Paper Reference(s)

9801/01

Edexcel

Mathematics

Advanced Extension Award

Tuesday 25 June 2013 – Morning

Time: 3 hours

Materials required for examination	**Items included with question papers**
Answer book (AB16)	Nil
Graph paper (ASG2)	
Mathematical Formulae (Pink)	

Candidates may NOT use a calculator in answering this paper.

Instructions to Candidates

In the boxes on the answer book provided, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title (Mathematics), the paper reference (9801), your surname, initials and signature.
Check that you have the correct question paper.
Answers should be given in as simple a form as possible. e.g. $\frac{2\pi}{3}$, $\sqrt{6}$, $3\sqrt{2}$.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 100, of which 7 marks are for style, clarity and presentation.
There are 8 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
P41807A

P 4 1 8 0 7 A

W850/R9801/57570 5/5/5/4/

Turn over

PEARSON

1. In the binomial expansion of

$$\left(1+\frac{12n}{5}x\right)^n$$

the coefficients of x^2 and x^3 are equal and non-zero.

(a) Find the possible values of n.

(4)

(b) State, giving a reason, which value of n gives a valid expansion when $x = \frac{1}{2}$

(2)

(Total 6 marks)

---

2. (a) Use the formula for $\sin(A-B)$ to show that $\sin(90° - x) = \cos x$

(1)

(b) Solve for $0 < \theta < 360°$

$$2\sin(\theta + 17°) = \frac{\cos(\theta + 8°)}{\cos(\theta + 17°)}$$

(7)

(Total 8 marks)

---

3. The lines L_1 and L_2 have equations given by

$$L_1: \quad \mathbf{r} = \begin{pmatrix} -7 \\ 7 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix} \quad \text{and} \quad L_2: \quad \mathbf{r} = \begin{pmatrix} 7 \\ p \\ -6 \end{pmatrix} + \mu \begin{pmatrix} 10 \\ -4 \\ -1 \end{pmatrix}$$

where λ and μ are parameters and p is a constant.

The two lines intersect at the point C.

(a) Find

(i) the value of p,

(ii) the position vector of C.

(5)

(b) Show that the point B with position vector $\begin{pmatrix} -13 \\ 11 \\ -4 \end{pmatrix}$ lies on L_2.

(1)

The point A with position vector $\begin{pmatrix} -7 \\ 7 \\ 1 \end{pmatrix}$ lies on L_1.

(c) Find $\cos(\angle ACB)$, giving your answer as an exact fraction.

(3)

The line L_3 bisects the angle ACB.

(d) Find a vector equation of L_3.

(4)

(Total 13 marks)

4. A sequence of positive integers $a_1, a_2, a_3, \ldots$ has rth term given by

$$a_r = 2^r - 1$$

(a) Write down the first 6 terms of this sequence.

(1)

(b) Verify that $a_{r+1} = 2a_r + 1$

(1)

(c) Find $\sum_{r=1}^{n} a_r$

(3)

(d) Show that $\frac{1}{a_{r+1}} < \frac{1}{2} \times \frac{1}{a_r}$

(1)

(e) Hence show that $1 + \frac{1}{3} + \frac{1}{7} + \frac{1}{15} + \frac{1}{31} + \ldots < 1 + \frac{1}{3} + \left(\frac{1}{7} + \frac{\frac{1}{2}}{7} + \frac{\frac{1}{4}}{7} + \ldots\right)$

(2)

(f) Show that $\frac{31}{21} < \sum_{r=1}^{\infty} \frac{1}{a_r} < \frac{34}{21}$

(5)

(Total 13 marks)

---

5. In this question u and v are functions of x. Given that $\int \mathrm{u}\,\mathrm{d}x$, $\int \mathrm{v}\,\mathrm{d}x$ and $\int \mathrm{uv}\,\mathrm{d}x$ satisfy

$$\int \mathrm{uv}\,\mathrm{d}x = \left(\int \mathrm{u}\,\mathrm{d}x\right) \times \left(\int \mathrm{v}\,\mathrm{d}x\right) \qquad \mathrm{uv} \neq 0$$

(a) show that $1 = \frac{\int \mathrm{u}\,\mathrm{d}x}{\mathrm{u}} + \frac{\int \mathrm{v}\,\mathrm{d}x}{\mathrm{v}}$

(3)

Given also that $\frac{\int \mathrm{u}\,\mathrm{d}x}{\mathrm{u}} = \sin^2 x$,

(b) use part (a) to write down an expression, in terms of x, for $\frac{\int \mathrm{v}\,\mathrm{d}x}{\mathrm{v}}$,

(1)

(c) show that

$$\frac{1}{\mathrm{u}} \frac{\mathrm{du}}{\mathrm{d}x} = \frac{1 - 2\sin x \cos x}{\sin^2 x}$$

(3)

(d) hence use integration to show that $\mathrm{u} = A\mathrm{e}^{-\cot x}\operatorname{cosec}^2 x$, where A is an arbitrary constant.

(6)

(e) By differentiating $\mathrm{e}^{\tan x}$ find a similar expression for v.

(2)

(Total 15 marks)

6. (a) Starting from $\left[f(x) - \lambda g(x)\right]^2 \geqslant 0$ show that λ satisfies the quadratic inequality

$$\left(\int_a^b [g(x)]^2 \, dx\right)\lambda^2 - 2\left(\int_a^b f(x)g(x) \, dx\right)\lambda + \int_a^b [f(x)]^2 \, dx \geqslant 0$$

where a and b are constants and λ can take any real value.

(2)

(b) Hence prove that

$$\left[\int_a^b f(x)g(x) \, dx\right]^2 \leqslant \left[\int_a^b [f(x)]^2 \, dx\right] \times \left[\int_a^b [g(x)]^2 \, dx\right]$$

(3)

(c) By letting f(x) = 1 and g(x) = $\left(1 + x^3\right)^{\frac{1}{2}}$ show that

$$\int_{-1}^{2} \left(1 + x^3\right)^{\frac{1}{2}} \, dx \leqslant \frac{9}{2}$$

(4)

(d) Show that $\displaystyle\int_{-1}^{2} x^2 \left(1 + x^3\right)^{\frac{1}{4}} \, dx = \frac{12\sqrt{3}}{5}$

(3)

(e) Hence show that

$$\frac{144}{55} \leqslant \int_{-1}^{2} \left(1 + x^3\right)^{\frac{1}{2}} \, dx$$

(4)

(Total 16 marks)

7.

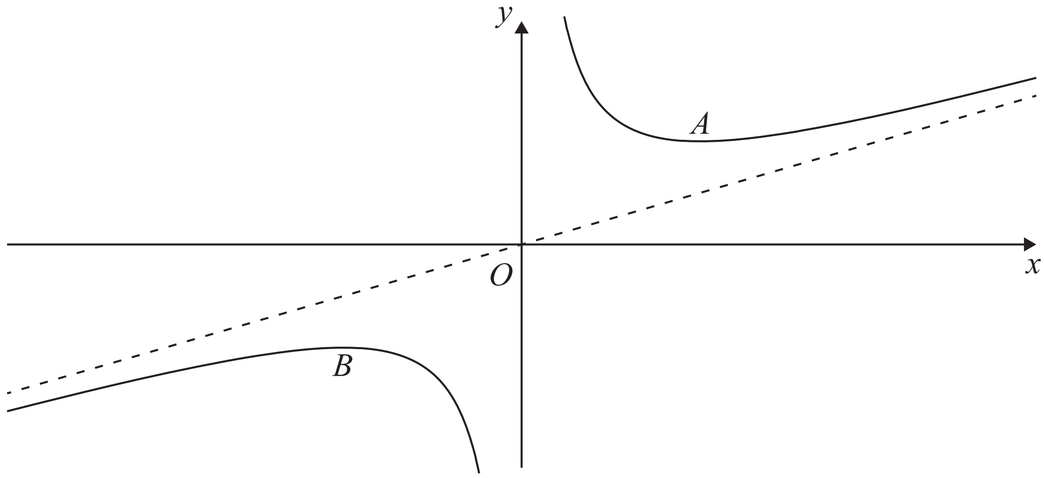

Figure 1

Figure 1 shows a sketch of the curve C_1 with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{x}{3} + \frac{12}{x} \qquad x \neq 0$$

The lines $x = 0$ and $y = \frac{x}{3}$ are asymptotes to C_1. The point A on C_1 is a minimum and the point B on C_1 is a maximum.

(a) Find the coordinates of A and B.

(4)

There is a normal to C_1, which does not intersect C_1 a second time, that has equation $x = k$, where $k > 0$.

(b) Write down the value of k.

(1)

The point $P\ (\alpha, \beta)$, $\alpha > 0$ and $\alpha \neq k$, lies on C_1. The normal to C_1 at P does not intersect C_1 a second time.

(c) Find the value of α, leaving your answer in simplified surd form.

(5)

(d) Find the equation of this normal.

(3)

The curve C_2 has equation $y = |\mathrm{f}(x)|$

(e) Sketch C_2 stating the coordinates of any turning points and the equations of any asymptotes.

(4)

The line with equation $y = mx + 1$ does not touch or intersect C_2.

(f) Find the set of possible values for m.

(5)

(Total 22 marks)

FOR STYLE, CLARITY AND PRESENTATION: 7 MARKS
TOTAL FOR PAPER: 100 MARKS

END

BLANK PAGE

BLANK PAGE