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GCSE

Mathematics A (1MA0)

Scheme of work (one-year)

Edexcel GCSE in Mathematics A (1MA0)

For first teaching from September 2010

Issue 2

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A PEARSON COMPANY

Contents

This scheme of work is Issue 2. Key changes to text book references are sidelined. We will inform centres to any changes to the issue. The latest issue can be found on the Edexcel website: www.edexcel.com

Introduction

This scheme of work is based on a two term model over one year for both Foundation and Higher tier students.

It can be used directly as a scheme of work for the GCSE Mathematics A specification (1MA0).

The scheme of work is structured so each topic contains:

- Module number
- Estimated teaching time, however this is only a guideline and can be adapted according to individual teaching needs
- Tier
- Contents
- Prior knowledge
- Objectives for students at the end of the module
- Ideas for differentiation and extension activities
- Notes for general mathematical teaching points and common misconceptions
- Resources

Updates will be available via a link from the Edexcel mathematics website (www.maths10.co.uk).

References to Edexcel published student books, both linear and post – 16 titles, for the course are given in a table, under the heading Resources, at the end of each module.

For example:

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Foundation Student book	1.1 – 1.5, 2.1 – 2.6, 3.6, 4.6
Edexcel GCSE Mathematics 16+ Student Book	1.1 – 1.5

TF support indicates that further material is in the Edexcel GCSE Mathematics 16+ Teacher Resource File.

n/a indicates a topic is not included in the post-16 textbook.

GCSE Mathematics A (1MA0)

Foundation Tier

Linear
Scheme of work

Foundation course overview

The table below shows an overview of modules in the Linear Foundation tier scheme of work.

Teachers should be aware that the estimated teaching hours are approximate and should be used as a guideline only.

Module number	Title	Estimated teaching hours
1	Number	4
2	Decimals and rounding	4
3	Fractions	3
4	Using a calculator	2
5	Percentages	4
6	Ratio and proportion	3
7	Algebra 1	4
8	Algebra 2	2
9	Sequences	2
10	Graphs 1	3
11	Linear equations and inequalities	5
12	Graphs 2	4
13	Formulae	3
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15	Angles 1	3
16	Angles 2	5
17	Perimeter and area of 2-D shapes	4
18	Circles	3
19	Constructions and loci	2
20	3-D shapes	4
21	Transformations	4
22	Pythagoras' theorem	4
23	Measure	3
24	Collecting and recording data	3
25	Processing, representing and interpreting data	4
26	Averages and range	3
27	Line diagrams and scatter graphs	2
28	Probability	5
	Total	**95 HOURS**

Module 1 **Time: 3 – 5 hours**

GCSE Tier: Foundation

Contents: Number

N b	Order integers
N u	Approximate to specified or appropriate degrees of accuracy
N a	Add, subtract, multiply and divide integers
N c	Use the concepts and vocabulary of factor (divisor), multiple, common factor, Highest Common Factor (HCF), Lowest Common Multiple (LCM), prime number and prime factor decomposition

PRIOR KNOWLEDGE

The ability to order numbers

An appreciation of place value

Experience of the four operations using whole numbers

Knowledge of integer complements to 10 and to 100

Knowledge of strategies for multiplying and dividing whole numbers by 2, 4, 5 and 10

OBJECTIVES

By the end of the module the student should be able to:

- use and order integers
- write numbers in words and write numbers from words
- add and subtract integers
- recall all multiplication facts to 10×10, and use them to derive quickly the corresponding division facts multiply or divide any number by powers of 10
- multiply and divide integers
- add, subtract, multiply and divide (negative) integers
- round whole numbers to the nearest: 10, 100, 1000
- recognise even and odd numbers
- identify factors, multiples and prime numbers
- find the prime factor decomposition of positive integers
- find the common factors and common multiples of two numbers
- find the Lowest Common Multiple (LCM) and Highest Common Factor (HCF) of two numbers
- recall integer squares up to 15×15 and the corresponding square roots
- recall the cubes of 2, 3, 4, 5 and 10
- find squares and cubes
- find square roots and cube roots

DIFFERENTIATION AND EXTENSION

Directed number work with multi-step calculations

Try investigations with digits 3, 7, 5 and 2 and challenge students to find the biggest number, smallest odd number, the largest sum or product etc

Calculator exercise to check factors of larger numbers

Use prime factors to find LCM

Use a number square to find primes (sieve of Eratosthenes)

Use division tests, eg when a number is divisible by 3 etc

NOTES

Students should present all working clearly

For non-calculator methods, students should ensure that remainders are shown as evidence of working

Try different methods from traditional ones, eg Russian or Chinese methods for multiplication

Incorporate Functional Elements where appropriate

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Foundation Student book	1.1 – 1.12, 5.5
Edexcel GCSE Mathematics 16+ Student Book	1.1 – 1.3, TF support

Module 2 **Time: 3 – 5 hours**

GCSE Tier: Foundation

Contents: Decimals and rounding

N b	Order decimals
N a	Add, subtract, multiply and divide any number
N d	Use the terms square, positive and negative square root, cube and cube root
N j	Use decimal notation
N q	Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
N u	Approximate to specified or appropriate degrees of accuracy

PRIOR KNOWLEDGE

The concept of a decimal

The four operations

OBJECTIVES

By the end of the module the student should be able to:

- understand place value, identifying the values of the digits
- write decimals in order of size
- add and subtract decimals
- multiply and divide decimal numbers by integers and decimal numbers
- round decimals to the nearest integer, a given number of decimal places or to one significant figure
- know that, eg $13.5 \div 0.5 = 135 \div 5$
- check their answers by rounding, and know that, eg $9.8 \times 17.2 \approx 10 \times 17$
- check calculations by rounding, eg $29 \times 31 \approx 30 \times 30$
- check answers by inverse calculation, eg if $9 \times 23 = 207$ then $207 \div 9 = 23$

DIFFERENTIATION AND EXTENSION

Practise long multiplication and division without using a calculator

Mental maths problems with negative powers of 10, eg 2.5×0.01, 0.001

Directed number work with decimal numbers

Use decimals in real-life problems as much as possible, eg Best Buys

Use functional examples such as entry into theme parks, cost of holidays, sharing the cost of a meal

Money calculations that require rounding answers to the nearest penny

Multiply and divide decimals by decimals with more than 2 decimal places

Round answers to appropriate degrees of accuracy to suit the context of the question

Calculator exercise to find squares, cubes and square roots of larger numbers (using trial and improvement)

NOTES

Advise students not to round decimals, used in calculations, until stating the final answer

For non-calculator methods ensure that remainders are shown as evidence of working

Students need to be clear about the difference between decimal places and significant figures

Link decimals to Statistics and Probability, eg the mean should not be rounded, the probability of all events occurring is equal to 1

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	5.1, 5.11
Edexcel GCSE Mathematics 16+ Student Book	1.2, 1.4 – 1.6, TF support

Module 3 **Time: 2 – 4 hours**

GCSE Tier: Foundation

Contents: Fractions

N h	Understand equivalent fractions, simplify a fraction by cancelling all common factors
N i, a	Add, subtract, multiply and divide fractions
N b	Order rational numbers
N j	Use decimal notation and understand that decimals and fractions are equivalent
N o	Write one number as a fraction of another

PRIOR KNOWLEDGE

Multiplication facts

Ability to find common factors

A basic understanding of fractions as being 'parts of a whole unit'

Use of a calculator with fractions

OBJECTIVES

By the end of the module the student should be able to:

- visualise a fraction diagrammatically
- understand a fraction as part of a whole
- recognise and write fractions used in everyday situations
- write one number as a fraction of another
- write a fraction in its simplest form and find equivalent fractions
- compare the sizes of fractions using a common denominator
- write an improper fraction as a mixed number
- find fractions of amounts
- multiply and divide fractions
- add and subtract fractions by using a common denominator
- convert between fractions and decimals

DIFFERENTIATION AND EXTENSION

Careful differentiation is essential as this topic is dependent on the student's ability

Relate simple fractions to percentages and vice versa

Work with improper fractions and mixed numbers, eg divide 5 pizzas between 3 people

Solve word problems involving fractions and in real-life problems, eg finding a perimeter from a shape with fractional side lengths

Link fractions with probability questions

NOTES

Regular revision of fractions is essential

Demonstrate how to use the fraction button on a calculator, in order to check solutions

Use real-life examples whenever possible

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	8.1 – 8.8, 10.1
Edexcel GCSE Mathematics 16+ Student Book	Chapter 2, TF support

Module 4 **Time: 1 – 3 hours**

GCSE Tier: Foundation

Contents: Using a calculator

N k Recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals

N q Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations

N v Use calculators effectively and efficiently

PRIOR KNOWLEDGE

Four operations

Rounding

OBJECTIVES

By the end of the module the student should be able to:

- convert fractions into decimals
- recognise that some fractions are recurring decimals
- find reciprocals
- understand 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal because division by zero is undefined)
- interpret the answer on a calculator display
- use calculators effectively and efficiently

DIFFERENTIATION AND EXTENSION

Convert a recurring decimal into a fraction

More complex calculations

NOTES

Students should show what is entered into their calculator, not just the answer

Students should write down the 'full' calculator answer before rounding

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	10.1 – 10.5
Edexcel GCSE Mathematics 16+ Student Book	Chapter 3

Module 5 **Time: 3 – 5 hours**

GCSE Tier: Foundation

Contents: Percentages

N l	Understand that 'percentage' means 'number of parts per 100' and use this to compare proportions
N m	Use percentages
N o	Interpret fractions, decimals and percentages as operators
N v	Use calculators effectively and efficiently to find percentages

PRIOR KNOWLEDGE

Four operations of number

The concepts of a fraction and a decimal

Number complements to 10 and multiplication tables

Awareness that percentages are used in everyday life

OBJECTIVES

By the end of the module the student should be able to:

- understand that a percentage is a fraction in hundredths
- convert between fractions, decimals and percentages
- calculate the percentage of a given amount
- use decimals to find quantities
- use percentages in real-life situations
  - VAT
  - value of profit or loss
  - simple interest
  - income tax
- find a percentage of a quantity in order to increase or decrease
- use percentages as multipliers
- write one number as a percentage of another number
- use percentages to solve problems

DIFFERENTIATION AND EXTENSION

Consider fractions as percentages of amounts, eg 12.5% = 0.125 = $\frac{1}{8}$

Consider percentages which convert to recurring decimals, eg $33\frac{1}{3}$%,

and situations which lead to percentages of more than 100%

Use fraction, decimal and percentage dominos or follow me cards

Investigate the many uses made of percentages, particularly in the media

Practise the ability to convert between different forms

Use a mixture of calculator and non-calculator methods

Use ideas for wall display; students make up their own poster to explain, say, a holiday reduction

Use functional skills questions to look at questions in context

Combine multipliers to simplify a series of percentage changes

Problems which lead to the necessity of rounding to the nearest penny, eg real-life contexts

Investigate comparisons between simple and compound interest calculations

NOTES

Use Functional Elements questions using fractions, eg $\frac{1}{4}$ off the list price when comparing different sale prices

Keep using non-calculator methods, eg start with 10%, then 1% in order to find the required percentages, although encourage students to use a calculator for harder percentages

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Foundation Student book	19.1 – 19.4
Edexcel GCSE Mathematics 16+ Student Book	4.1 – 4.2, 4.4, TF support

Module 6 **Time: 2 – 4 hours**

GCSE Tier: Foundation

Contents: Ratio and proportion

N p	Use ratio notation, including reduction to its simplest form and its various links to fraction notation
N t	Divide a quantity in a given ratio
N q	Understand and use number operations and inverse operations

PRIOR KNOWLEDGE

Using the four operations

Ability to recognise common factors

Knowledge of fractions

OBJECTIVES

By the end of the module the student should be able to:

- understand what is meant by ratio and use ratios
- write a ratio in its simplest form and find an equivalent ratio
- solve a ratio problem in context, eg recipes
- share a quantity in a given ratio
- solve problems involving money conversions, eg £s to Euros etc

DIFFERENTIATION AND EXTENSION

Consider maps: draw a plan of the school

Further problems involving scale drawing, eg find the real distance in metres between two points on 1 : 40000 map

Plan a housing estate with variety of different sized houses

Currency calculations using foreign exchange rates

Link ratios and proportion to Functional Elements, eg investigate the proportion of different metals in alloys, the ingredients needed for recipes for fewer or more people, mixing cement, planting forests, comparing prices of goods here and abroad, Best buy type questions

NOTES

Students often find ratios with 3 parts difficult

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	24.1 – 24.4
Edexcel GCSE Mathematics 16+ Student Book	5.1 – 5.2

Module 7 **Time: 3 – 5 hours**

GCSE Tier: Foundation

Contents: Algebra 1

A a	Distinguish the different roles played by letter symbols in algebra, using the correct notation
A b	Distinguish in meaning between the words 'equation', 'formula' and 'expression'
A c	Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors
A f	Substitute positive and negative numbers into expressions

PRIOR KNOWLEDGE

Experience of using a letter to represent a number

Ability to use negative integers with the four operations

OBJECTIVES

By the end of the module the student should be able to:

- use notation and symbols correctly
- write an expression
- simplify algebraic expressions in one or more variables, by adding and subtracting like terms
- simplify expressions
- multiply a single algebraic term over a bracket
- factorise algebraic expressions by taking out common factors
- understand the difference between the words 'equation', 'formula', and 'expression'
- substitute positive and negative numbers into expressions

DIFFERENTIATION AND EXTENSION

Look at patterns in games like 'frogs', eg Total moves $= R \times G + R + G$

Look at methods to understand expressions, eg there are 'b' boys and 'g' girls in a class, what is the total 't' number of students in the class

Further work, such as collecting like terms involving negative terms, collecting terms where each term may consist of more than one letter eg $3ab + 4ab$

NOTES

Emphasise correct use of symbolic notation, eg $3x$ rather than $3 \times x$

Students should present all work neatly and use the appropriate algebraic vocabulary

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	4.1 – 4.9, 9.5 – 9.6
Edexcel GCSE Mathematics 16+ Student Book	6.1 – 6.5, TF support

Module 8 **Time: 1 – 3 hours**

GCSE Tier: Foundation

Contents: Algebra 2

N e	Use index notation for squares, cubes and powers of 10
N f	Use the index laws for multiplication and division of integer powers
N q	Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
A c	Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors

PRIOR KNOWLEDGE

Number complements to 10 and multiplication/division facts

Recognition of basic number patterns

Experience of classifying integers

Squares and cubes

Experience of using a letter to represent a number

Ability to use negative numbers with the four operations

OBJECTIVES

By the end of the module the student should be able to:

- use index notation for squares and cubes
- use index notation for powers of 10
- find the value of calculations using indices
- use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, and of powers of a power
- use simple instances of index laws
- use brackets and the hierarchy of operations (BIDMAS)

DIFFERENTIATION AND EXTENSION

Further work on indices to include negative and/or fractional indices

Use various investigations leading to generalisations, eg:

Indices – cell growth, paper folding

Brackets – pond borders $4n + 4$ or $4(n + 1)$, football league matches $n^2 - n$ or $n\ (n - 1)$

NOTES

Any of the work in this module can be reinforced easily by using it as 'starters' or 'plenaries'

For extension, work could introduce simple ideas on standard form

Use everyday examples that lead to generalisations

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	4.6 – 4.7, 9.1 – 9.6
Edexcel GCSE Mathematics 16+ Student Book	1.7, 6.3 – 6.4, 6.6

Module 9 **Time: 1 – 3 hours**

GCSE Tier: Foundation

Contents: Sequences

A i Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence

A j Use linear expressions to describe the n^{th} term of an arithmetic sequence

PRIOR KNOWLEDGE

Know about odd and even numbers

Recognise simple number patterns, eg 1, 3, 5...

Writing simple rules algebraically

Raise numbers to positive whole number powers

Substitute into simple expressions

OBJECTIVES

By the end of the module the student should be able to:

- recognise and generate simple sequences of odd or even numbers
- continue a sequence derived from diagrams
- use a calculator to produce a sequence of numbers
- find the missing numbers in a number pattern or sequence
- find the n^{th} term of a number sequence
- use number machines
- use the n^{th} number of an arithmetic sequence
- find whether a number is a term of a given sequence

DIFFERENTIATION AND EXTENSION

Match-stick problems

Use practical real-life examples like 'flower beds'

Sequences of triangle numbers, Fibonacci numbers etc

Extend to quadratic sequences whose n^{th} term is $an^2 + b$ and link to square numbers

NOTES

Emphasise good use of notation, eg $3n$ means $3 \times n$

When investigating linear sequences, students should be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the n^{th} term

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	13.1, 13,4
Edexcel GCSE Mathematics 16+ Student Book	Chapter 7

Module **10** **Time: 2 – 4 hours**

GCSE Tier: **Foundation**

Contents: **Graphs 1**

A k Use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information

A l Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients

PRIOR KNOWLEDGE

Directed numbers

Parallel and perpendicular lines

Experience of plotting points in all quadrants

Substitution into simple formulae

OBJECTIVES

By the end of the module the student should be able to:

- use axes and coordinates to specify points in all four quadrants in 2-D
- identify points with given coordinates
- identify coordinates of given points (NB: Points may be in the first quadrant or all four quadrants)
- find the coordinates of points identified by geometrical information in 2-D
- find the coordinates of the midpoint of a line segment, AB, given the coordinates of A and B
- draw, label and put suitable scales on axes
- recognise that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane
- plot and draw graphs of functions
- plot and draw graphs of straight lines of the form $y = mx + c$, when values are given for m and c
- find the gradient of a straight line from a graph

DIFFERENTIATION AND EXTENSION

There are plenty of sources of good material here such as drawing animal pictures with coordinates, games like Connect 4 using coordinates

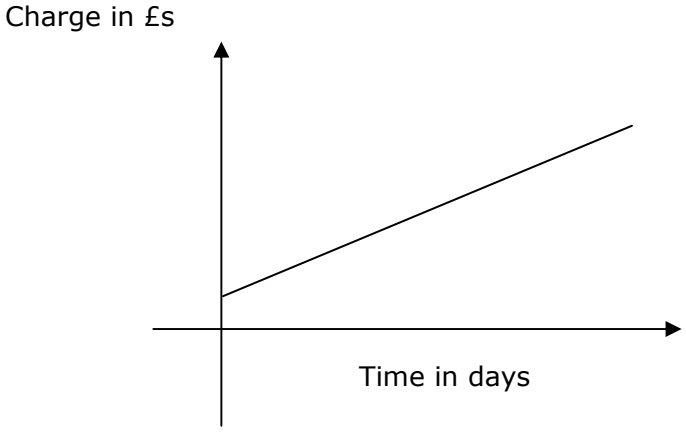
This topic can be delivered in conjunction with the properties of quadrilaterals

Plot graphs of the form $y = mx + c$ where students have to generate their own table and set out their own axes

Use a spreadsheet to generate straight-line graphs, posing questions about the gradient of lines

Use a graphical calculator or graphical ICT package to draw straight-line graphs

For hire of a skip the intercept is delivery charge and the gradient is the cost per day



NOTES

Emphasis that clear presentation of graphs with axes correctly labelled is important

Careful annotation should be encouraged. Students should label the coordinate axes and write the equation of the line on the graph

Cover horizontal and vertical line graphs as students often forget these ($x = c$ and $y = c$)

Link graphs and relationships in other subject areas, eg science and geography

Interpret straight-line graphs in Functional Elements

Link conversion graphs to converting metric and imperial units and equivalents

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Foundation Student book	15.1 – 15.7
Edexcel GCSE Mathematics 16+ Student Book	8.1 – 8.3, TF support

Module 11 **Time: 4 – 6 hours**

GCSE Tier: Foundation

Contents: Linear equations and inequalities

A d	Set up and solve simple equations
A h	Use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them
A g	Solve linear inequalities in one variable and represent the solution set on a number line
N u	Approximate to a specified or appropriate degree of accuracy
N v	Use calculators effectively and efficiently
N q	Understand and use number operations and the relationships between them, including inverse operations and the hierarchy of operations

PRIOR KNOWLEDGE

Experience of finding missing numbers in calculations

The idea that some operations are the reverse of each other

An understanding of balancing

Experience of using letters to represent quantities

Be able to draw a number line

An understanding of fractions and negative numbers

Substituting numbers into algebraic expressions

Dealing with decimals on a calculator

Comparing/ordering decimals

OBJECTIVES

By the end of the module the student should be able to:

- set up simple equations
- rearrange simple equations
- solve simple equations
- solve linear equations in one unknown, with integer or fractional coefficients
- solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation
- solve linear equations which include brackets, those that have negative signs occurring anywhere in the equation, and those with a negative solution
- use linear equations to solve problems
- solve algebraic equations involving squares and cubes, eg $x^3 + 3x = 40$ using trial and improvement
- solve simple linear inequalities in one variable, and represent the solution set on a number line
- use the correct notation to show inclusive and exclusive inequalities

DIFFERENTIATION AND EXTENSION

Derive equations from practical situations (such as finding unknown angles in polygons or perimeter problems)

Solve equations where manipulation of fractions (including negative fractions) is required

Look at various calculator functions like 'square root' and 'cube root'.

Solve equations of the form $\frac{1}{x} = x^2 - 5$

NOTES

Remind students about work on linear patterns and sequences

Students need to realise that not all equations should be solved by 'trial and improvement' or by observation. The use of a formal method of solving equations is very important

Remind students of the need to set their work out clearly, keeping the equal signs in line

Students should be encouraged to use their calculator efficiently by using the 'replay' or ANS/EXE function keys

Advise students to take care when entering negative values to be squared

Students should write down all the digits on their calculator display and only round the final answer to the required degree of accuracy

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Foundation Student book	21.1 – 21.11
Edexcel GCSE Mathematics 16+ Student Book	9.1 – 9.8

Module 12 **Time: 3 – 5 hours**

GCSE Tier: Foundation

Contents: Graphs 2

A r	Construct linear functions from real-life problems and plotting their corresponding graphs
A s	Discuss, plot and interpret graphs (which may be non-linear) modelling real situations
A t	Generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions
N v	Use calculators effectively and efficiently

PRIOR KNOWLEDGE

Experience of plotting points in all quadrants

Experience of labelling axes and reading scales

Knowledge of metric units, eg 1 m = 100 cm

Know that 1 hour = 60 mins, 1 min = 60 seconds

Know how to find average speed

Know how to read scales, draw and interpret graphs

Substituting positive and negative numbers into algebraic expressions

Experience of dealing with algebraic expressions with brackets – BIDMAS

OBJECTIVES

By the end of the module the student should be able to:

- draw graphs representing 'real' examples like filling a bath/containers
- interpret and draw linear graphs, including conversion graphs, fuel bills etc
- solve problems relating to mobile phone bills with fixed charge and price per unit
- interpret non-linear graphs
- draw distance time graphs
- interpret distance time graphs and solve problems
- substitute values of x into a quadratic function to find the corresponding values of y
- draw graphs of quadratic functions
- find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function

DIFFERENTIATION AND EXTENSION

Use open-ended questions that test student awareness of what intersections mean, eg mobile phone bills

Use spreadsheets to generate straight-line graphs and pose questions about gradient of lines

Use ICT packages or graphical calculators to draw straight-line graphs and quadratic graphs

Make up a graph and supply the commentary for it

Use timetables to plan journeys

Students to draw simple cubic and $\frac{1}{x}$ graphs

Students to solve simultaneous equations graphically including a quadratic graph and a line

Students to solve simple projectile problems

NOTES

Clear presentation is important with axes clearly labelled

Students need to be able to recognise linear graphs and also be able to recognise when their graph is incorrect

Link graphs and relationships in other subject areas, eg science, geography

Students should have plenty of practice interpreting linear graphs for Functional Elements problems

The graphs of quadratic functions should be drawn freehand, and in pencil. Turning the paper often helps

Squaring negative integers may be a problem for some.

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	22.1 – 22.5
Edexcel GCSE Mathematics 16+ Student Book	8.4 – 8.6

Module 13 **Time: 2 – 4 hours**

GCSE Tier: Foundation

Contents: Formulae

A f Derive a formula

A f Substitute numbers into a formula

A f Change the subject of a formula

PRIOR KNOWLEDGE

An understanding of the mathematical meaning of the words expression, simplifying, formulae and equation

Experience of using letters to represent quantities

Substituting into simple expressions with words

Using brackets in numerical calculations and removing brackets in simple algebraic expressions

OBJECTIVES

By the end of the module the student should be able to:

- use formulae from mathematics and other subjects expressed initially in words and then using letters and symbols
- substitute numbers into a formula
- substitute positive and negative numbers into expressions such as $3x^2 + 4$ and $2x^3$
- derive a simple formula, including those with squares, cubes and roots
- find the solution to a problem by writing an equation and solving it
- change the subject of a formula

DIFFERENTIATION & EXTENSION

Use negative numbers in formulae involving indices

Various investigations leading to generalisations, eg the painted cube, frogs, pond borders

Relate to topic on graphs of real-life functions

More complex changing the subject, moving onto higher tier work

Apply changing the subject to physics formulae, eg speed, density, equations of motion

NOTES

Emphasise the need for good algebraic notation

Show a linear equation first and follow the same steps to rearrange a similarly structured formula

Link with Functional Elements problems in real-life problems

Link with formulae for area and volume

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	28.1 – 28.6
Edexcel GCSE Mathematics 16+ Student Book	Chapter 10

Module 14 **Time: 2 – 4 hours**

GCSE Tier: Foundation

Contents: 2-D shapes

GM d Recall the properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus

GM e Recognise reflection and rotation symmetry of 2-D shapes

GM f Understand congruence and similarity

GM i Distinguish between the centre, radius, chord, diameter, circumference, tangent, arc, sector and segment

GM t Measure and draw lines and angles

GM u Draw triangles and other 2-D shapes using a ruler and a protractor

GM v Draw circles and arcs to a given radius

PRIOR KNOWLEDGE

Basic idea of shape and symmetry

OBJECTIVES

By the end of the module the student should be able to:

- recall the properties and definitions of special types of quadrilaterals, including symmetry properties
- list the properties of each, or identify (name) a given shape
- draw sketches of shapes
- name all quadrilaterals that have a specific property
- identify quadrilaterals from everyday usage
- classify quadrilaterals by their geometric properties
- understand congruence
- identify shapes which are congruent
- understand similarity
- identify shapes which are similar, including all circles or all regular polygons with equal number of sides
- make accurate drawing of triangles and other 2-D shapes using a ruler and a protractor
- recall the definition of a circle and identify and draw parts of a circle
- draw a circle or arc given its radius (or a circle given its diameter)
- recognise reflection symmetry of 2-D shapes
- identify and draw lines of symmetry on a shape
- draw or complete diagrams with a given number of lines of symmetry
- recognise rotation symmetry of 2-D shapes
- identify the order of rotational symmetry of a 2-D shape
- draw or complete diagrams with a given order of rotational symmetry

DIFFERENTIATION AND EXTENSION

Practical activities help with the understanding of the properties – games like 'Guess who I am?'

Investigate Rangoli Patterns, which are a good source of display work

Ask students to find their own examples of symmetry, similarity and congruence in real life

NOTES

Equations of lines of symmetry are covered later in the course

Reinforce accurate drawing skills and measurement

Use tracing paper or mirrors to assist with symmetry questions

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	6.1 – 6.8
Edexcel GCSE Mathematics 16+ Student Book	11.1 – 11.2, 11.4 – 11.5, TF support

Module 15 **Time: 2 – 4 hours**

GCSE Tier: Foundation

Contents: Angles 1

GM a Recall and use properties of angles at a point, angles at a point on a straight line (including right angles), perpendicular lines and vertically opposite angles

GM b Understand and use the angle properties of triangles and intersecting lines

GM t Measure and draw lines and angles

PRIOR KNOWLEDGE

An understanding of angles as a measure of turning

The ability to use a ruler and a protractor

OBJECTIVES

By the end of the module the student should be able to:

- distinguish between acute, obtuse, reflex and right angles
- name angles
- use letters to identify points, lines and angles
- use two letter notation for a line and three letter notation for an angle
- measure and draw angles, to the nearest degree
- estimate sizes of angles
- measure and draw lines, to the nearest mm
- use geometric language appropriately
- distinguish between scalene, equilateral, isosceles and right-angled triangles
- understand and use the angle properties of triangles
- find a missing angle in a triangle, using the angle sum of a triangle is 180º
- use the side/angle properties of isosceles and equilateral triangles
- find the size of missing angles at a point or at a point on a straight line
- recall and use properties of:
  - angles at a point
  - angles at a point on a straight line, including right angles
  - vertically opposite angles
- give reasons for calculations
- recall and use properties of perpendicular lines
- mark perpendicular lines on a diagram

DIFFERENTIATION AND EXTENSION

Explore other angle properties in triangles, parallel lines or quadrilaterals, in preparation for future topics

NOTES

Students should make sure that drawings are neat, accurate and labelled

Give students a lot of practice drawing angles, including reflex angles, and encourage students to check their drawings

Angles should be accurate to within 2° and lengths accurate to the nearest mm

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Foundation Student book	2.1 – 2.2, 2.3 – 2.8, 6.1
Edexcel GCSE Mathematics 16+ Student Book	11.1, TF support

Module 16 **Time: 4 – 6 hours**

GCSE Tier: Foundation

Contents: Angles 2

GM b Understand and use the angle properties of parallel and intersecting lines, triangles and quadrilaterals

GM c Calculate and use the sums of the interior and exterior angles of polygons

GM r Understand and use bearings

GM v Use straight edge and a pair of compasses to carry out constructions

GM m Use scale drawings

PRIOR KNOWLEDGE

Know that angles in a triangle add up to 180º

Know that angles at a point on a straight line sum to 180°

Know that a right angle = 90°

Measure and draw lines and angles

OBJECTIVES

By the end of the module the student should be able to:

- give reasons for angle calculations
- understand and use the angle properties of quadrilaterals
- use the fact that the angle sum of a quadrilateral is 360
- calculate and use the sums of the interior angles of polygons
- use geometrical language appropriately and recognise and name pentagons, hexagons, heptagons, octagons and decagons
- know, or work out, the relationship between the number of sides of a polygon and the sum of its interior angles
- know that the sum of the exterior angles of any polygon is 360°
- calculate the size of each exterior/interior angle of a regular polygon
- understand tessellations of regular and irregular polygons
- tessellate combinations of polygons
- explain why some shapes tessellate and why other shapes do not
- understand and use the angle properties of parallel lines
- mark parallel lines on a diagram
- find missing angles using properties of corresponding and alternate angles
- understand the proof that the angle sum of a triangle is 180°
- understand the proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices
- use three-figure bearings to specify direction
- mark on a diagram the position of point B given its bearing from point A

- give a bearing between the points on a map or scaled plan
- given the bearing of point A from point B, work out the bearing of B from A
- make an accurate scale drawing from a diagram
- use and interpret scale drawings

DIFFERENTIATION AND EXTENSION

Use the angle properties of triangles to find missing angles in combinations of triangles and rectangles

Explore other properties in triangles, quadrilaterals and parallel lines

Study Escher drawings (possibly cross curricular with Art).

Ask students to design their own tessellation, and explain why their shapes tessellate

NOTES

All diagrams should be presented neatly and accurately

Students should have plenty of practice drawing examples to illustrate the properties of various shapes

For bearings and scaled drawings, angles should be correct to 2° and lines accurate to 2 mm

Use of tracing paper helps with tessellations

Consider real-life examples of tessellations

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	7.1 – 7.9
Edexcel GCSE Mathematics 16+ Student Book	11.1 – 11.3, 11.6 – 11.8, TF support

Module 17 **Time: 3 – 5 hours**

GCSE Tier: Foundation

Contents: Perimeter and Area of 2-D shapes

GM x Calculate perimeters and areas of shapes made from triangles and rectangles

PRIOR KNOWLEDGE

Names of triangles and quadrilaterals

Knowledge of the properties of rectangles, parallelograms and triangles

Concept of perimeter and area

Units of measurement

Four operations of number

OBJECTIVES

By the end of the module the student should be able to:

- measure shapes to find perimeters and areas
- find the perimeter of rectangles and triangles
- find the perimeter of compound shapes
- find the area of a rectangle and triangle
- recall and use the formulae for the area of a triangle, rectangle and a parallelogram
- calculate areas of compound shapes made from triangles and rectangles
- find the area of a trapezium
- solve a range of problems involving areas including cost of carpet type questions

DIFFERENTIATION AND EXTENSION

Further problems involving combinations of shapes

Use practical examples from functional papers on topics such as returfing a garden, carpeting a room, laying carpet tiles on a floor

Perimeter questions could use skirting board, wallpaper, planting a border of a garden

NOTES

Discuss the correct use of language and units, particularly when method marks are for the correct unit of measure

Ensure that students can distinguish between perimeter and area

Practical examples help to clarify the concepts, eg floor tiles etc

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	14.1, 14.3 – 14.4
Edexcel GCSE Mathematics 16+ Student Book	Chapter 13

Module 18 **Time: 2 – 4 hours**

GCSE Tier: Foundation

Contents: Circles

GM z Find circumferences and areas

N u Approximate to a specified or appropriate degree of accuracy

N v Use calculators effectively and efficiently

PRIOR KNOWLEDGE

The ability to substitute numbers into formulae

OBJECTIVES

By the end of the module the student should be able to:

- find circumferences of circles and areas enclosed by circles
- recall and use the formulae for the circumference of a circle and the area enclosed by a circle
- use $\pi \approx 3.142$ or use the π button on a calculator
- find the perimeters and areas of semicircles and quarter circles

DIFFERENTIATION AND EXTENSION

Use more complex 2-D shapes, eg (harder) sectors of circles

Approximate π as $\frac{22}{7}$

Work backwards to find the radius/diameter given the circumference/area

Apply to real-life contexts with laps of running tracks and average speeds

Harder problems involving multi-stage calculations

Define a circle by using the language of loci

NOTES

All working should be clearly and accurately presented

Students should use a pencil to draw all diagrams

A sturdy pair of compasses is essential

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	17.1 – 17.3
Edexcel GCSE Mathematics 16+ Student Book	Chapter 14

Module 19 **Time: 1 – 3 hours**

GCSE Tier: Foundation

Contents: Constructions and loci

GM v Use a straight edge and a pair of compasses to carry out constructions

GM w Construct loci

PRIOR KNOWLEDGE

Knowledge of types of triangle

Knowledge of the difference between a line and a region

OBJECTIVES

By the end of the module the student should be able to:

- use straight edge and a pair of compasses to do standard constructions such as
  - construct a triangle
  - construct an equilateral triangle
  - understand, from the experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not
  - construct the perpendicular bisector of a given line
  - construct the perpendicular from a point to a line
  - construct the bisector of a given angle
  - construct angles of 60º, 90º, 30º, 45º
  - draw parallel lines
  - construct diagrams of everyday 2-D situations involving rectangles, triangles, perpendicular and parallel lines
- draw and construct loci from given instructions
  - a region bounded by a circle and an intersecting line
  - a given distance from a point and a given distance from a line
  - equal distances from 2 points or 2 line segments
  - regions which may be defined by 'nearer to' or 'greater than'
  - find and describe regions satisfying a combination of loci
- construct a regular hexagon inside a circle

DIFFERENTIATION AND EXTENSION

Try to do this module as practically as possible using real life-situations, eg horses tethered to ropes, mobile phone, masts etc

Use the internet to source ideas for this module

Use loci problems that require a combination of loci

NOTES

All constructions should be presented neatly and accurately

A sturdy pair of compasses is essential

Construction lines should not be erased as they carry valuable method marks

All lines should be correct to within 2 mm and angles correct to 2°

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Foundation Student book	18.1 – 18.3
Edexcel GCSE Mathematics 16+ Student Book	Chapter 15, TF support

Module 20 **Time: 3 – 5 hours**

GCSE Tier: Foundation

Contents: 3-D shapes

GM k	Use 2-D representations of 3-D shapes
GM x	Calculate the surface area of a 3-D shape
GM z	Find the surface area of a cylinder
GM aa	Find the volume of a cylinder
GM aa	Calculate volumes of right prisms and shapes made from cubes and cuboids
GM n	Understand the effect of enlargement for perimeter, area and volume of shapes and solids
GM p	Convert between units and area measures
GM p	Converting between metric volume measures, including cubic centimetres and cubic metres

PRIOR KNOWLEDGE

The names of standard 2-D and 3-D shapes

Concept of volume

Concept of prism

Experience of constructing cubes or cuboids from multi-link

Identify and name common solids: cube, cuboid, cylinder, prism, pyramid, sphere and cone

Experience of multiplying and dividing by powers of 10

OBJECTIVES

By the end of the module the student should be able to:

- know the terms face, edge and vertex
- use 2-D representations of 3-D shapes
- use isometric grids
- draw nets and show how they fold to make a 3-D solid
- understand and draw front and side elevations and plans of shapes made from simple solids
- draw a sketch of the 3-D solid, given the front and side elevations and the plan of a solid
- find the surface area and volume of a cylinder
- find volumes of shapes by counting cubes
- recall and use formulae for the volume of cubes and cuboids
- calculate the volumes of right prisms and shapes made from cubes and cuboids
- find the surface area of a 3-D shape
- understand how enlargement changes perimeters and areas
- understand how enlargement affects volume

- convert between metric units of area
- convert between units of volume and capacity (1 ml = 1cm^3)

DIFFERENTIATION AND EXTENSION

Make solids using equipment such as clixi or multi-link

Draw on isometric paper shapes made from multi-link

Build shapes using cubes from 2-D representations

Euler's theorem

A useful topic for a wall display - students tend to like drawing 3-D shapes and add interest by using a mixture of colours in the elevations

Look at 'practical' examples with fish tanks / filling containers and finding the number of small boxes that fit into a large box

Further problems involving a combination of shapes

NOTES

Accurate drawing skills need to be reinforced

Some students find visualising 3-D objects difficult, so using simple models will help

Discuss the correct use of language and units. Remind students that there is often a mark attached to writing down the correct unit

Use practical problems to enable the students to understand the difference between perimeter, area and volume

Use Functional Elements problems, eg filling a water tank, optimisation type questions etc

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	20.1 -20.7
Edexcel GCSE Mathematics 16+ Student Book	Chapter 16

Module 21 **Time: 3 – 5 hours**

GCSE Tier: Foundation

Contents: Transformations

GM l Describe and transform 2-D shapes using single or combined rotations, reflections, translations or enlargements by a positive scale factor and distinguish properties that are preserved under particular transformations

PRIOR KNOWLEDGE

Recognition of basic shapes

An understanding of the concept of rotation, reflection and enlargement

Coordinates in four quadrants

Equations of lines parallel to the coordinate axes and $y = \pm x$

OBJECTIVES

By the end of the module the student should be able to:

- describe and transform 2-D shapes using single translations
- understand that translations are specified by a distance and direction (using a vector)
- translate a given shape by a vector
- describe and transform 2-D shapes using single rotations
- understand that rotations are specified by a centre and an (anticlockwise) angle
- find the centre of rotation
- rotate a shape about the origin, or any other point
- describe and transform 2-D shapes using single reflections
- understand that reflections are specified by a mirror line
- identify the equation of a line of symmetry
- describe and transform 2-D shapes using enlargements by a positive scale factor
- understand that an enlargement is specified by a centre and a scale factor
- scale a shape on a grid (without a centre specified)
- draw an enlargement
- enlarge a given shape using (0, 0) as the centre of enlargement
- enlarge shapes with a centre other than (0, 0)
- find the centre of enlargement
- recognise that enlargements preserve angle but not length
- identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides
- describe and transform 2-D shapes using combined rotations, reflections, translations, or enlargements
- understand that distances and angles are preserved under rotations, reflections and translations, so that any shape is congruent under any of these transformations
- describe a transformation

DIFFERENTIATION AND EXTENSION

Use squared paper to enlarge cartoon characters to make a display

Use kaleidoscope patterns to illustrate transformations

NOTES

Emphasise that students should describe transformations fully

Diagrams should be drawn in pencil

Tracing paper can be useful for rotations

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Foundation Student book	23.2 – 23.6
Edexcel GCSE Mathematics 16+ Student Book	Chapter 17

Module 22 **Time: 3 – 5 hours**

GCSE Tier: Foundation

Contents: Pythagoras' Theorem

GM g Understand, recall and use Pythagoras' theorem in 2-D

A k Calculate the length of a line segment

N u Approximate to specified or appropriate degrees of accuracy

N v Use calculators effectively and efficiently

PRIOR KNOWLEDGE

Knowledge of square and square roots

Knowledge of types of triangle

OBJECTIVES

By the end of this module students should be able to:

- understand and recall Pythagoras' theorem
- use Pythagoras' theorem to find the hypotenuse
- use Pythagoras' theorem to find a side
- use Pythagoras' theorem to find the length of a line segment from a coordinate grid
- apply Pythagoras' theorem to practical situations

DIFFERENTIATION AND EXTENSION

See exemplar question involving times taken to cross a field as opposed to going around the edge

Use examples that involve finding the area of isosceles triangles

Try to find examples with ladders on walls, area of a sloping roof etc

Introduce 3-D Pythagoras (moving towards Higher Tier)

NOTES

A useful way of remembering Pythagoras' theorem is *'Square it, square it, add/subtract it, square root it'*

Students should not forget to state units for the answers

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	27.1 – 27.4
Edexcel GCSE Mathematics 16+ Student Book	18.1 – 18.2

Module 23 **Time: 2 – 4 hours**

GCSE Tier: Foundation

Contents: Measure

GM o	Interpret scales on a range of measuring instruments, and recognise the inaccuracy of measurements
GM p	Convert measurements from one unit to another
GM p	Convert between speed measures
GM q	Make sensible estimates of a range of measures
GM s	Understand and use compound measures
N u	Approximate to specified or appropriate degree of accuracy
SP e	Extract data from printed tables and lists

PRIOR KNOWLEDGE

An awareness of the imperial system of measures

Strategies for multiplying and dividing by 10 (for converting metric units)

Knowledge of metric units eg 1 m = 100 cm

Know that 1 hour = 60 mins, 1 min = 60 seconds

Experience of multiplying and dividing by powers of 10, eg $100 \times 100 = 10\,000$, $10\,000 \div 10 = 1000$

OBJECTIVES

By the end of the module the student should be able to:

- interpret scales on a range of measuring instruments, including mm, cm, m, km, m*l*, c*l*, *l*, mg, g, kg, tonnes, °C, seconds, minutes, hours, days, weeks, months and years
- indicate given values on a scale
- read times and work out time intervals
- convert between 12–hour and 24–hour hour clock times
- read bus and train timetables and plan journeys
- know that measurements using real numbers depend upon the choice of unit
- convert units within one system
- make sensible estimates of a range of measures in everyday settings
- choose appropriate units for estimating or carrying out measurements
- convert metric units to metric units
- convert imperial units to imperial units
- convert between metric and imperial measures

- know rough metric equivalents of pounds, feet, miles, pints and gallons, ie

  Metric Imperial

  1 kg = 2.2 pounds

  1 litre = 1.75 pints

  4.5 *l* = 1 gallon

  8 km = 5 miles

  30 cm = 1 foot
- estimate conversions
- use the relationship between distance, speed and time to solve problems
- convert between metric units of speed, eg km/h to m/s
- recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction

DIFFERENTIATION & EXTENSION

This could be made a practical activity, by collecting assorted everyday items and weighing and measuring to check the estimates of their length, weights and volume

Use the internet to find the weight, volume and height of large structures such as buildings, aeroplanes and ships

Use conversions for height and weight of students, cars, bridges. Combine with simple scales such as 1 cm to 1 m for classrooms, playing fields, bedrooms and ask them to draw a plan of their ideal design for their bedrooms including the furniture

Convert imperial units to metric units, eg mph into km/h which would remind students that 5 miles = 8 km

NOTES

Measurement is essentially a practical activity

Use a range of everyday objects to bring reality to lessons

Use Functional Elements as a source of practical activities

All working out should be shown with multiplication or division by powers of 10

Use the distance/speed/time triangle (ie **D**rink **S**ome **T**ea)

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Foundation Student book	11.1 – 11.6
Edexcel GCSE Mathematics 16+ Student Book	Chapter 12, TF support

Module 24 **Time: 2 – 4 hours**

GCSE Tier: Foundation

Contents: Collecting and recording data

SP a Understand and use statistical problem-solving process/handling data cycle

SP b Identify possible sources of bias

SP c Design an experiment or survey

SP d Design data-collection sheets distinguishing between different types of data

SP e Extract data from printed tables and lists

SP f Design and use two-way tables for discrete and grouped data

PRIOR KNOWLEDGE

An understanding of the importance of statistics in our society, and of why data needs to be collected

Experience of simple tally charts

Some idea about different types of graphs

Experience of inequality notation

OBJECTIVES

By the end of the module the student should be able to:

- specify the problem and plan
- decide what data to collect and what statistical analysis is needed
- collect data from a variety of suitable primary and secondary sources
- use suitable data collection techniques
- process and represent the data
- interpret and discuss the data
- design and use data-collection sheets for grouped, discrete and continuous data
- collect data using various methods
- sort, classify and tabulate data and discrete or continuous quantitative data
- group discrete and continuous data into class intervals of equal width
- understand how sources of data may be biased
- identify which primary data they need to collect and in what format, including grouped data
- consider fairness
- design a question for a questionnaire
- criticise questions for a questionnaire
- extract data from lists and tables
- understand sample and population
- design and use two-way tables for discrete and grouped data
- use information provided to complete a two way table

DIFFERENTIATION AND EXTENSION

Students carry out a statistical investigation of their own, including designing an appropriate means of gathering the data

Some guidance needs to be given to stop students choosing limited investigations, eg favourite football team

NOTES

For Functional Elements activities, it is worth collecting data at different times of the day, eg to compare types of shopper in a centre. Get data from holiday brochures to compare resorts for temperature, rainfall and type of visitor

Emphasise the differences between primary and secondary data. Mayfield High data can be used as an example of secondary data

Discuss sample size and mention that a census is the whole population. In the UK, the Census is held every year that ends in '1', so the next census is in 2011

If students are collecting data as a group, then they should use the same procedure

Emphasise that continuous data is data that is measured, eg temperature

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	3.1 – 3.5
Edexcel GCSE Mathematics 16+ Student Book	19.1 – 19.3

Module 25 **Time: 3 – 5 hours**

GCSE Tier: Foundation

Contents: Processing, representing and interpreting data

SP g Produce charts and diagrams for various data types

SP i Interpret a wide range of graphs and diagrams and draw conclusions

SP l Compare distributions and make inferences

PRIOR KNOWLEDGE

An understanding of why data needs to be collected and some idea about different types of graphs

Measuring and drawing angles

Fractions of simple quantities

OBJECTIVES

By the end of the module the student should be able to:

- draw:
  - pictograms
  - composite bar charts
  - comparative and dual bar charts
  - frequency polygons
  - pie charts
  - histograms with equal class intervals
  - frequency diagrams for grouped discrete data
  - line graphs
- interpret:
  - composite bar charts
  - comparative and dual bar charts
  - pie charts
  - frequency polygons
- from pictograms, bar charts, line graphs, frequency polygons and histograms with equal class intervals:
  - read off frequency values
  - calculate total population
  - find greatest and least values
- recognise simple patterns and characteristic relationships in bar charts, line graphs and frequency polygons
- use dual or comparative bar charts to compare distributions
- understand that the frequency represented by corresponding sectors in two pie charts is dependent on the total populations represented by each of the pie charts

- from pie charts:
  - find the total frequency
  - find the size of each category

DIFFERENTIATION AND EXTENSION

Students carry out a statistical investigation of their own and use an appropriate means of displaying the results

Use a spreadsheet to draw different types of graphs

Collect examples of charts and graphs in the media which have been misused, and discuss the implications

Use this module to revise frequency and tally tables

Practise the ability to divide by 20, 30, 40, 60 etc

This can be delivered as a practical module that could lead to wall display- remind students about bias, eg only asking their friends which band they like

Compare pie charts for, eg boys and girls, to identify similarities and differences

Ask students to combine two pie charts

NOTES

Reiterate that clear presentation with axes correctly labelled is important, and to use a ruler to draw straight lines

Make comparisons between previously collected data

Encourage students to work in groups and present their charts (useful display material for classrooms/corridors)

Use Excel Graph wizard

Consider Functional Elements, eg by comparing rainfall charts, distributions of ages in cinemas etc

Angles for pie charts should be accurate to within 2°

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	12.1 – 12.6, 25.1
Edexcel GCSE Mathematics 16+ Student Book	21.1 – 21.4, 21.7, TF support

Module 26 **Time: 2 – 4 hours**

GCSE Tier: Foundation

Contents: Averages and range

SP h Calculate median, mean, range, mode and modal class

SP l Compare distributions and make inferences

SP u Use calculators efficiently and effectively, including statistical functions

SP g Draw ordered stem and leaf diagrams

SP i Draw conclusions from graphs and diagrams

PRIOR KNOWLEDGE

Addition and subtraction

Different statistical diagrams

OBJECTIVES

By the end of the module the student should be able to:

- calculate the mean, mode, median and range for discrete data
- calculate the mean of a small data set, using the appropriate key on a scientific calculator
- recognise the advantages and disadvantages between measures of average
- compare the mean and range of two distributions
- calculate the mean, mode, median and range from an ordered stem and leaf diagram
- draw and interpret an ordered stem and leaf diagram
- calculate the mean, median and mode from a frequency table
- find the modal class and the interval containing the median for continuous data
- estimate the mean of grouped data using the mid-interval value

DIFFERENTIATION AND EXTENSION

Students to find the mean for grouped continuous data with unequal class intervals

Students to collect continuous data and decide on appropriate (equal) class intervals; then find measures of average

Students to use the statistical functions on a calculator or a spreadsheet to calculate the mean for continuous data

NOTES

Ask class to do their own survey with data collection sheets, eg to find the average number of children per family in the class

The internet and old coursework tasks are a rich source of data to work with, eg *Second-Hand Car Sales*, *Mayfield High* data etc

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	16.1 – 16.7
Edexcel GCSE Mathematics 16+ Student Book	Chapter 20

Module 27 **Time: 1 – 3 hours**

GCSE Tier: Foundation

Contents: Line diagrams and scatter graphs

SP g, i Draw and interpret scatter diagrams

SP k Recognise correlation and draw and/or use lines of best fit by eye, understanding what these represent

SP j Look at data to find patterns and exceptions

PRIOR KNOWLEDGE

Plotting coordinates and scale

An understanding of the concept of a variable

Recognition that a change in one variable can affect another

Linear graphs

OBJECTIVES

By the end of the module the student should be able to:

- draw and interpret a line graph
- draw and interpret a scatter graph
- look at data to find patterns and exceptions
- distinguish between positive, negative and zero correlation using lines of best fit
- interpret correlation in terms of the problem and the relationship between two variables
- understand that correlation does not imply causality
- draw a line of best fit by eye and understand what it represents
- use a line of best fit to predict values of one variable given values of the other variable

DIFFERENTIATION & EXTENSION

Vary the axes required on a scatter graph to suit the ability of the class

Students to carry out a statistical investigation of their own including designing an appropriate means of gathering the data, and an appropriate means of displaying the results, eg height and length of arm

Use a spreadsheet, or other software, to produce scatter diagrams/lines of best fit

Investigate how the line of best fit is affected by the choice of scales on the axes, eg use car data with age and price of the same make of car

NOTES

Statistically, the line of best fit should pass through the point representing the mean of the data

Students should label all axes clearly and use a ruler to draw all straight lines

Remind students that the line of best fit does not necessarily go through the origin of the graph

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	25.1 – 25.5
Edexcel GCSE Mathematics 16+ Student Book	Chapter 22

Module 28 **Time: 4 - 6 hours**

GCSE Tier: Foundation

Contents: Probability

SP m	Understand and use the vocabulary of probability and probability scale
SP n	Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency
SP o	List all outcomes for single events, and for two successive events, in a systematic way and derive relative probabilities
SP p	Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
SP s	Compare experimental data and theoretical probabilities
SP t	Understand that if they repeat an experiment, they may – and usually will – get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics

PRIOR KNOWLEDGE

Fractions, decimals and percentages

Ability to read from a two-way table

Use and draw two-way tables

OBJECTIVES

By the end of the module the student should be able to:

- distinguish between events which are: impossible, unlikely, even chance, likely, and certain to occur
- mark events and/or probabilities on a probability scale of 0 to 1
- write probabilities in words, fractions, decimals and percentages
- find the probability of an event happening using theoretical probability
- use theoretical models to include outcomes using dice, spinners, coins
- list all outcomes for single events systematically
- list all outcomes for two successive events systematically
- use and draw sample space diagrams
- find the probability of an event happening using relative frequency
- compare experimental data and theoretical probabilities
- compare relative frequencies from samples of different sizes
- find probabilities from a two-way table

- estimate the number of times an event will occur, given the probability and the number of trials
- add simple probabilities
- identify different mutually exclusive outcomes and know the sum of the probabilities of all the outcomes is 1
- use 1 – p as the probability of an event not occurring
- find a missing probability from a list or table

DIFFERENTIATION AND EXTENSION

Use this as an opportunity for practical work

Experiments with dice and spinners

Show sample space for outcomes of throwing two dice (36 outcomes)

Use 'the horse race'/drawing pins/let students make their own biased dice and find experimental probability

NOTES

Students should express probabilities as fractions, percentages or decimals

Probabilities written as fractions do not need to be cancelled to their simplest form

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Foundation Student book	26.1 – 26.7
Edexcel GCSE Mathematics 16+ Student Book	23.1 – 23.4, TF support

Foundation course objectives (1MA0)

Number

N a	Add, subtract, multiply and divide any number
N a	Add, subtract, multiply and divide any integer < 1
N b	Order integers
N b	Order rational numbers
N c	Use the concepts and vocabulary of factor (divisor), multiple, common factor, Highest Common Factor (HCF), Least Common Multiple (LCM), prime number and prime factor decomposition
N d	Use the terms square, positive and negative square root, cube and cube root
N e	Use index notation for squares, cubes and powers of 10
N f	Use index laws for multiplication and division of integer powers
N h	Understand equivalent fractions, simplify a fraction by cancelling all common factors
N h	Understand equivalent fractions in the context of 'hundredths'
N i	Add and subtract fractions
N j	Use decimal notation and recognise that each terminating decimal is a fraction
N j	Use decimal notation and understand that decimals and fractions are equivalent
N k	Recognise that recurring decimals are exact fractions, and that some exact fractions are recurring
N l	Understand that 'percentage' means 'number of parts per 100' and use this to compare proportions
N m	Use percentages
N o	Interpret fractions, decimals and percentages as operators
N p	Use ratio notation, including reduction to its simplest form and its various links to fraction notation
N q	Understand and use number operations and inverse operations
N q	Understand and use number operations and the relationships between them, including inverse operations and the hierarchy of operations
N t	Divide a quantity in a given ratio
N u	Approximate to specified or appropriate degrees of accuracy
N v	Use calculators effectively and efficiently

Algebra

A a	Distinguish the different roles played by letter symbols in algebra
A b	Distinguish the meaning between the words 'equation', 'formula' and 'expression'
A c	Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors
A d	Set up and solve simple equations
A f	Derive a formula, substitute numbers into a formula and change the subject of a formula
A g	Solve linear inequalities in one variable and represent the numbers on a number line
A h	Use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them
A i	Generate terms of a sequence using term-to-term and position to-term definitions of the sequence
A j	Use linear expressions to describe the nth term of an arithmetic sequence
A k	Calculate the length of a line segment
A k	Use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information
A l	Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients
A r	Construct linear functions from real-life problems and plotting their corresponding graphs
A s	Discuss, plot and interpret graphs (that may be non-linear) that model real situations
A s	Interpret distance time graphs
A t	Generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions

Geometry and Measures

GM a	Recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines, and vertically opposite angles
GM b	Understand and use the angle properties of triangles
GM b	Understand and use the angle properties of parallel lines and quadrilaterals
GM c	Calculate and use the sums of the interior and exterior angles of polygons
GM d	Recall the properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus
GM e	Recognise reflection and rotation symmetry of 2-D shapes
GM f	Understand congruence and similarity
Gm g	Understand, recall and use Pythagoras' theorem in 2-D
GM i	Distinguish between the centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
GM k	Use 2-D representations of 3-D shapes
GM l	Describe and transform 2-D shapes using single or combined rotations, reflections, translations, or enlargements by a positive scale factor and distinguish properties that are preserved under particular transformations
GM m	Use and interpret maps and scale drawings
GM n	Understand the effect of enlargement for perimeter, area and volume of shapes and solids
GM o	Interpret scales on a range of measuring instruments, and recognise the inaccuracy of measurements
GM o	Use correct notation for time 12- and 24- hour clock
GM p	Convert measurements from one unit to another
GM p	Convert between metric area measures
GM p	Convert between speed measures
GM p	Convert between volume measures, including cubic centimetres and cubic metres
GM q	Make sensible estimates of a range of measures
GM r	Understand and use bearings
GM s	Understand and use compound measures
GM t	Measure and draw lines and angles
GM u	Draw triangles and other 2-D shapes using a ruler and protractor
GM v	Use straight edge and a pair of compasses to carry out constructions
GM w	Construct loci
GM x	Calculate perimeters and areas of shapes made from triangles and rectangles

GM x	Calculate the surface area of a 3-D shape
GM z	Find circumferences and areas of circles
GM aa	Calculate volumes of right prisms and shapes made from cubes and cuboids

Statistics and Probability

SP a	Understand and use statistical problem solving process/handling data cycle
SP b	Identify possible sources of bias
SP c	Design an experiment or survey
SP d	Design data-collection sheets distinguishing between different types of data
SP e	Extract data from printed tables and lists
SP e	Read timetables
SP f	Design and use two-way tables for discrete and grouped data
SP g	Produce charts and diagrams for various data types
SP g	Produce ordered stem and leaf diagrams
SP h	Calculate median, mean, range, mode and modal class
SP i	Interpret pie charts
SP i	Interpret a wide range of graphs and diagrams and draw conclusions
SP i	Draw conclusions from diagrams
SP j	Look at data to find patterns and exceptions
SP k	Recognise correlation and draw and/or use lines of best fit by eye, understanding what these represent
SP l	Compare distributions and make inferences
SP m	Understand and use the vocabulary of probability and probability scale
SP n	Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency
SP o	List all outcomes for single events, and for two successive events, in a systematic way and derive relative probabilities
SP p	Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
SP s	Compare experimental data and theoretical probabilities
SP t	Understand that if they repeat an experiment, they may – and usually will – get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics
SP u	Use calculators efficiently and effectively, including statistical functions

GCSE Mathematics A (1MA0)

Higher Tier

Linear
Scheme of work

Higher course overview

The table below shows an overview of modules in the Linear Higher tier scheme of work.

Teachers should be aware that the estimated teaching hours are approximate and should be used as a guideline only.

Module number	Title	Estimated teaching hours
1	Integers and decimals	5
2	Fractions	2
3	Fractions, decimals and percentages	4
4	Ratio and proportion	3
5	Index notation and surds	3
6	Algebra	5
7	Formulae and linear equations	4
8	Linear graphs	4
9	Simultaneous equations, quadratic equations and graphs	5
10	Trial and improvement	2
11	Further graphs and functions	3
12	Transformations of functions	2
13	Shape and angle	3
14	Construction and loci	3
15	Perimeter and area	4
16	Pythagoras and trigonometry	4
17	Surface area and volume	4
18	Transformations	4
19	Similarity and congruence	3
20	Circle theorems	3
21	Sine and cosine rules	3
22	Vectors	3
23	Measures and compound measures	3
24	Collecting data	2
25	Displaying data	4
26	Averages and range	4
27	Probability	3
	Total	**92 HOURS**

Module 1 **Time: 4 – 6 hours**

GCSE Tier: Higher

Contents: Integers and decimals

N a Add, subtract, multiply and divide whole numbers, integers and decimals

N b Order integers and decimals

N c Use the concepts and vocabulary of factor (divisor), multiple, common factor, Highest Common Factor, Lowest Common Multiple, prime number and prime factor decomposition

N d Use the terms square, positive and negative square root, cube and cube root

N e Use index notation for squares, cubes and powers of 10

N j Use decimal notation

N k Recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals

N q Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations

N s Calculate upper and lower bounds

N u Approximate to specified or appropriate degrees of accuracy, including a given power of 10, number of decimal places and significant figures

N v Use calculators effectively and efficiently

PRIOR KNOWLEDGE

The ability to order numbers and the appreciation of place value

Experience of the four operations using whole numbers

Knowledge of integer complements to 10 and 100 and multiplication facts to 10×10

Knowledge of strategies for multiplying and dividing whole numbers by 10

Concept of a decimal

OBJECTIVES

By the end of the module the student should be able to:

- understand and order integers and decimals
- identify factors, multiples and prime numbers
- find the prime factor decomposition of positive integers
- find the common factors and common multiples of two numbers
- find the Highest Common Factor (HCF) and the Lowest Common Multiple (LCM)
- recall integer squares from 2×2 to 15×15 and the corresponding square roots
- recall the cubes of 2, 3, 4, 5 and 10 and cube roots
- use index notation for squares and cubes
- use brackets and the hierarchy of operations (BIDMAS)

- understand and use positive numbers and negative integers, both as positions and translations on a number line
- add, subtract, multiply and divide integers, negative numbers and decimals
- round whole numbers to the nearest 10, 100, 1000,
- round decimals to appropriate numbers of decimal places or significant figures
- multiply and divide by any number between 0 and 1
- check their calculations by rounding, eg $29 \times 31 \approx 30 \times 30$
- check answers to a division sum using multiplication, eg use inverse operations
- multiply and divide decimal numbers by whole numbers and decimal numbers (up to 2 decimal places)
- know that eg $13.5 \div 0.5 = 135 \div 5$
- convert between recurring decimals and exact fractions and use proof
- calculate the upper and lower bounds of calculations, particularly when working with measurements
- find the upper and lower bounds of calculations involving perimeter, areas and volumes of 2-D and 3-D shapes
- find the upper and lower bounds in real-life situations using measurements given to appropriate degrees of accuracy
- give the final answer to an appropriate degree of accuracy following an analysis of the upper and lower bounds of a calculation

DIFFERENTIATION AND EXTENSION

Teachers may want to check that students have the appropriate skills, eg give students five digits such as 2, 5, 7, 8 and 1. They then need to find:

1) the largest even number

2) the smallest number in the 5 times table

3) the largest answer $\square\,\square\,\square + \square\,\square$

4) the smallest answer to $\square\,\square\,\square - \square\,\square$

Practise long multiplication and division without using a calculator

Estimate answers to calculations involving the four rules

Work with mental maths problems with negative powers of 10: 2.5×0.01, 0.001

Directed number work with two or more operations, or with decimals

Use decimals in real-life problems

Introduce standard form for very large and small numbers

Money calculations that require rounding answers to the nearest penny

Multiply and divide decimals by decimals (more than 2 decimal places)

Calculator exercise to check factors of larger numbers

Further work on indices to include negative and/or fractional indices

Use prime factors to find LCM and square roots

Plenty of investigative work for squares like 'half time' scores

Use a number square to find primes (sieve of Eratosthenes)

Calculator exercise to find squares, cubes and square roots of larger numbers (using trial and improvement)

NOTES

The expectation for most students doing Higher tier is that some of this material can be delivered or reinforced during other topics. For example, rounding with significant figures could be done with trigonometry

Students should present all working clearly with decimal points in line; and show that all working

For non-calculator methods, make sure that remainders and carrying are shown

Amounts of money should always be rounded to the nearest penny where necessary.

Make sure students are absolutely clear about the difference between significant figures and decimal places

Extend to multiplication of decimals and/or long division of integers

Try different methods from the traditional ones, eg Russian or Chinese methods for multiplication etc

Give lots of Functional Elements examples

All the work in this unit is easily reinforced by starters and plenaries

Calculators should only be used when appropriate

Encourage students to learn square, cube, prime and common roots for the non-calculator examination

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	1.1 – 1.5, 4.2 – 4.10, 25.1, 25.3
Edexcel GCSE Mathematics 16+ Student Book	1.2 – 1.7, 3.1, 12.3, TF support

Module 2 **Time: 1 – 3 hours**

GCSE Tier: Higher

Contents: Fractions

N h	Understand equivalent fractions, simplify a fraction by cancelling all common factors
N i, a	Add, subtract, multiply and divide fractions
N b	Order rational numbers
N v	Use a calculator effectively and efficiently
N o	Use fractions as operators

PRIOR KNOWLEDGE:

Multiplication facts

Ability to find common factors

A basic understanding of fractions as being 'parts of a whole unit'

Use of a calculator with fractions

OBJECTIVES

By the end of the module the student should be able to:

- find equivalent fractions
- compare the sizes of fractions
- write a fraction in its simplest form
- find fractions of an amount
- express a given number as a fraction of another number
- convert between mixed numbers and improper fractions
- add, subtract, multiply and divide fractions
- multiply and divide fractions including mixed numbers
- solve problems using fractions

DIFFERENTIATION AND EXTENSION

Could introduce 'hundredths' at this stage

Solve word problems involving fractions

Improper fractions can be introduced by using real-world examples, eg dividing 5 pizzas equally amongst 3 people

Careful differentiation is essential for this topic dependent on the student's ability

Use a calculator to change fractions into decimals and look for patterns

Work with improper fractions and mixed numbers

Multiplication and division of fractions to link with probability

Recognising that any fraction whose denominator has only 2 and/or 5 as a prime factor of the denominator can be written as a terminating decimal

Introduce algebraic fractions

NOTES

Constant revision of this topic is needed

Use fraction button on the calculator to check solutions

Link with Probability calculations using AND and OR Laws

Use fractions for calculations involving compound units

Use Functional Elements questions and examples using fractions, eg $\frac{1}{4}$ off the list price when comparing different sale prices

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Higher Student book	3.1 – 3.4, 4.1
Edexcel GCSE Mathematics 16+ Student Book	Chapter 2

Module 3 **Time: 3 – 5 hours**

GCSE Tier: Higher

Contents: Fractions, decimals and percentages

N j	Use decimal notation and recognise that each terminating decimal is a fraction
N k	Recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals
N l	Understand that 'percentage' means 'number of parts per 100' and use this to compare proportions
N m	Use percentage and repeated proportional change
N o	Interpret fractions, decimals and percentages as operators
N v	Use calculators effectively and efficiently
N q	Use reverse percentage calculations

PRIOR KNOWLEDGE

Four operations of number

The concepts of a fraction and a decimal

Awareness that percentages are used in everyday life

OBJECTIVES

By the end of the module the student should be able to:

- understand that a percentage is a fraction in hundredths
- convert between fractions, decimals and percentages
- write one number as a percentage of another number
- calculate the percentage of a given amount
- find a percentage increase/decrease of an amount
- find a reverse percentage, eg find the original cost of an item given the cost after a 10% deduction
- use a multiplier to increase by a given percent over a given time, eg $1.1^8 \times 64$ increases 64 by 10% over 8 years
- calculate simple and compound interest

DIFFERENTIATION & EXTENSION

Find fractional percentages of amounts, without using a calculator, eg 0.825%

Combine multipliers to simplify a series of percentage changes

Percentages which convert to recurring decimals, eg $33\frac{1}{3}$%, and situations which lead to percentages of more than 100%

Problems which lead to the necessity of rounding to the nearest penny (eg real-life contexts)

Comparisons between simple and compound interest calculations

NOTES

Emphasise the Functional Elements in this topic, use real-world problems involving fractions, decimals and percentages

Amounts of money should always be rounded to the nearest penny where necessary, except where such rounding is premature, eg in successive calculations like in compound interest

In preparation for this unit, students should be reminded of basic percentages and recognise their fraction and decimal equivalents

Link with probability calculations using AND and OR Laws

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	14.1 – 14.5
Edexcel GCSE Mathematics 16+ Student Book	4.1 – 4.4, TF support

Module 4 **Time: 2 – 4 hours**

GCSE Tier: Higher

Contents: Ratio and proportion

N p	Use ratio notation, including reduction to its simplest form and its various links to fraction notation
N q	Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
N t	Divide a quantity in a given ratio
N n	Understand and use direct and indirect proportion
A u	Direct and indirect proportion (algebraic)

PRIOR KNOWLEDGE:

Fractions

OBJECTIVES

By the end of the module the student should be able to:

- use ratios
- write ratios in their simplest form
- divide a quantity in a given ratio
- solve a ratio problem in a context
- solve word problems
- calculate an unknown quantity from quantities that vary in direct or inverse proportion
- set up and use equations to solve word and other problems involving direct proportion or inverse proportion and relate algebraic solutions to graphical representation of the equations

DIFFERENTIATION AND EXTENSION

Harder problems involving multi-stage calculations

Relate ratios to Functional Elements situations, eg investigate the proportions of the different metals in alloys and the new amounts of ingredients for a recipe for different numbers of guests

NOTES

Students often find ratios with three parts difficult

Link ratios given in different units to metric and imperial units

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	16.1 – 16.5, 27.1 – 27.5
Edexcel GCSE Mathematics 16+ Student Book	Chapter 5

Module 5 **Time: 2 – 4 hours**

GCSE Tier: Higher

Contents: Index notation and surds

N e	Use index notation for squares, cubes and powers of 10
N f	Use index laws for multiplication and division of integer (negative and fractional) powers
N g	Interpret, order and calculate with numbers written in standard form
N v	Use calculators effectively and efficiently
N q	Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
N r	Calculate with surds

PRIOR KNOWLEDGE

Knowledge of squares, square roots, cubes and cube roots

Fractions and algebra

OBJECTIVES

By the end of the module the student should be able to:

- use index notation for integer powers of 10
- use standard form, expressed in conventional notation
- be able to write very large and very small numbers presented in a context in standard form
- convert between ordinary and standard form representations
- interpret a calculator display using standard form
- calculate with standard form
- use index laws to simplify and calculate the value of numerical expressions involving
- multiplication and division of integer negative and fractional powers, and powers of a power
- find the value of calculations using indices
- use index laws to simplify and calculate numerical expressions involving powers, eg $(2^3 \times 2^5) \div 2^4$, 4^0, $8^{-2/3}$
- know that, eg $x^3 = 64 \Rightarrow x = 8^{2/3}$
- understand that the inverse operation of raising a positive number to a power of n is raising the result of this operation to the power $\frac{1}{n}$
- rationalise the denominator, eg $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ and, eg write $(\sqrt{18} + 10) \div \sqrt{2}$ in the form $p + q\sqrt{2}$
- use calculators to explore exponential growth and decay
- write $\sqrt{8}$ in the form $2\sqrt{2}$
- write $(3 - \sqrt{3})^2$ in the form $a + b\sqrt{3}$

DIFFERENTIATION AND EXTENSION

Explain the difference between rational and irrational numbers as an introduction to surds

Prove that $\sqrt{2}$ is irrational

Revise the difference of two squares to show why we use, for example, $(\sqrt{3} - 2)$ as the multiplier to rationalise $(\sqrt{3} + 2)$

Rationalise the denominator, eg $\frac{1}{\sqrt{3}-1} = \left(\frac{\sqrt{3}+1}{2}\right)$

Link to work on circle measures (involving π) and Pythagoras calculations in exact form.

NOTES

Link simplifying surds to collecting together like terms, eg $3x + 2x = 5x$, $3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$

Stress it is better to write answers in exact form, eg $\frac{1}{3}$ is better than 0.333333…..

A-Level C1 textbooks are a good source of extension questions on surd manipulation, some of which are algebraic

Useful generalisation to learn is $\sqrt{x} \times \sqrt{x} = x$

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Higher Student book	1.5, 25.1 – 25.4
Edexcel GCSE Mathematics 16+ Student Book	1.8 – 1.9, 3.2

Module 6 **Time: 4 – 6 hours**

GCSE Tier: Higher

Contents: Algebra

A a	Distinguish between the different roles played by letter symbols in algebra, using the correct notation
A b	Distinguish in meaning of the words 'equation', 'formula', 'identity' and 'expression'
A c	Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors, multiplying two linear expressions, factorise quadratic expressions including the difference of two squares and simplify rational expressions
A c	Simplify expressions using rules of indices
A i	Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence
A j	Use linear expressions to describe the *n*th term of an arithmetic sequence

PRIOR KNOWLEDGE

Experience of using a letter to represent a number

Ability to use negative numbers with the four operations

Recall and use BIDMAS

Recognise simple number patterns, eg 1, 3, 5...

Writing simple rules algebraically

OBJECTIVES

By the end of the module the student should be able to:

- use notation and symbols correctly
- write an expression
- select an expression/identity/equation/formula from a list
- manipulate algebraic expressions by collecting like terms
- simplify expressions using index laws
- use index laws for integer, negative, and fractional powers and powers of a power
- recognise sequences of odd and even numbers
- generate simple sequences of numbers, squared integers and sequences derived from diagrams
- describe the term-to-term definition of a sequence in words
- identify which terms cannot be in a sequence
- generate specific terms in a sequence using the position-to-term and term-to-term rules
- find and use the *n*th term of an arithmetic sequence
- multiply a single term over a bracket

- factorise algebraic expressions by taking out common factors
- expand the product of two linear expressions
- factorise quadratic expressions, including using the difference of two squares
- simplify rational expressions by cancelling, adding, subtracting and multiplying

DIFFERENTIATION AND EXTENSION

This topic can be used as a reminder of the KS3 curriculum and could be introduced via investigative material, eg frogs, handshakes, patterns in real life, formulae

Use examples where generalisation skills are required

Extend the above ideas to the 'equation' of the straight line, $y = mx + c$

Practise factorisation where the factor may involve more than one variable

Sequences and nth term formula for triangle numbers, Fibonacci numbers etc

Prove a sequence cannot have odd numbers for all values of n

Extend to quadratic sequences whose nth term is $an^2 + bn + c$

NOTES

There are plenty of old exam papers with matching tables testing knowledge of the 'Vocabulary of Algebra' (see Emporium website)

Emphasise good use of notation, eg $3n$ means $3 \times n$

When investigating linear sequences, students should be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the nth term

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	2.1 – 2.6, 9.1 – 9.4, 13.6, 32.1 – 32.3
Edexcel GCSE Mathematics 16+ Student Book	6.1, 6.3 – 6.8, 7.1 – 7.2, TF support

Module 7 **Time: 3 – 5 hours**

GCSE Tier: Higher

Contents: Formulae and linear equations

A f	Derive a formula, substitute numbers into a formula and change the subject of a formula
A d	Set up and solve simple equations
A g	Solve linear inequalities in one variable, and represent the solution set on a number line

PRIOR KNOWLEDGE

Experience of finding missing numbers in calculations

The idea that some operations are the reverse of each other

An understanding of balancing

Experience of using letters to represent quantities

Understand and recall BIDMAS

OBJECTIVES

By the end of the module the student should be able to:

- derive a formula
- use formulae from mathematics and other subjects
- substitute numbers into a formula
- substitute positive and negative numbers into expressions such as $3x^2 + 4$ and $2x^3$
- set up linear equations from word problems
- solve simple linear equations
- solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation
- solve linear equations that include brackets, those that have negative signs occurring anywhere in the equation, and those with a negative solution
- solve linear equations in one unknown, with integer or fractional coefficients
- solve simple linear inequalities in one variable, and represent the solution set on a number line
- use the correct notation to show inclusive and exclusive inequalities
- change the subject of a formula including cases where the subject is on both sides of the original formula, or where a power of the subject appears

DIFFERENTIATION AND EXTENSION

Use negative numbers in formulae involving indices

Use investigations to lead to generalisations

Apply changing the subject to $y = mx + c$

Derive equations from practical situations (such as finding unknown angles in polygons or perimeter problems)

NOTES

Emphasise good use of notation, eg $3ab$ means $3 \times a \times b$

Students need to be clear on the meanings of the words expression, equation, formula and identity

Remind students that not all linear equations can easily be solved by either observation or trial and improvement, and hence the use of a formal method is important

Students can leave their answers in fractional form where appropriate

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Higher Student book	2.2, 13.1 – 13.5, 19.1 – 19.8
Edexcel GCSE Mathematics 16+ Student Book	9.1 – 9.4, 9.7 – 9.8, 10.1 – 10.2, 10.4

Module 8 **Time: 3 – 5 hours**

GCSE Tier: Higher

Contents: Linear graphs

A k	Use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information
A l	Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients
A m	Understand that the form $y = mx + c$ represents a straight line and that m is the gradient of the line and c is the value of the y-intercept
A n	Understand the gradients of parallel lines
A g	Solve linear inequalities in two variables, and represent the solution set on a coordinate grid
A r	Construct linear functions from real-life problems and plot their corresponding graphs
A s	Interpret graphs of linear functions

PRIOR KNOWLEDGE

Substitute positive and negative numbers into algebraic expressions

Rearrange to change the subject of a formula

OBJECTIVES

By the end of the module the student should be able to:

- draw, label and scale axes
- use axes and coordinates to specify points in all four quadrants in 2-D and 3-D
- identify points with given coordinates
- identify coordinates of given points (NB: Points may be in the first quadrant or all four quadrants)
- find the coordinates of points identified by geometrical information in 2-D and 3-D
- find the coordinates of the midpoint of a line segment, AB, given the coordinates of A and B
- recognise that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane
- draw and interpret straight-line graphs for real-life situations such as:
  - ready reckoner graphs
  - conversion graphs
  - fuel bills, eg gas and electric
  - fixed charge (standing charge) and cost per unit
- plot and draw graphs of straight lines with equations of the form $y = mx + c$
- find the gradient of a straight line from a graph

- analyse problems and use gradients to interpret how one variable changes in relation to another
- interpret and analyse a straight-line graph
- understand that the form $y = mx + c$ represents a straight line
- find the gradient of a straight line from its equation
- explore the gradients of parallel lines and lines perpendicular to each other
- write down the equation of a line parallel or perpendicular to a given line
- use the fact that when $y = mx + c$ is the equation of a straight line then the gradient of a line parallel to it will have a gradient of m and a line perpendicular to this line will have a gradient of $-\frac{1}{m}$
- interpret and analyse a straight-line graph and generate equations of lines parallel and perpendicular to the given line
- show the solution set of several inequalities in two variables on a graph

DIFFERENTIATION AND EXTENSION

Students should find the equation of the line through two given points

Students should find the equation of the perpendicular bisector of the line segment joining two given points

Use Functional Elements in terms of mobile phone bills

Use a spreadsheet to generate straight-line graphs, posing questions about the gradient of lines

Use a graphical calculator or graphical ICT package to draw straight-line graphs

Link to scatter graphs and correlation

Cover lines parallel to the axes ($x = c$ and $y = c$), as students often forget these

NOTES

Careful annotation should be encouraged. Students should label the coordinate axes and origin and write the equation of the line

Students need to recognise linear graphs and hence when data may be incorrect

Link to graphs and relationships in other subject areas, ie science, geography etc

Link conversion graphs to converting metric and imperial units

A-Level C1 textbooks can be a good source of extension questions on this topic

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	15.1 – 15.6, 19.4, 23.10
Edexcel GCSE Mathematics 16+ Student Book	8.1 – 8.5, TF support

Module 9 **Time: 4 – 6 hours**

GCSE Tier: Higher

Contents: Simultaneous equations, quadratic equations and graphs

A c	Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors, factorising quadratic expressions, and difference of two squares
A d	Set up and solve simultaneous equations in two unknowns
A t	Generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions
A r	Construct linear, quadratic and other functions from real-life problems and plot their corresponding graphs
A e	Solve quadratic equations
A o	Find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions

PRIOR KNOWLEDGE

An introduction to algebra

Substitution into expressions/formulae

Linear functions and graphs

Solving equations

OBJECTIVES

By the end of the module the student should be able to:

- find the exact solutions of two simultaneous equations in two unknowns
- use elimination or substitution to solve simultaneous equations
- interpret a pair of simultaneous equations as a pair of straight lines and their solution as the point of intersection
- set up and solve a pair of simultaneous equations in two variables
- generate points and plot graphs of simple quadratic functions, then more general quadratic functions
- find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function
- find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions
- solve simple quadratic equations by factorisation and completing the square
- solve simple quadratic equations by using the quadratic formula

- select and apply algebraic and graphical techniques to solve simultaneous equations where one is linear and one quadratic
- solve equations involving algebraic fractions which lead to quadratic equations

DIFFERENTIATION & EXTENSION

Clear presentation of workings is essential

Use open-ended questions that test student awareness of what intersections mean for mobile phone bills

Students to solve two simultaneous equations with fractional coefficients

Students to solve two simultaneous equations with second order terms, eg equations in x and y^2

Students to derive the quadratic equation formula by completing the square

Use graphical calculators or ICT graph package where appropriate

Show how the value of '$b^2 - 4ac$' can be useful in determining if the quadratic factorises or not (i.e. square number)

Extend to properties of the discriminant and roots

NOTES

Build up the algebraic techniques slowly

Link the graphical solutions with linear graphs and changing the subject, and use practical examples, eg projectiles

Emphasise that inaccurate graphs could lead to inaccurate solutions; encourage substitution of answers to check they are correct

Some students may need additional help with factorising

Students should be reminded that factorisation should be tried before the formula is used

In problem-solving, one of the solutions to a quadratic equation may not be appropriate

There may be a need to remove the HCF (numerical) of a trinomial before factorising to make the factorisation easier

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	22.1 – 22.9, 22.1 – 22.12
Edexcel GCSE Mathematics 16+ Student Book	8.6, 9.9

Module 10 **Time: 1 – 3 hours**

GCSE Tier: Higher

Contents: Trial and Improvement

A h Use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them

N u Approximate to specified or appropriate degrees of accuracy including a number of decimal places and significant figures

N v Use calculators effectively and efficiently

PRIOR KNOWLEDGE

Substituting numbers into algebraic expressions

Dealing with decimals on a calculator

Ordering decimals

OBJECTIVES

By the end of the module the student should be able to:

- use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them
- Understand the connections between changes of sign and location of roots

DIFFERENTIATION AND EXTENSION

Solve equations of the form $\frac{1}{x} = x^2 - 5$ (revise changing the subject of the formula)

NOTES

Look at 'practical examples'. A room is 2 m longer than it is wide. If its area is 30 m² what is its perimeter?

Students should be encouraged to use their calculators efficiently – by using the 'replay' or ANS/EXE functions

The square/cube function on a calculator may not be the same for different makes

Take care when entering negative values to be squared (always use brackets)

Students should write down all the digits on their calculator display and only round the final answer declared to the degree of accuracy

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	21.5
Edexcel GCSE Mathematics 16+ Student Book	9.6

Module 11 **Time: 2 – 4 hours**

GCSE Tier: Higher

Contents: Further graphs and functions

A o Find the intersection points of the graphs of a linear and quadratic function

A p Draw, sketch, recognise graphs of simple cubic functions,

the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$,

the function $y = k^x$ for integer values of x and simple positive values of k,

the trigonometric functions $y = \sin x$ and $y = \cos x$

A q Construct the graphs of simple loci

PRIOR KNOWLEDGE:

Linear functions 1

Quadratic functions

OBJECTIVES

By the end of the module the student should be able to:

- plot and recognise cubic, reciprocal, exponential and circular functions $y = \sin x$ and $y = \cos x$, within the range -360º to +360º
- find the values of p and q in the function $y = pq^x$ given the graph of $y = pq^x$
- match equations with their graphs
- recognise the characteristic shapes of all these functions
- construct the graphs of simple loci including the circle $x^2 + y^2 = r^2$ for a circle of radius r centred at the origin of the coordinate plane
- find the intersection points of a given straight line with a circle graphically
- select and apply construction techniques and understanding of loci to draw graphs based on circles and perpendiculars of lines
- solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns, one of which is linear in each unknown, and the other is linear in one unknown and quadratic in the other, or where the second equation is of the form $x^2 + y^2 = r^2$
- draw and plot a range of mathematical functions

DIFFERENTIATION AND EXTENSION

Explore the function $y = e^x$ (perhaps relate this to $y = \ln x$)

Explore the function $y = \tan x$

Find solutions to equations of the circular functions $y = \sin x$ and $y = \cos x$ over more than one cycle (and generalise)

This work should be enhanced by drawing graphs on graphical calculators and appropriate software

Complete the square for quadratic functions and relate this to transformations of the curve $y = x^2$

NOTES

Make sure students understand the notation $y = f(x)$. Start by comparing $y = x^2$ with $y = x^2 + 2$ before mentioning $y = f(x) + 2$ etc

Graphical calculators and/or graph drawing software will help to underpin the main ideas in this unit

Link with trigonometry and curved graphs

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Higher Student book	21.2 – 21.4, 22.10 -22.12, 29.3
Edexcel GCSE Mathematics 16+ Student Book	n/a

Module 12 **Time: 1 – 3 hours**

GCSE Tier: Higher

Contents: Transformations of functions

A v Transformation of functions

PRIOR KNOWLEDGE:

Transformations

Using f(x) notation

Graphs of simple functions

OBJECTIVES

By the end of the module the student should be able to:

- apply to the graph of $y = \text{f}(x)$ the transformations $y = \text{f}(x) + a$, $y = \text{f}(ax)$, $y = \text{f}(x + a)$, $y = a\text{f}(x)$ for linear, quadratic, sine and cosine functions, f(x)
- select and apply the transformations of reflection, rotation, enlargement and translation of functions expressed algebraically
- interpret and analyse transformations of functions and write the functions algebraically

DIFFERENTIATION AND EXTENSION

Complete the square of quadratic functions and relate this to transformations of the curve $y = x^2$

Use a graphical calculator/software to investigate transformations

Investigate curves which are unaffected by particular transformations

Investigate simple relationships such as $\sin(180 - x) = \sin x$, and $\sin(90 - x) = \cos x$

NOTES

Make sure students understand the notation $y = \text{f}(x)$, start by comparing $y = x^2$ with $y = x^2 + 2$ before mentioning $y = \text{f}(x) + 2$ etc

Graphical calculators and/or graph drawing software will help to underpin the main ideas in this unit

Link with trigonometry and curved graphs

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Higher Student book	30.2 – 30.4
Edexcel GCSE Mathematics 16+ Student Book	n/a

Module 13 **Time: 2 – 4 hours**

GCSE Tier: Higher

Contents: Shape and angle

GM a Recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines, and opposite angles at a vertex

GM b Understand and use the angle properties of parallel lines, triangles and quadrilaterals

GM c Calculate and use the sums of the interior and exterior angles of polygons

GM d Recall the properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus

GM r Understand and use bearings

PRIOR KNOWLEDGE

An understanding of angle as a measure of turning

The ability to use a protractor to measure angles

An understanding of the concept of parallel lines

OBJECTIVES

By the end of the module the student should be able to:

- recall and use properties of:
  - angles at a point
  - angles at a point on a straight line
  - perpendicular lines
  - vertically opposite angles
- understand and use the angle properties of parallel lines
- mark parallel lines on a diagram
- use the properties of corresponding and alternate angles
- distinguish between scalene, isosceles, equilateral, and right-angled triangles
- understand and use the angle properties of triangles
- use the angle sum of a triangle is 180º
- understand and use the angle properties of intersecting lines
- recognise and classify quadrilaterals
- understand and use the angle properties of quadrilaterals
- give reasons for angle calculations
- explain why the angle sum of a quadrilateral is 360º
- understand the proof that the angle sum of a triangle is 180º
- understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles of the other two vertices
- use the size/angle properties of isosceles and equilateral triangles
- recall and use these properties of angles in more complex problems

- understand, draw and measure bearings
- calculate bearings and solve bearings problems
- calculate and use the sums of the interior angles of polygons
- use geometric language appropriately and recognise and name pentagons, hexagons, heptagons, octagons and decagons
- use the angle sums of irregular polygons
- calculate and use the angles of regular polygons
- use the sum of the interior angles of an n sided polygon
- use the sum of the exterior angles of any polygon is 360º
- use the sum of the interior angle and the exterior angle is 180º
- find the size of each interior angle or the size of each exterior angle or the number of sides of a regular polygon
- understand tessellations of regular and irregular polygons and combinations of polygons
- explain why some shapes tessellate when other shapes do not

DIFFERENTIATION AND EXTENSION

Use triangles to find the angle sums of polygons

Use the angle properties of triangles to find missing angles in combinations of triangles

Harder problems involving multi-step calculations

Link with symmetry and tessellations

NOTES

Most of this is KS3, so can be treated as an opportunity for groups of students to present parts of the module to the rest of the class. They could be encouraged to make resources, eg follow me cards, puzzles etc for the others to do

Angles in polygons could be investigated algebraically

The tessellation can be done as a cross curricular project with Art (Escher) and is good for wall displays

Use lots of practical drawing examples to help illustrate properties of various shapes

Explain that diagrams used in examinations are seldom drawn accurately

Use tracing paper to show which angles in parallel lines are equal

Encourage students to always give their reasons in problems and 'quote' the angle fact/theorem used

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	5.1 – 5.7
Edexcel GCSE Mathematics 16+ Student Book	11.1 – 11.3, 11.6 – 11.7, TF support

Module 14 **Time: 2 – 4 hours**

GCSE Tier: Higher

Contents: Constructions and Loci

GM v Use a straight edge and a pair of compasses to carry out constructions

GM w Construct loci

GM m Use and interpret maps and scale drawings

PRIOR KNOWLEDGE

The ability to use a pair of compasses to draw a line of a given length

The special names of triangles (and angles)

An understanding of the terms perpendicular, parallel and arc

OBJECTIVES

By the end of the module students should be able to:

- use straight edge and a pair of compasses to do standard constructions
- construct triangles including an equilateral triangle
- understand, from the experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not
- construct the perpendicular bisector of a given line
- construct the perpendicular from a point to a line
- construct the perpendicular from a point on a line
- construct the bisector of a given angle
- construct angles of 60º, 90º, 30º and 45º
- draw parallel lines
- draw circles and arcs to a given radius
- construct a regular hexagon inside a circle
- construct diagrams of everyday 2-D situations involving rectangles, triangles, perpendicular and parallel lines
- draw and construct diagrams from given information
- construct: – a region bounded by a circle and an intersecting line
- a given distance from a point and a given distance from a line
- equal distances from 2 points or 2 line segments
- regions which may be defined by 'nearer to' or 'greater than'
- find and describe regions satisfying a combination of loci
- use and interpret maps and scale drawings
- read and construct scale drawings drawing lines and shapes to scale
- estimate lengths using a scale diagram

DIFFERENTIATION & EXTENSION

Solve loci problems that require a combination of loci

Relate to real life examples including horses tethered in fields or mobile phone masts and signal coverage

NOTES

All working should be presented clearly, and accurately

A sturdy pair of compasses is essential

Construction lines should not be erased as they carry method marks

Could use construction to link to similarity and congruence

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	12.1 – 12.6
Edexcel GCSE Mathematics 16+ Student Book	11.8, Chapter 15, TF support

Module 15 **Time: 3 – 5 hours**

GCSE Tier: Higher

Contents: Perimeter and area

GM x Calculate perimeters and areas of shapes made from triangles and rectangles and other shapes

GM z Find circumferences and areas of circles

N r Use π in an exact calculation

PRIOR KNOWLEDGE

Names of triangles, quadrilaterals and polygons

Concept of perimeter and area

Units of measurement

Substitute numbers into formulae

The ability to give answers to an appropriate degree of accuracy

OBJECTIVES

By the end of the module the student should be able to:

- measure sides of a shape to work out perimeter or area
- find the perimeter of rectangles and triangles
- recall and use the formulae for the area of a triangle, rectangle and parallelogram
- find the area of a trapezium
- calculate perimeter and area of compound shapes made from triangles, rectangles and other shapes
- find circumferences of circles and areas enclosed by circles
- recall and use the formulae for the circumference of a circle and the area enclosed by a circle
- use $\pi \approx 3.142$ or use the π button on a calculator
- give an exact answer to a question involving the area or the circumference of a circle in terms of π
- find the perimeters and areas of semicircles and quarter circles
- calculate the lengths of arcs and the areas of sectors of circles

DIFFERENTIATION & EXTENSION

Calculate areas and volumes using formulae

Using compound shape methods to investigate areas of other standard shapes such as parallelograms, trapeziums and kites

Emphasise the Functional Elements here with carpets for rooms, tiles for walls, turf for gardens as well as wall paper and skirting board problems

Further problems involving combinations of shapes

Practical activities, eg using estimation and accurate measuring to calculate perimeters and areas of classroom/corridor floors

NOTES

Discuss the correct use of language and units

Ensure that students can distinguish between perimeter, area and volume

Practical experience is essential to clarify these concepts.

There are many Functional Elements questions which can be applied to this topic area, eg floor tiles, optimisation type questions, which pack of tiles give the best value?

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	10.1 – 10.3, 23.1 – 23.2
Edexcel GCSE Mathematics 16+ Student Book	13.1 – 13.2, Chapter 14

Module 16 **Time: 3 – 5 hours**

GCSE Tier: Higher

Contents: Pythagoras and Trigonometry

GM g	Use Pythagoras' theorem in 2-D and 3-D
GM h	Use the trigonometric ratios to solve 2-D and 3-D problems
N v	Use calculators effectively and efficiently, including trigonometrical functions
N u	Approximate to specified or appropriate degrees of accuracy including a given, number of decimal places and significant figures
A k	Find the length of a line segment

PRIOR KNOWLEDGE

Some understanding of similar triangles

Able to use a calculator to divide numbers

Mensuration – perimeter and area

Formulae

OBJECTIVES

By the end of the module the student should be able to:

- understand, recall and use Pythagoras' theorem in 2-D, then in 3-D problems
- calculate the length of a line segment in 2-D
- give an answer in the use of Pythagoras' theorem as $\sqrt{13}$
- recall and use the trigonometric ratios to solve 2-D and 3-D problems
- find angles of elevation and angles of depression
- understand the language of planes, and recognise the diagonals of a cuboid
- calculate the length of a diagonal of a cuboid
- find the angle between a line and a plane (but not the angle between two planes or between two skew lines)

DIFFERENTIATION AND EXTENSION

Look at Functional Elements exemplar material

Harder problems involving multi-stage calculations

Organise a practical surveying lesson to find the heights of buildings/trees around your school grounds. All you need is a set of tape measures (or trundle wheels) and clinometers

NOTES

Students should be encouraged to become familiar with one make of calculator

Calculators should be set to *degree* mode

Emphasise that scale drawings will score no marks for this type of question

A useful mnemonic for remembering trigonometric ratios is 'Sir Oliver's Horse, Came Ambling Home, To Oliver's Aunt' or 'SOH/CAH/TOA'; but students often enjoy making up their own

Calculated angles should be given to at least 1 decimal place and sides are determined by the units used or accuracy asked for in the question.

Students should not forget to state the units for the answers.

The angle between two planes or two skew lines is not required

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Higher Student book	20.2 – 20.5, 25.4, 29.1 – 29.2, 29.9
Edexcel GCSE Mathematics 16+ Student Book	Chapter 18

Module 17 **Time: 3 – 5 hours**

GCSE Tier: Higher

Contents: Surface area and volume

GM k	Use 2-D representations of 3-D shapes
GM x	Find the surface area of simple shapes (prisms) using the formulae for triangles and rectangles, and other shapes
GM aa	Calculate volumes of right prisms and shapes made from cubes and cuboids
GM z	Find the surface area of a cylinder
GM bb	Solve mensuration problems involving more complex shapes and solids
GM p	Convert measures from one unit to another
N r	Use π for answers in exact calculations

PRIOR KNOWLEDGE

Construction and loci

Names of 3-D shapes

Concept of volume

Knowledge of area

An ability to give answers to a degree of accuracy

Experience of changing the subject of a formula

OBJECTIVES

By the end of the module the student should be able to:

- use 2-D representations of 3-D shapes
- use isometric grids
- draw nets and show how they fold to make a 3-D solid
- understand and draw front and side elevations and plans of shapes made from simple solids
- given the front and side elevations and the plan of a solid, draw a sketch of the 3-D solid
- know and use formulae to calculate the surface areas and volumes of cuboids and right-prisms and shapes made from cuboids
- find the surface area of simple shapes (prisms) using the formulae for triangles and rectangles, and other shapes
- find the surface area of a cylinder
- convert between metric units of area
- solve a range of problems involving surface area and volume, eg given the volume and length of a cylinder, find the radius find the volume of a cylinder
- convert between volume measures, including cubic centimetres and cubic metres
- solve problems involving more complex shapes and solids, including segments of circles and frustums of cones

- find the surface area and volumes of compound solids constructed from cubes, cuboids, cones, pyramids, spheres, hemispheres, cylinders, eg solids in everyday use
- convert between units of capacity and volume
- find the area of a segment of a circle given the radius and length of the chord

DIFFERENTIATION & EXTENSION

Additional work using algebraic expressions

Find surface area and volume of a sphere and cone (using standard formulae)

Convert between less familiar units, eg cm^3 to mm^3, cm^3 to litres

Use Functional Elements type questions, eg fitting boxes in crates

Use density/volume/mass questions

Find the volume of a cylinder given its surface area, leaving the answer in terms of l

Find the volume of a right hexagonal pyramid of side x and height h (researching the method for finding the volume of any pyramid)

Make solids using equipment such as clixi or multi-link with different coloured cubes.

Draw on isometric paper shapes made from multi-link

Construct combinations of 2-D shapes to make nets of 3-D shapes

An excellent topic for wall display.

Extend to Planes of Symmetry for 3-D solids

Discover Euler's formula for solids

Investigate how many small boxes can be packed into a larger box, as a Functional Elements type example

This result is known as **Euler's formula**. An illustration of the formula on some below.

Name	Image	Vertices *V*	Edges *E*	Faces *F*	Euler characteristic: *V* − *E* + *F*
Tetrahedron		4	6	4	**2**
Hexahedron or cube		8	12	6	**2**
Octahedron		6	12	8	**2**
Dodecahedron		20	30	12	**2**
Icosahedron		12	30	20	**2**

NOTES

'Now! I Know Pi' is a good way to learn the approximate value (the number of letters of each word and the ! is the decimal point)

Also 'Cherry Pie Delicious' is $C = \pi D$ and 'Apple Pies are too' is $A = \pi r^2$

Answers in terms of π may be required or final answers rounded to the required degree of accuracy

Need to constantly revise the expressions for area/volume of shapes

Students should be aware of which formulae are on the relevant page on the exam paper and which they need to learn

All working should be presented clearly, and accurately

A sturdy pair of compasses is essential

Accurate drawing skills need to be reinforced

Some students find visualising 3-D objects difficult – simple models will assist

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Higher Student book	10.4 – 10.8, 23.1, 23.3 – 23.9
Edexcel GCSE Mathematics 16+ Student Book	Chapter 16

Module 18 **Time: 3 – 5 hours**

GCSE Tier: Higher

Contents: Transformations

GM e Recognise reflection and rotation symmetry of 2-D shapes

GM l Describe and transform 2-D shapes using single or combined rotations, reflections, translations, or enlargements by a positive, fractional or negative scale factor and distinguish properties that are preserved under particular transformations

PRIOR KNOWLEDGE

Recognition of basic shapes

An understanding of the concept of rotation, reflection and enlargement

Coordinates in four quadrants

Linear equations parallel to the coordinate axes

OBJECTIVES

By the end of the module the student should be able to:

- translate a given shape by a vector
- recognise rotation and reflection of 2-D shapes
- state the line symmetry as a simple algebraic equation
- recognise rotation symmetry of 2-D shapes
- identify the order of rotational symmetry of a 2-D shape
- understand rotation as a (anti clockwise) turn about a given centre
- reflect shapes in a given mirror line; parallel to the coordinate axes and then $y = x$ or $y = -x$
- enlarge shapes by a given scale factor from a given point; using positive, negative and fractional scale factors
- find the centre of enlargement
- understand that images produced by translation, rotation and reflection are congruent to the object
- describe and transform 2-D shapes using single rotations
- understand that rotations are specified by a centre and an (anticlockwise) angle
- find the centre of rotation
- rotate a shape about the origin, or any other point
- describe and transform 2-D shapes using combined rotations, reflections, translations or enlargements
- use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations
- distinguish properties that are preserved under particular transformations
- recognise that enlargements preserve angle but not length, linking to similarity
- describe a transformation

DIFFERENTIATION AND EXTENSION

The tasks set can be extended to include further combinations of transformations

Students could research glide reflection

NOTES

Emphasise that students should describe the given transformation fully

Diagrams should be drawn carefully

The use of tracing paper is allowed in the examination (although students should not have to rely on the use of tracing paper to solve problems)

RESOURCES

Textbook	**References**
Edexcel GCSE Mathematics A Linear Higher Student book	17.1 – 17.5
Edexcel GCSE Mathematics 16+ Student Book	Chapter 17

Module 19 **Time: 2 – 4 hours**

GCSE Tier: Higher

Contents: Similarity and congruence

GM f Understand congruence and similarity

GM n Understand and use the effect of enlargement for perimeter, area and volume of shapes and solids

PRIOR KNOWLEDGE

Ratio

Proportion

Area and Volume

OBJECTIVES

By the end of the module the student should be able to:

- understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and a pair of compasses constructions
- understand similarity of triangles and of other plane figures, and use this to make geometric inferences
- provide formal geometric proof of similarity of two given triangles
- recognise that all corresponding angles in similar figures are equal in size when the lengths of sides are not
- understand the effect of enlargement for perimeter, area and volume of shapes and solids
- understand that enlargement does not have the same effect on area and volume
- use simple examples of the relationship between enlargement and areas and volumes of simple shapes and solids
- use the effect of enlargement on areas and volumes of shapes and solids
- know the relationships between linear, area and volume scale factors of mathematically similar shapes and solids

DIFFERENTIATION AND EXTENSION

This could be introduced practically or by investigation of simple shapes such as squares, rectangles, circles (reminder of formula), cuboids, cylinders etc

Link with tessellations and enlargements

Link with similar areas and volumes

Harder problems in congruence

Relate this unit to circle theorems

NOTES

All working should be presented clearly and accurately

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	8.1, 8.4 – 8.5, 26.1 – 26.3
Edexcel GCSE Mathematics 16+ Student Book	n/a

Module 20 **Time: 2 – 4 hours**

GCSE Tier: Higher

Contents: Circle theorems

GM i Distinguish between centre, radius, chord, diameter, circumference, tangent, arc, sector and segment

GM j Understand and construct geometrical proofs using circle theorems

PRIOR KNOWLEDGE:

Recall the words centre, radius, diameter and circumference

Experience of drawing circles with compasses

OBJECTIVES

By the end of the module the student should be able to:

- recall the definition of a circle and identify (name) and draw the parts of a circle
- understand related terms of a circle
- draw a circle given the radius or diameter
- understand and use the fact that the tangent at any point on a circle is perpendicular to the radius at that point
- understand and use the fact that tangents from an external point are equal in length
- find missing angles on diagrams
- give reasons for angle calculations involving the use of tangent theorems
- prove and use the:
  - the fact that the angle subtended by an arc at the centre of a circle is twice
  - the angle subtended at any point on the circumference
  - the fact that the angle in a semicircle is a right angle
  - the fact that angles in the same segment are equal
  - the fact that opposite angles of a cyclic quadrilateral sum to 180º
  - the fact that alternate segment theorem
  - the fact that the perpendicular line from the centre of a circle to a chord bisects the chord

DIFFERENTIATION AND EXTENSION

Harder problems involving multi-stage angle calculations

Intersecting chord theorems

NOTES

Any proof required will be in relation to a diagram, not purely by reference to a named theorem

Reasoning needs to be carefully constructed as 'Quality of Written Communication' marks are likely to be allocated to proofs

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	31.2 – 31.4
Edexcel GCSE Mathematics 16+ Student Book	n/a

Module 21 **Time: 2 – 4 hours**

GCSE Tier: Higher

Contents: Sine and cosine rules

GM h Use the sine and cosine rules to solve 2-D and 3-D problems

GM y Calculate the area of a triangle using $\frac{1}{2}$ *ab* sin *C*

PRIOR KNOWLEDGE:

Trigonometry

Bearings

Formulae

OBJECTIVES

By the end of the module the student should be able to:

- calculate the unknown lengths, or angles, in non right-angle triangles using the sine and cosine rules in 2-D and 3-D
- calculate the area of triangles given two lengths and an included angle

DIFFERENTIATION AND EXTENSION

Use these rules to solve problems in 3-D and decide if it is easier to extract right-angle triangles to use 'normal' trigonometry

Stress that the cosine rule is only used when we have SAS (and we need to find the side opposite the angle given) or when we are given SSS (then we use the re-arranged version to find any angle) [else we use the sine rule]

NOTES

Reminders of simple geometrical facts may be helpful, eg angle sum of a triangle, the shortest side is opposite the smallest angle

Show the form of the cosine rule in the formula page and re-arrange it to show the form which finds missing angles

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	29.4 – 29.9
Edexcel GCSE Mathematics 16+ Student Book	n/a

Module 22 **Time: 2 – 4 hours**

GCSE Tier: Higher

Contents: Vectors

GM cc Use vectors to solve problems

PRIOR KNOWLEDGE:

Vectors to describe translations

Geometry of triangles and quadrilaterals

OBJECTIVES

By the end of the module the student should be able to:

- Understand that $2\mathbf{a}$ is parallel to $\mathbf{a}$ and twice its length
- Understand that $\mathbf{a}$ is parallel to $-\mathbf{a}$ and in the opposite direction
- Use and interpret vectors as displacements in the plane (with an associated direction)
- Use standard vector notation to combine vectors by addition, eg
- $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ and $\mathbf{a} + \mathbf{b} = \mathbf{c}$
- Calculate, and represent graphically, the sum of two vectors, the difference of two vectors and the scalar multiple of a vector
- Represent vectors, and combinations of vectors, in 2-D
- Solve geometrical problems in 2-D using vector methods

DIFFERENTIATION AND EXTENSION

Harder geometric proof, eg show that the medians of a triangle intersect at a single point

Illustrate use of vectors by showing 'Crossing the flowing River' example or navigation examples.

Vector problems in 3-D (for the most able)

Use **i** and **j** (and **k**) notation

NOTES

Students often find the pictorial representation of vectors more difficult than the manipulation of column vectors

Geometry of a hexagon provides a rich source of parallel, reverse and multiples of vectors.

Link with like terms and brackets when simplifying

Show there is more than one route round a geometric shape, but the answer simplifies to the same vector

Remind students to underline vectors or they will be regarded as just lengths with no direction

Some extension questions can be found in Mechanics 1 textbooks

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	31.1 – 33.5
Edexcel GCSE Mathematics 16+ Student Book	n/a

Module 23 **Time: 2 – 4 hours**

GCSE Tier: Higher

Contents: Measures and compound measures

GM o	Interpret scales on a range of measuring instruments and recognise the inaccuracy of measurements
GM p	Convert measurements from one unit to another
GM q	Make sensible estimates of a range of measures
GM s	Understand and use compound measures
A r	Draw distance-time graphs
A s	Discuss, plot and interpret graphs (which may be non-linear) modelling real situations

PRIOR KNOWLEDGE

Knowledge of metric units, eg 1 m = 100 cm etc

Know that 1 hour = 60 mins, 1 min = 60 seconds

Experience of multiplying by powers of 10, eg $100 \times 100 = 10\,000$

OBJECTIVES

By the end of the module the student should be able to:

- convert between units of measure in the same system. (NB: Conversion between imperial units will be given, metric equivalents should be known)
- know rough metric equivalents of pounds, feet, miles, pints and gallons, ie

  Metric Imperial
  1 kg = 2.2 pounds
  1 litre = 1.75 pints
  4.5 *l* = 1 gallon
  8 km = 5 miles
  30 cm = 1 foot
- convert between imperial and metric measures
- use the relationship between distance, speed and time to solve problems
- convert between metric units of speed, eg km/h to m/s
- construct and interpret distance time graphs
- know that density is found by mass ÷ volume
- use the relationship between density, mass and volume to solve problems, eg find the mass of an object with a given volume and density
- convert between metric units of density, eg kg/m³ to g/cm³ (ie convert between metric units of volume)

DIFFERENTIATION AND EXTENSION

Perform calculations on a calculator by using standard form

Convert imperial units to metric units, eg mph into km/h

Help students to recognise the problem they are trying to solve by the unit measurement given, eg km/h is a unit of speed as it is a distance divided by a time

Mention other units (not on course) like hectares

NOTES

Use a formula triangle to help students see the relationship between the variables for speed and density

Borrow a set of electronic scales and a Eureka Can from Physics for a practical density lesson.

Look up densities of different elements from the internet.

Link converting area and volume units to similar shapes

Draw a large grid made up of 100 by 100 cm squares to show what 1 square metre looks like

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	7.1 – 7.4, 15.6
Edexcel GCSE Mathematics 16+ Student Book	8.5, 12.1 – 12.2, 16.5

Module 24 **Time: 1 – 3 hours**

GCSE Tier: Higher

Contents: Collecting data

SP a Understand and use statistical problem-solving process/handling data cycle

SP b Identify possible sources of bias

SP c Design an experiment or survey

SP d Design data-collection sheets distinguishing between different types of data

SP e Extract data from printed tables and lists

SP f Design and use two-way tables for discrete and grouped data

PRIOR KNOWLEDGE

An understanding of the importance of a knowledge of statistics in our society, and of why data needs to be collected

Experience of simple tally charts

Experience of inequality notation

OBJECTIVES

By the end of the module the student should be able to:

- specify the problem and plan
- decide what data to collect and what statistical analysis is needed
- collect data from a variety of suitable primary and secondary sources
- use suitable data collection techniques
- process and represent the data
- interpret and discuss the data
- discuss how data relates to a problem, identify possible sources of bias and plan to minimise it
- understand how different sample sizes may affect the reliability of conclusions drawn
- identify which primary data they need to collect and in what format, including grouped data
- consider fairness
- understand sample and population
- design a question for a questionnaire
- criticise questions for a questionnaire
- design an experiment or survey
- select and justify a sampling scheme and a method to investigate a population, including random and stratified sampling
- use stratified sampling
- design and use data-collection sheets for grouped, discrete and continuous data
- collect data using various methods

- sort, classify and tabulate data and discrete or continuous quantitative data
- group discrete and continuous data into class intervals of equal width
- extract data from lists and tables
- design and use two-way tables for discrete and grouped data
- use information provided to complete a two-way table

DIFFERENTIATION AND EXTENSION

Students to carry out a statistical investigation of their own, including designing an appropriate means of data collection

Some guidance needs to be given to stop students from choosing limited investigations, eg favourite football team

Get data from holiday brochures to compare resorts for temperature, rainfall and type of visitor

Investigation into other sampling schemes, such as cluster, systematic and quota sampling

NOTES

Students may need reminding about the correct use of tallies

Emphasise the differences between primary and secondary data

Discuss sample size and mention that a census is the whole population

In the UK the census takes place every year that ends in a '1' (2011 is the next census)

If students are collecting data as a group, they should all use the same procedure

Emphasise that continuous data is data that is measured, eg temperature

Mayfield High data from coursework task can be used to collect samples and can be used to make comparisons in the next module, to introduce stratified sampling and to link with future statistics modules

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	6.1 – 6.8
Edexcel GCSE Mathematics 16+ Student Book	Chapter 19

Module 25 **Time: 3 – 5 hours**

GCSE Tier: Higher

Contents: Displaying data

SP g	Produce charts and diagrams for various data types
SP i	Interpret a wide range of graphs and diagrams and draw conclusions
SP j	Look at data to find patterns and exceptions
SP k	Recognise correlation and draw and/or use lines of best fit by eye, understanding what these represent
SP l	Compare distributions and make inferences

PRIOR KNOWLEDGE:

An understanding of the different types of data: continuous; discrete

Experience of inequality notation

An ability to multiply a number by a fraction

Use a protractor to measure and draw angles

OBJECTIVES

By the end of the module the student should be able to:

- produce composite bar charts, comparative and dual bar charts, pie charts, histograms with equal or unequal class intervals, frequency diagrams for grouped discrete data, scatter graphs, line graphs, frequency polygons for grouped data and grouped frequency tables for continuous data
- interpret composite bar charts, comparative and dual bar charts, pie charts, scatter graphs, frequency polygons and histograms
- recognise simple patterns, characteristics and relationships in line graphs and frequency polygons
- find the median from a histogram or any other information from a histogram, such as the number of people in a given interval
- from line graphs, frequency polygons and frequency diagrams: read off frequency values, calculate total population, find greatest and least values
- from pie charts: find the total frequency and find the frequency represented by each sector
- from histograms: complete a grouped frequency table and understand and define frequency density
- present findings from databases, tables and charts
- look at data to find patterns and exceptions, explain an isolated point on a scatter graph
- draw lines of best fit by eye, understanding what these represent
- use a line of best fit, or otherwise, to predict values of one variable given values of the other variable
- distinguish between positive, negative and zero correlation using lines of best fit
- understand that correlation does not imply causality

- appreciate that correlation is a measure of the strength of the association between two variables and that zero correlation does not necessarily imply 'no relationship', but merely 'no linear relationship'
- compare distributions and make inferences, using the shapes of distributions and measures of average and spread, including median and quartiles
- understand that the frequency represented by corresponding sectors in two pie charts is dependent upon the total populations represented by each of the pie charts
- use dual or comparative distributions

DIFFERENTIATION AND EXTENSION

Students should carry out a statistical investigation of their own and use an appropriate means of displaying the results

Use a spreadsheet/ICT to draw different types of graphs

NOTES

Collect examples of charts and graphs in the media which are misleading or wrongly interpreted, and discuss the implications

Students should clearly label all axes on graphs and use a ruler to draw straight lines

Many students enjoy drawing statistical graphs for classroom displays. Include Functional Elements in this topic with regard to holiday data, energy charts etc

Angles for pie charts should be accurate to within 2°. Ask students to check each others' charts

Make comparisons between previously collected data, eg Mayfield boys vs girls or Yr 7 vs Yr 8

Encourage students to work in groups and present their charts – display work in classroom/corridors

Use Excel Graph wizard

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	18.1 – 18.2, 18.4 – 18.7, 24.1 – 18.7, 24.1 – 24.5
Edexcel GCSE Mathematics 16+ Student Book	21.1 – 21.5, 21.7, 22.1

Module 26 **Time: 3 – 5 hours**

GCSE Tier: Higher

Contents: Averages and range

SP h	Calculate median, mean, range, quartiles and interquartile range, mode and modal class and interval containing the median
SP g	Produce charts and diagrams for various data types
SP i	Interpret a wide range of graphs and diagrams and draw conclusions
SP l	Compare distributions and make inferences
SP u	Use calculators efficiently and effectively, including statistical functions

PRIOR KNOWLEDGE:

Knowledge of finding the mean for small data sets

An ability to find the midvalue of two numbers

OBJECTIVES

By the end of the module the student should be able to:

- calculate mean, mode, median and range for small data sets
- recognise the advantages and disadvantages between measures of average
- produce ordered stem and leaf diagrams and use them to find the range and averages
- calculate averages and range from frequency tables (use Σx and Σfx)
- find modal class and interval containing the median
- estimate the mean for large data sets with grouped data (and understand that it is an estimate)
- find quartile and interquartile range from data
- draw and interpret cumulative frequency tables and graphs
- use cumulative frequency graphs to find median, quartiles and interquartile range
- draw box plots from a cumulative frequency graph and from raw data
- compare the measures of spread between a pair of box plots/cumulative frequency graphs
- interpret box plots to find, for example, median, quartiles, range and interquartile range
- find the median from a histogram
- compare distributions and make inferences, using the shapes of distributions and measures of average and spread, including median and quartiles
- find the mode, median, range and interquartile range, as well as the greatest and least values from the stem and leaf diagrams

DIFFERENTIATION AND EXTENSION

Use statistical functions on calculators and spreadsheets

Use statistical software to calculate the mean for grouped data sets

Estimate the mean for data sets with ill-defined class boundaries

Investigate the effect of combining class intervals on estimating the mean for grouped data sets

Students should understand that finding an *estimate for the mean* of grouped data is not a guess

Pose the question: 'Investigate if the average number of children per family is 2.4, are the families represented in your class representative of the whole population?'

Discuss occasions when one average is more appropriate, and the limitations of each average

Possibly mention standard deviation (not on course, but good for further comparison of data sets with similar means)

NOTES

Collect data from class – children per family etc. Extend to different classes, year groups or secondary data from the internet. (Previous coursework tasks are a rich source of data to work with, eg *Second-Hand Car Sales*)

Compare distributions and make inferences, using the shapes of distributions and measures of average and spread, eg 'boys are taller on average but there is a much greater spread in heights'. (Use data collected from previous investigations or Mayfield High data)

Students tend to select modal class but identify it by the frequency rather than the class itself

Explain that the median of grouped data is not necessarily from the middle class interval

Stem and leaf diagrams must have a key and show how to find the median and mode from a stem and leaf diagram

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	11.1 – 11.7, 18.3, 18.5, 18.8 – 18.10
Edexcel GCSE Mathematics 16+ Student Book	21.6 – 21.7, Chapter 20

Module 27 **Time: 2 – 4 hours**

GCSE Tier: Higher

Contents: Probability

SP m Understand and use the vocabulary of probability and probability scale

SP n Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency

SP o List all outcomes for single events, and for two successive events, in a systematic way and derive relative probabilities

SP p Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1

SP q Know when to add or multiply two probabilities: when A and B are mutually exclusive, then the probability of A or B occurring is P(A) + P(B), whereas when A and B are independent events, the probability of A and B occurring is P(A) × P(B)

SP r Use tree diagrams to represent outcomes of compound events, recognising when events are independent

SP s Compare experimental data and theoretical probabilities

SP t Understand that if they repeat an experiment, they may, and usually will, get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics

PRIOR KNOWLEDGE

An understand that a probability is a number between 0 and 1

Know how to add and multiply fractions and decimals

Experience of expressing one number as a fraction of another number

Recognise the language of probability, eg words such as likely, certain, impossible

OBJECTIVES

By the end of the module the student should be able to:

- write probabilities using fractions, percentages or decimals
- understand and use estimates or measures of probability, including relative frequency
- use theoretical models to include outcomes using dice, spinners, coins etc
- find the probability of successive events, such as several throws of a single dice
- estimate the number of times an event will occur, given the probability and the number of trials
- list all outcomes for single events, and for two successive events, systematically
- use and draw sample space diagrams
- add simple probabilities, eg from sample space diagrams
- identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1

- use $1 - p$ as the probability of an event not occurring where p is the probability of the event occurring
- find a missing probability from a list or table
- understand conditional probabilities
- understand selection with or without replacement
- draw a probability tree diagram based on given information
- use a tree diagram to calculate conditional probability
- compare experimental data and theoretical probabilities
- compare relative frequencies from samples of different sizes

DIFFERENTIATION AND EXTENSION

An opportunity for practical examples, eg P(pin up) for a drawing pin, the 'horse' race, the national lottery

Show that each cluster of branches on a tree diagram adds up to 1

Explain that if two objects are chosen, then this is the same as one event followed by another event without replacement

Show that it is often easier to solve a problem involving multiple outcomes, by considering the *opposite* event and subtracting from 1, eg 'at least' two reds, 'at least' two beads of a different colour etc

Experiments with dice and spinners

Show sample space for outcomes of throwing 2 dice.

Stress that there are 36 outcomes (students will initially guess its 12 outcomes for 2 dice)

Binomial probabilities (H or T)

Do a question 'with' and then repeat it 'without' replacement. Good idea to show the contents of the bag and physically remove the object to illustrate the change of probability fraction for the second selection

NOTES

Students should express probabilities as fractions, percentages or decimals

Fractions do not need to be cancelled to their lowest terms. This makes it easier to calculate tree diagram probabilities, eg easier to add like denominators

RESOURCES

Textbook	References
Edexcel GCSE Mathematics A Linear Higher Student book	28.1 – 28.7
Edexcel GCSE Mathematics 16+ Student Book	Chapter 23

Higher course objectives (1MA0)

Number

N a	Add, subtract, multiply and divide whole numbers integers and decimals
N a	Multiply and divide fractions
N b	Order integers and decimals
N b	Order rational numbers
N c	Use the concepts and vocabulary of factor (divisor), multiple, common factor, Highest Common Factor, Least Common Multiple, prime number and prime factor decomposition
N d	Use the terms square, positive and negative square root, cube and cube root
N e	Use index notation for squares, cubes and powers of 10
N f	Use index laws for multiplication and division of integer powers
N f	Use index laws for multiplication and division of integer, fractional and negative powers
N g	Interpret, order and calculate with numbers written in standard index form
N h	Understand equivalent fractions, simplify a fraction by cancelling all common factors
N i	Add and subtract fractions
N j	Use decimal notation and recognise that each terminating decimal is a fraction
N k	Recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals
N l	Understand that 'percentage' means 'number of parts per 100' and use this to compare proportions
N m	Use percentage, repeated proportional change
N n	Understand and use direct and indirect proportion
N o	Interpret fractions, decimals and percentages as operators
N p	Use ratio notation, including reduction to its simplest form and its various links to fraction notation
N q	Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
N r	Use π in an exact calculation
N r	Calculations with surds
N r	Use surds in exact calculations
N s	Calculate upper and lower bounds
N t	Divide a quantity in a given ratio
N u	Approximate to specified or appropriate degrees of accuracy including a given power of ten, number of decimal places and significant figures
N v	Use a calculator efficiently and effectively

Algebra

A a	Distinguish the different roles played by letter symbols in algebra, using the correct notation
A b	Distinguish in meaning between the words 'equation', 'formula', 'identity' and 'expression'
A c	Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors, multiplying two linear expressions, factorise quadratic expressions including the difference of two squares and simplify rational expressions
A d	Set up and solve simple equations
A d	Set up and solve simple equations including simultaneous equations in two unknowns
A e	Solve quadratic equations
A f	Derive a formula, substitute numbers into a formula and change the subject of a formula
A g	Solve linear inequalities in one variable, and represent the solution set on a number line
A g	Solve linear inequalities in two variables, and represent the solution set on a coordinate grid
A h	Using systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them
A i	Generate terms of a sequence using term-to-term and position to-term definitions of the sequence
A j	Use linear expressions to describe the nth term of an arithmetic sequence
A k	Use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information
A l	Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients
A m	Understand that the form $y = mx + c$ represents a straight line and that m is the gradient of the line and c is the value of the y- intercept
A n	Understand the gradients of parallel lines
A o	Find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions
A p	Draw, sketch, recognise graphs of simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, the function $y = kx^n$ for integer values of x and simple positive values of k, the trigonometric functions $y = \sin x$ and $y = \cos x$
A q	Construct the graphs of simple loci
A r	Construct linear functions from real-life problems and plot their corresponding graphs
A r	Construct linear, quadratic and other functions from real-life problems and plot their corresponding graphs

A r	Construct distance time graphs
A s	Discuss, plot and interpret graphs (which may be non-linear) modelling real situations
A t	Generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions
A u	Direct and indirect proportion (algebraic)
A v	Transformation of functions

Geometry

GM a	Recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines, and opposite angles at a vertex
GM b	Understand and use the angle properties of parallel lines, triangles and quadrilaterals
GM c	Calculate and use the sums of the interior and exterior angles of polygons
GM d	Recall the properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus
GM e	Recognise reflection and rotation symmetry of 2-D shapes
GM f	Understand congruence and similarity
GM g	Use Pythagoras' theorem in 2-D and 3-D
GM h	Use the trigonometric ratios and the sine and cosine rules to solve 2-D and 3-D problems
GM i	Distinguish between centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
GM j	Understand and construct geometrical proofs using circle theorems
GM k	Use 2-D representations of 3-D shapes
GM l	Describe and transform 2-D shapes using single or combined rotations, reflections, translations, or enlargements by a positive, fractional or negative scale factor and distinguish properties that are preserved under particular transformations
GM m	Use and interpret maps and scale drawings
GM n	Understand and use the effect of enlargement for perimeter, area and volume of shapes and solids
GM o	Interpret scales on a range of measuring instruments and recognise the inaccuracy of measurements
GM p	Convert measurements from one unit to another
GM p	Convert between volume measures, including cubic centimetres and cubic metres
GM q	Make sensible estimates of a range of measures
GM r	Understand and use bearing
GM s	Understand and use compound measures
GM t	Measure and draw lines and angles
GM u	Draw triangles and other 2-D shapes using ruler and protractor
GM v	Use straight edge and a pair of compasses to carry out constructions
GM w	Construct loci
GM x	Calculate perimeters and areas of shapes made from triangles and rectangles or other shapes
GM y	Calculate the area of a triangle using $\frac{1}{2}\ ab \sin C$

GM z	Find circumferences and areas of circles
GM z	Find surface area of a cylinder
GM aa	Calculate volumes of right prisms and shapes made from cubes and cuboids
GM bb	Solve mensuration problems involving more complex shapes and solids
GM cc	Use vectors to solve problems

Statistics and Probability

SP a	Understand and use statistical problem solving process/handling data cycle
SP b	Identify possible sources of bias
SP c	Design an experiment or survey
SP d	Design data-collection sheets distinguishing between different types of data
SP e	Extract data from printed tables and lists
SP f	Design and use two-way tables for discrete and grouped data
SP g	Produce charts and diagrams for various data types
SP g	Produce charts and diagrams for various data types
SP h	Calculate median, mean, range, quartiles and interquartile range, mode and modal class
SP i	Interpret a wide range of graphs and diagrams and draw conclusions
SP j	Look at data to find patterns and exceptions
SP k	Recognise correlation and draw and/or use lines of best fit by eye, understanding what these represent
SP l	Compare distributions and make inferences
SP m	Understand and use the vocabulary of probability and probability scale
SP n	Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency
SP o	List all outcomes for single events, and for two successive events, in a systematic way and derive relative probabilities
SP p	Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
SP q	Know when to add or multiply two probabilities: when A and B are mutually exclusive, then the probability of A or B occurring is P(A) + P(B), whereas when A and B are independent events, the probability of A and B occurring is P(A) × P(B)
SP r	Use tree diagrams to represent outcomes of compound events, recognising when events are independent
SP s	Compare experimental data and theoretical probabilities
SP t	Understand that if they repeat an experiment, they may, and usually will, get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics
SP u	Use calculators efficiently and effectively, including statistical functions

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