Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced Subsidiary

Wednesday 18 January 2006 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Lilac or Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 6 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. A particle P of mass 0.5 kg is released from rest at time $t = 0$ and falls vertically through a liquid. The motion of P is resisted by a force of magnitude $2v$ N, where v m s^{-1} is the speed of v at time t seconds.

(a) Show that $5\frac{\mathrm{d}v}{\mathrm{d}t} = 49 - 20v$.

(2)

(b) Find the speed of P when $t = 1$.

(5)

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2. A small smooth sphere S of mass m is attached to one end of a light inextensible string of length $2a$. The other end of the string is attached to a fixed point A which is at a distance $a\sqrt{3}$ from a smooth vertical wall. The sphere S hangs at rest in equilibrium. It is then projected horizontally towards the wall with a speed $\sqrt{\left(\frac{37ga}{5}\right)}$.

(a) Show that S strikes the wall with speed $\sqrt{\left(\frac{27ga}{5}\right)}$.

(4)

Given that the loss in kinetic energy due to the impact with the wall is $\frac{3mga}{5}$,

(b) find the coefficient of restitution between S and the wall.

(7)

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3. Two ships P and Q are moving with constant velocity. At 3 p.m., P is 20 km due north of Q and is moving at 16 km h^{-1} due west. To an observer on ship P, ship Q appears to be moving on a bearing of 030° at 10 km h^{-1}. Find

(a) (i) the speed of Q,

(ii) the direction in which Q is moving, giving your answer as a bearing to the nearest degree,

(6)

(b) the shortest distance between the ships,

(3)

(c) the time at which the two ships are closest together.

(3)

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4. A particle P of mass m is suspended from a fixed point by a light elastic spring. The spring has natural length a and modulus of elasticity $2m\omega^2 a$, where ω is a positive constant. At time $t = 0$ the particle is projected vertically downwards with speed U from its equilibrium position. The motion of the particle is resisted by a force of magnitude $2m\omega v$, where v is the speed of the particle. At time t, the displacement of P downwards from its equilibrium position is x.

(*a*) Show that $\dfrac{d^2x}{dt^2} + 2\omega\dfrac{dx}{dt} + 2\omega^2 x = 0.$

(5)

Given that the solution of this differential equation is $x = e^{-\omega t}(A \cos \omega t + B \sin \omega t)$, where A and B are constants,

(*b*) find A and B.

(4)

(*c*) Find an expression for the time at which P first comes to rest.

(3)

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5. Two smooth uniform spheres A and B have equal radii. Sphere A has mass m and sphere B has mass km. The spheres are at rest on a smooth horizontal table. Sphere A is then projected along the table with speed u and collides with B. Immediately before the collision, the direction of motion of A makes an angle of 60° with the line joining the centres of the two spheres. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(*a*) Show that the speed of B immediately after the collision is $\dfrac{3u}{4(k+1)}$.

(6)

Immediately after the collision the direction of motion of A makes an angle arctan $(2\sqrt{3})$ with the direction of motion of B.

(*b*) Show that $k = \frac{1}{2}$.

(6)

(*c*) Find the loss of kinetic energy due to the collision.

(4)

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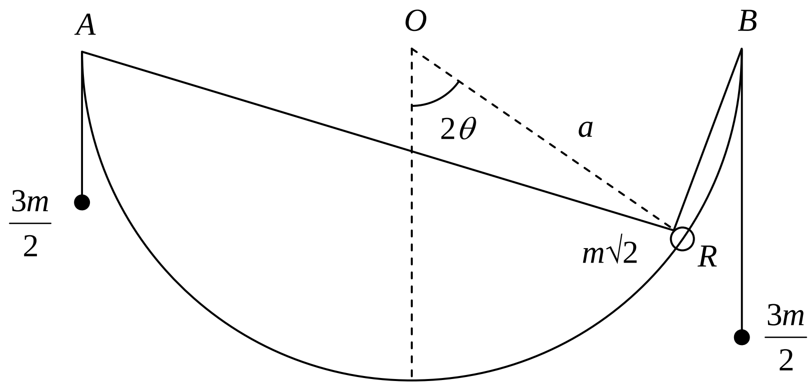
6. **Figure 1**



A smooth wire with ends A and B is in the shape of a semi-circle of radius a. The mid-point of AB is O. The wire is fixed in a vertical plane and hangs below AB which is horizontal. A small ring R, of mass $m\sqrt{2}$, is threaded on the wire and is attached to two light inextensible strings. The other end of each string is attached to a particle of mass $\frac{3m}{2}$. The particles hang vertically under gravity, as shown in Figure 1.

(*a*) Show that, when the radius OR makes an angle 2θ with the vertical, the potential energy, V, of the system is given by

$$V = \sqrt{2}mga(3\cos\theta - \cos 2\theta) + \text{constant}.$$

(7)

(*b*) Find the values of θ for which the system is in equilibrium.

(6)

(*c*) Determine the stability of the position of equilibrium for which $\theta > 0$.

(4)

TOTAL FOR PAPER: 75 MARKS

END