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Mark Scheme (Results)
January 2008

GCE

GCE Mathematics (6664/01)

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January 2008
6664 Core Mathematics C2
Mark Scheme

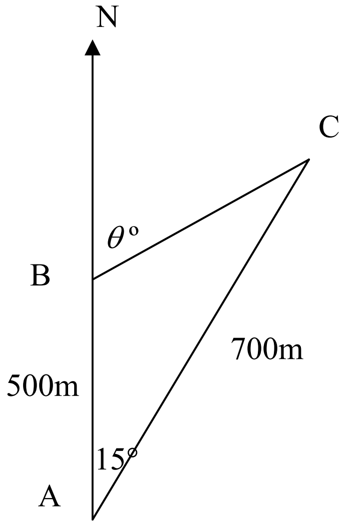
Question Number	Scheme	Marks
1. a)i)	f(3) = $3^3 - 2 \times 3^2 - 4 \times 3 + 8$; = 5	M1; A1
ii)	f(–2) = (– 8 – 8 + 8 + 8) = 0 (B1 on Epen, but A1 in fact)	A1 (3)
	M1 is for attempt at either f(3) or f(–3) in (i) or f(–2) or f(2) in (ii).	
(b)	$[(x + 2)](x^2 - 4x + 4)$ (= 0 not required) [must be seen or used in (b)]	M1 A1
	$(x + 2)(x - 2)^2$ (= 0) (can imply previous 2 marks)	M1
	Solutions: $x = 2$ or -2 (both) or (–2, 2, 2) A1 (4)	**[7]**
Notes: (a)	No working seen: Both answers correct scores full marks One correct ;M1 then A1B0 or A0B1, whichever appropriate.	
	<u>Alternative (Long division)</u> Divide by (x – 3) OR (x + 2) to get $x^2 + ax + b$, *a* may be zero [M1] $x^2 + x - 1$ and $+5$ seen i.s.w. (or "remainder = 5") [A1] $x^2 - 4x + 4$ and 0 seen (or "no remainder") [B1]	
(b)	First M1 requires division by a found factor ; e.g $(x + 2)$, $(x - 2)$ or what candidate thinks is a factor to get $(x^2 + ax + b)$, a may be zero. First A1 for $[(x + 2)](x^2 - 4x + 4)$ or $(x - 2)(x^2 - 4)$ Second M1:attempt to factorise their found quadratic. (or use formula correctly) [Usual rule: $x^2 + ax + b = (x + c)(x + d)$, *where* $\lvert cd \rvert = \lvert b \rvert$.] **N.B.** Second A1 is for solutions, not factors <u>*Alternative (first two marks)*</u> $(x+2)(x^2 + bx + c) = x^3 + (2+b)x^2 + (2b+c)x + 2c = 0$ and then compare with $x^3 - 2x^2 - 4x + 8 = 0$ to find *b* and *c*. [M1] $b = -4$, $c = 4$ [A1] <u>*Method of grouping*</u> $x^3 - 2x^2 - 4x + 8 = x^2(x - 2)$,$4(x \pm 2)$ M1; = $x^2(x - 2) - 4(x - 2)$ A1 [= $(x^2 - 4)(x - 2)$] = $(x+2)(x-2)^2$ M1 Solutions: $x = 2$, $x = -2$ both A1	
2. (a)	Complete method, using terms of form ar^k, to find r	M1
	[e.g. **Dividing** $ar^6 = 80$ by $ar^3 = 10$ to find r ; $r^6 - r^3 = 8$ is M0]	
	$r = 2$	A1 (2)
(b)	Complete method for finding a	M1
	[e.g. Substituting value for *r* into equation of form ar^k = 10 or 80 and finding a value for *a*.]	

	($8a = 10$) $a = \frac{5}{4} = 1\frac{1}{4}$ (equivalent single fraction or 1.25) A1 (2)
(c)	Substituting their values of *a* and *r* into **correct** formula for sum. M1 $S = \frac{a(r^n - 1)}{r - 1} = \frac{5}{4}(2^{20} - 1)$ (= 1310718.75) 1 310 719 (only this) A1 (2) **[6]**
Notes:	(a) M1: Condone errors in powers, e.g. $ar^4 = 10$ and/or $ar^7 = 80$, A1: For $r = 2$, allow even if $ar^4 = 10$ and $ar^7 = 80$ used (just these) (M mark can be implied from numerical work, if used correctly) (b) M1: Allow for numerical approach: e.g. $\frac{10}{r_c^{\ 3}} \leftarrow \frac{10}{r_c^{\ 2}} \leftarrow \frac{10}{r_c} \leftarrow 10$ In (a) and (b) correct answer, with no working, allow both marks. (c) Attempt 20 terms of series and add is M1 (correct last term 655360) If formula **not** quoted, errors in applying their *a* and/or *r* is M0 Allow full marks for correct answer with no working seen.
3. (a)	$\left(1 + \frac{1}{2}x\right)^{10} = 1 + \underline{\binom{10}{1}\left(\frac{1}{2}x\right) + \binom{10}{2}\left(\frac{1}{2}x\right)^2 + \binom{10}{3}\left(\frac{1}{2}x\right)^3}$ M1 A1 $= 1 + 5x$; + $\frac{45}{4}$ (or 11.25)x^2 + $15x^3$ (coeffs need to be these, i.e, simplified) A1; A1 (4) [Allow A1A0, if totally correct with unsimplified, single fraction coefficients)
(b)	$(1 + \frac{1}{2} \times 0.01)^{10} = 1 + 5(0.01) + (\frac{45}{4}\ or\ 11.25)(0.01)^2 + 15(0.01)^3$ M1 A1√ = 1 + 0.05 + 0.001125 + 0.000015 = 1.05114 cao A1 (3) **[7]**
Notes:	(a) For M1 first A1: Consider underlined expression only. M1 Requires correct structure for at least two of the three terms: (i) Must be attempt at binomial coefficients. (ii) Must have increasing powers of x , (iii) May be listed, need not be added; *this applies for all marks*. First A1: Requires all three correct terms but need not be simplified, allow 1^{10} etc, ${}^{10}C_2$ etc, and condone omission of brackets around powers of ½x Second A1: Consider as B1 for 1 + 5x (b) For M1: Substituting their (0.01) into their (a) result First A1 (f.t.): Substitution of (0.01) into their 4 termed expression in (a) Answer with no working scores no marks (calculator gives this answer)

4.	(a)	$3\sin^2\theta - 2\cos^2\theta = 1$	
		$3\sin^2\theta - 2(1 - \sin^2\theta) = 1$ (M1: Use of $\sin^2\theta + \cos^2\theta = 1$)	M1
		$3\sin^2\theta - 2 + 2\sin^2\theta = 1$	
		$5\sin^2\theta = 3$ cso AG	A1 (2)
	(b)	$\sin^2\theta = \frac{3}{5}$, so $\sin\theta = (\pm)\sqrt{0.6}$	M1
		Attempt to solve both $\sin\theta$ = +.. and $\sin\theta$ = – (may be implied by later work)	M1
		$\theta = 50.7685°$ awrt $\theta = 50.8°$ (dependent on first M1 only)	A1
		θ $(= 180° - 50.7685_c°)$; $= 129.23...°$ awrt $129.2°$	M1; A1 √
		[f.t. dependent on first M and 3rd M]	
		$\sin\theta = -\sqrt{0.6}$	
		$\theta = 230.785°$ and $309.23152°$ awrt $230.8°$, $309.2°$ (both)	M1A1 (7)
			[9]
Notes:		(a) N.B: AG; need to see at least one line of working after substituting $\cos^2\theta$.	
		(b) First M1: Using $5\sin^2\theta = 3$ to find value for $\sin\theta$ or θ	
		Second M1: Considering the – value for $\sin\theta$. (usually later)	
		First A1: Given for awrt 50.8°. Not dependent on second M.	
		Third M1: For $(180 - 50.8_c)°$, need not see written down	
		Final M1: **Dependent** on second M (but may be implied by answers)	
		For (180 + candidate's 50.8)° **or** $(360 - 50.8_c)°$ **or equiv.**	
		Final A1: Requires both values. (no follow through)	
		[Finds $\cos^2\theta = k$ (k = 2/5) and so $\cos\theta = (\pm)$...M1, then mark equivalently]	

5.	<u>Method 1</u> (Substituting a = 3b into second equation at some stage)	
	Using a law of logs correctly (anywhere) e.g. $\log_3 ab = 2$	M1
	Substitution of 3*b* for *a* (or a/3 for b) e.g. $\log_3 3b^2 = 2$	M1
	Using base correctly on correctly derived $\log_3 p = q$ e.g. $3b^2 = 3^2$	M1
	First correct value $b = \sqrt{3}$ (allow $3^{1/2}$)	A1
	Correct method to find other value (dep. on at least first M mark)	M1
	Second answer $a = 3b = 3\sqrt{3}$ or $\sqrt{27}$	A1
	<u>Method 2</u> (Working with two equations in $\log_3 a$ and $\log_3 b$)	
	" Taking logs" of first equation and " separating" $\log_3 a = \log_3 3 + \log_3 b$ $(= 1 + \log_3 b)$	M1
	Solving simultaneous equations to find $\log_3 a$ or $\log_3 b$ [$\log_3 a = 1\frac{1}{2}$, $\log_3 b = \frac{1}{2}$]	M1
	Using base correctly to find a or b	M1
	Correct value for *a* or *b* $a = 3\sqrt{3}$ or $b = \sqrt{3}$	A1
	Correct method for second answer, dep. on first M; correct second answer [Ignore negative values]	M1;A1**[6]**
Notes:	Answers must be exact; decimal answers lose both A marks There are several variations on Method 1, depending on the stage at which *a* = 3*b* is used, but they should all mark as in scheme. In this method, the first three method marks on Epen are for (i) First M1: correct use of log law, (ii) Second M1: substitution of *a* = 3*b*, (iii) Third M1: requires using base correctly on correctly derived $\log_3 p = q$	

6.



(a)

$BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700 \cos 15°$ — M1 A1

(= 63851.92…)

BC = 253 awrt — A1 (3)

(b)

$\dfrac{\sin B}{700} = \dfrac{\sin 15}{\text{candidate's } BC}$ — M1

$\sin B = \sin 15 \times 700 / 253_c = 0.716..$ and giving an **obtuse** B (134.2°) dep — M1

θ = 180° – candidate's angle B (Dep. on first M only, B can be acute) — M1

θ = 180 – 134.2 = (0)45.8 (allow 46 or awrt 45.7, 45.8, 45.9) — A1 (4) **[7]**

[46 needs to be from correct working]

Notes:

(a) If use cos 15° = ….., then A1 not scored until written as BC² = … correctly

Splitting into 2 triangles BAX and CAX, where X is foot of perp. from B to AC

Finding value for *BX* and *CX* and using Pythagoras — M1

$BC^2 = (500\sin 15°)^2 + (700 - 500\cos 15°)^2$ — A1

BC = 253 awrt — A1

(b) Several alternative methods: (Showing the M marks, 3rd M dep. on first M))

(i) $\cos B = \dfrac{500^2 + \text{candidate's}BC^2 - 700^2}{2\text{x}500\text{xcandidate's}BC}$ or $700^2 = 500^2 + BC_c{}^2 - 2x500xBC_c$ M1

Finding angle B M1, then M1 as above

(ii) 2 triangle approach, as defined in notes for (a)

$\tan CBX = \dfrac{700 - valueforAX}{valueforBX}$ — M1

Finding value for $\angle CBX$ ($\approx 59°$) — M1

$\theta = [180° - (75° + candidate's\ \angle CBX)]$ — M1

(iii) Using sine rule (or cos rule) to find C first:

Correct use of sine or cos rule for C M1, Finding value for C M1

Either B =180° – (15° + candidate's C) or θ = (15° + candidate's C) M1

(iv) $700\cos 15° = 500 + BC\cos\theta$ M2 {first two Ms earned in this case}

Solving for θ; θ = 45.8 (allow 46 or5.7, 45.8, 45.9 M1;A1

7 (a)	Either solving $0 = x(6 - x)$ and showing $x = 6$ (and $x = 0$) or showing (6,0) (and $x = 0$) satisfies $y = 6x - x^2$ [allow for showing x = 6]	B1 (1)
(b)	Solving $2x = 6x - x^2$ $(x^2 = 4x)$ to x = .. $x = 4$ (and $x = 0$) Conclusion: when $x = 4$, $y = 8$ and when $x = 0$, $y = 0$,	M1 A1 A1 (3)
(c)	(Area =) $\int_{(0)}^{(4)} (6x - x^2)\,dx$ Limits not required Correct integration $3x^2 - \frac{x^3}{3}$ (+ c) Correct use of correct limits on their result above (see notes on limits) [" $3x^2 - \frac{x^3}{3}$ "]^4 – [" $3x^2 - \frac{x^3}{3}$ "]$_0$ with limits substituted $[= 48 - 21\frac{1}{3} = 26\frac{2}{3}]$ Area of triangle = 2 × 8 =16 (Can be awarded even if no M scored, i.e. B1) Shaded area = ± (area under curve – area of triangle) applied correctly $(= 26\frac{2}{3} - 16) = 10\frac{2}{3}$ (awrt 10.7)	M1 A1 M1 A1 M1 A1 (6)**[10]**
Notes	(b) In scheme first A1: need only give $x = 4$ If *verifying approach* used: Verifying (4,8) satisfies both the line and the curve M1(attempt at both), Both shown successfully A1 For final A1, (0,0) needs to be mentioned ; accept " clear from diagram" (c) **Alternative** Using Area = $\pm \int_{(0)}^{(4)} \{(6x - x^2); \ -2x\}\,dx$ approach (i) If candidate integrates separately can be marked as main scheme If combine to work with = $\pm \int_{(0)}^{(4)} (4x - x^2)\,dx$, **first** M mark and **third** M mark $= (\pm)\,[2x^2 - \frac{x^3}{3}\ (+ c)\ ]$ A1, Correct use of correct limits on their result **second** M1, Totally correct, unsimplified ± expression (may be implied by correct ans.) A1 10⅔ A1 [Allow this if, having given - 10⅔, they correct it] M1 *for correct use of correct limits*: Must substitute correct limits for their strategy into a changed expression and subtract, either way round, e.g $\pm\{[\ ]^4 - [\ ]_0\}$ If a long method is used, e,g, finding three areas, this mark only gained for correct strategy and all limits need to be correct for this strategy. Use of trapezium rule: M0A0MA0,possibleA1for triangle M1(if correct application of trap. rule from $x = 0$ to $x = 4$) A0	

8 (a)	$(x-6)^2 + (y-4)^2 = ;\ 3^2$	B1; B1 (2)
(b)	Complete method for MP: $= \sqrt{(12-6)^2+(6-4)^2}$	M1
	$= \sqrt{40}$ (= 6.325)	A1
	[These first two marks can be scored if seen as part of solution for (c)]	
	Complete method for $\cos\theta$, $\sin\theta$ or $\tan\theta$	M1
	e.g. $\cos\theta = \frac{\text{MT}}{\text{MP}} = \frac{3}{candidate's\sqrt{40}}$ (= 0.4743) (θ = 61.6835°)	
	[If TP = 6 is used, then M0]	
	θ = 1.0766 rad **AG**	A1 (4)
(c)	Complete method for area TMP; e.g. $= \frac{1}{2}\times 3\times\sqrt{40}\sin\theta$	M1
	$= \frac{3}{2}\sqrt{31}$ (= 8.3516..) allow awrt 8.35	A1
	Area (sector) MTQ $= 0.5\times 3^2\times 1.0766$ (= 4.8446…)	M1
	Area TPQ = candidate's (8.3516.. – 4.8446..)	M1
	= 3.507 awrt	A1 (5)
	[Note: 3.51 is A0]	**[11]**
Notes	(a) Allow 9 for 3^2.	
	(b) First M1 can be implied by $\sqrt{40}$	
	For second M1:	
	May find TP = $\sqrt{(\sqrt{40})^2 - 3^2} = \sqrt{31}$, then either	
	$\sin\theta = \frac{TP}{MP} = \frac{\sqrt{31}}{\sqrt{40}}$ (= 0.8803...) *or* $\tan\theta = \frac{\sqrt{31}}{3}$ (1.8859..) or cos rule	
	NB. Answer is given, but allow final A1 if all previous work is correct.	
	(c) First M1: (alternative) $\frac{1}{2}\times 3\times\sqrt{40-9}$	

9 (a)	(Total area) = $3xy + 2x^2$	B1
	(Vol:) $x^2y = 100$ $\left(y = \frac{100}{x^2}, xy = \frac{100}{x}\right)$	B1
	Deriving expression for area in terms of x only (Substitution, or clear use of, y or xy into expression for area)	M1
	(Area =) $\frac{300}{x} + 2x^2$ **AG**	A1 cso (4)
(b)	$\frac{dA}{dx} = -\frac{300}{x^2} + 4x$	M1A1
	Setting $\frac{dA}{dx} = 0$ and finding a value for correct power of x, for cand. M1 [$x^3 = 75$] $x = 4.2172$ awrt 4.22 (allow exact $\sqrt[3]{75}$)	A1 (4)
(c)	$\frac{d^2A}{dx^2} = \frac{600}{x^3} + 4$ = positive therefore minimum	M1A1 (2)
(d)	Substituting found value of x into (a) (Or finding y for found x and substituting both in $3xy + 2x^2$)	M1
	$\left[y = \frac{100}{4.2172^2} = 5.6228\right]$ Area = 106.707 awrt 107	A1 (2) **[12]**
Notes	(a) First B1: Earned for correct unsimplified expression, isw. (c) For M1: Find $\frac{d^2A}{dx^2}$ and explicitly consider its sign, state > 0 or "positive" A1: Candidate's $\frac{d^2A}{dx^2}$ must be correct for their $\frac{dA}{dx}$, sign must be + ve and conclusion "so minimum", (allow QED, √). (may be wrong x, or even no value of x found) <u>Alternative:</u> M1: Find value of $\frac{dA}{dx}$ on either side of "$x = \sqrt[3]{75}$" and consider sign A1: Indicate sign change of negative to positive for $\frac{dA}{dx}$, and conclude minimum. OR M1: Consider values of A on either side of "$x = \sqrt[3]{75}$" and compare with"107" A1: Both values greater than "$x = 107$" and conclude minimum. Allow marks for (c) and (d) where seen; even if part labelling confused.	