **B1**: This expression must be correct and in part (a) \( \frac{60}{\pi} \) is B0

(b) **B1**: Accept any equivalent correct form – may be on two or more lines.

**M1**: substitute their expression for \( h \) in terms of \( x \) into Area formula of the form \( kx^2 + cxh \)

**A1**: There should have been no errors in part (b) in obtaining this printed answer

(c) **M1**: At least one power of \( x \) decreased by 1  **A1** accept any equivalent correct answer

**M1**: Setting \( \frac{dA}{dx} = 0 \) and finding a value for \( x^3 \) (\( x^3 \) may be implied by answer). Allow \( \frac{dy}{dx} = 0 \)

**dM1**: Using **cube root** to find \( x \)

**A1**: For any equivalent correct answer (need 3sf or more) Correct answer implies previous M mark

(d) **M1**: Substitute the (+ve) \( x \) value found in (c) into equation for \( A \) and evaluate . **A1** is for 85 only

(e) **M1**: Complete method, usually one of the three listed in the scheme. For first method \( A''(x) \) must be attempted and sign considered

**A1**: Clear statements and conclusion. (numerical substitution of \( x \) is not necessary in first method shown, and \( x \) or calculation could be wrong but \( A''(x) \text{ must be correct} \). Must not see 85 substituted)
Question | Scheme | Marks
---|---|---
9 (a) | \[S_n = a + ar + (ar^2) + ... + ar^{n-1}\] and \[rS_n = ar + ar^2 + (ar^3) + ... + ar^n\]  
\[S_n - rS_n = a - ar^n\]  
\[S_n(1-r) = a(1-r^n)\]  
And so result \[S_n = \frac{a(1-r^n)}{(1-r)}\]  
(b) Divides one term by other (either way) to give \[r^2 = ...\] then square roots to give \[r = \sqrt{\frac{1.944}{5.4}}, \quad r = 0.6\] (ignore \(-0.6\))  
(c) Uses \[5.4 \div r^2\] or \[1.944 \div r^4\], to give \[a = \frac{15}{1-0.6}\] to obtain 37.5  
(d) Uses sum to infinity formula with values of \(a\) and \(r\) found earlier, provided \(r < 1\)  

Notes | Special Case | Common errors
---|---|---
(a) M1: Lists both of these sums \((S_n)\) may be omitted, \(rS_n\) (or \(rS\)) must be stated  
1st two terms must be correct in each series. Last term must be \(ar^{n-1}\) or \(ar^n\) in first series and the corresponding \(ar^n\) or \(ar^{n+1}\) in second series. Must be \(n\) and not a number. Reference made to other terms e.g. space or dots to indicate missing terms  
M1: Subtracts series for \(rS\) from series for \(S\) (or other way round) to give \(RHS = \pm (a - ar^n)\). This may have been obtained by following a pattern. If wrong power stated on line 1 M0 here. (Ignore LHS)M0M0M0A0  
dM1: Factorises both sides correctly—must follow from a previous M1 (It is possible to obtain M0M1M1A0 or M1M0M1A0) A1: completes the proof with no errors seen  
No errors seen: First line absolutely correct, omission of second line, third and fourth lines correct: M1M0M1A1  
See next sheet of common errors.  
Refer any attempts involving sigma notation, or any proofs by induction to team leader. Also attempts which begin with the answer and work backwards.  
(b) M1: Deduces \(r^2\) by dividing either term by other and attempts square root  
A1: any correct equivalent for \(r\) e.g. 3/5 Answer only is 2/2  
(Method 2) Those who find fourth term must use \(\sqrt{ab}\) and not \(\frac{1}{2}(a+b)\) then must use it in a division with given term to obtain \(r\) =  
(c) M1: May be done in two steps or more e.g. \(5.4 \div r\) then divided by \(r\) again  
A1ft: follow through their value of \(r\). Just \(a = 15\) with no wrong working implies M1A1  
(d) M1: States sum to infinity formula with values of \(a\) and \(r\) found earlier, provided \(|r| < 1\)  
A1: uses 15 and 0.6 (or 3/5) (This is not a ft mark) A1: 37.5 or exact equivalent  
(i) Fraction inverted in (b) \(r^i = \frac{5.4}{1.944}\) and \(r = 1\frac{2}{19}\), then correct ft gives M1A0 M1 A1ft M0A0A0 i.e. 3/7  
(ii) Uses \(r = 0.36\): (b)M0A0 (c)M1A1ft (d) M1A0A0 i.e. 3/7  
(iii) Uses \(ar^i = 5.4, \quad ar^i = 1.944\) Likely to have (b)M1A1 (c)M0A0 (d) M1A0A0 i.e.3/7