Mark Scheme (Results)

Summer 2015

Pearson Edexcel Advanced Extension Award in Mathematics (9801/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d... or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given
   - $\sqsubset$ or d... The second mark is dependent on gaining the first mark

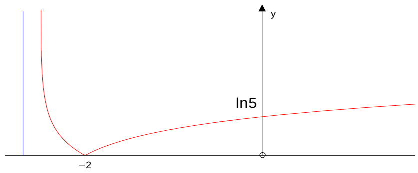
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

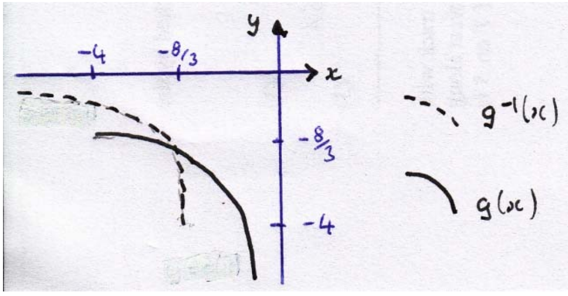
6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

AEA June 2015 Mark scheme

Question	Scheme	Marks	Notes
1. (a)	$x = -\frac{5}{2}$ Both branches $y > 0$ & cusp on –ve x-axis All 3 required	B1 B1 B1 (3)	Shape (Inc. cusp) Position Asymptote and int' with axes
(b)	$2x + 5 = 9$ so $\underline{x = 2}$ $\frac{1}{2x+5} = 9$ or $2x + 5 = \frac{1}{9}$ so $\underline{x} = \frac{-22}{9}$ (o.e.)	B1 M1 A1 (3) **[6]**	Correct equation with no ln
2. (a)	$-2 + 3 - 1 = 0$ so $(x + 1)$ is a factor	B1cso (1)	
(b)	$x^2 + 2x + 5 = \underline{x^2} + \underline{2x\sqrt{2x+3}} + \underline{2x+3}$ $1 = x\sqrt{2x+3}$ $0 = 2x^3 + 3x^2 - 1$ (Accept $2x^3 + 3x^2 = 1$ o.e.) $0 = (x+1)(2x^2 + x - 1)$ $0 = (x+1)(2x-1)(x+1)$ $\underline{x} = -1$ or $\frac{1}{2}$ Check -1: LHS = 2 RHS = 0 **so -1 is not a solution** Check $\frac{1}{2}$: LHS = $\sqrt{\frac{25}{4}} = \frac{5}{2}$ RHS=$\frac{1}{2} + \sqrt{4} = 2.5$ **(Only) solution is 0.5** [S- for treating $\sqrt{4}$ as $\pm$ 2 etc]	M1 M1 A1cso M1 A1 B1 M1 A1 (8) **[9]**	Attempt to square. 3 terms on RHS Prepare for final sq Div attempt. At least 2 correct terms of quadratic Correct factors and both roots Must reject -1 Attempts 0.5 in original or line 2 Only award if check is in <u>original</u>
3.	LHS = $\frac{\cos 2x}{\sin 2x} - \frac{\sin 78}{\cos 78}$ $\cos(2x + 78) = \frac{1}{2}(\sin 2x \cos 78 \sec x \sec 78)$ $[\cos(2x+78) =]$ $\frac{1}{2}[2\sin x \cos x \cos 78 \sec 78 \sec x]$ $\cos(2x + 78) = \sin x$ or $\cos(2x+78) = \cos(90 - x)$ $2x + 78 = 90 - x$ $\underline{x = 4}$ $2x + 78 = 270 + x$ $\underline{x = 192}$ $2x + 78 = 450 - x$ $\underline{x = 124}$ $2x + 78 = 810 - x$ $\underline{x = 244}$	M1 M1 M1 A1 M1 A1 A3/2/1 **[9]**	Cot and tan to sin and cos Use of $\cos(A + B)$ Use of $\sin 2x$ <u>and</u> some cancelling Non-trig eqn in x Allow 90 $\pm$ x Award A1 for each of these 3 solutions found. Extras inside the range -1 i.e allow upto 4 answers. If more than 4 then deduct 1 for each in range

Question	Scheme	Marks	Notes
4. (a)	$(4+y)^{\frac{1}{2}} = 2\left(1+\frac{y}{4}\right)^{\frac{1}{2}}$	M1	Correct prep or dealing with 4
	$=[2]\left(1+\frac{y}{8}+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(\frac{y}{4}\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{y}{4}\right)^3 ...\right)$	M1	Clear use of bin for 3rd or 4th terms. Condone missing 2
	$= 2+\frac{y}{4}-\frac{y^2}{64}+\frac{y^3}{512}$	A1	Allow o.e. for coefficients
		(3)	
ALT	$(4+y)^{\frac{1}{2}} = 4^{\frac{1}{2}}+\frac{1}{2}4^{-\frac{1}{2}}y+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}4^{-\frac{3}{2}}y^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}4^{-\frac{5}{2}}y^3$		1st M1 for the powers of 4
(b)	Let $y=5x+x^2$ so $2+\frac{5x}{4}+\frac{x^2}{4}-\frac{\left(25x^2+10x^3+[x^4]\right)}{64}+\frac{\left(125x^3...\right)}{512}$	M1	Some attempt to sub. for y Ignore higher order terms.
		M1	Clearly attempt x^2
	$=2+\frac{5x}{4}-\frac{9x^2}{64}+\frac{45x^3}{512}$ (*)	A1cso	
		(3)	
ALT	$\left(4+5x+x^2\right)^{\frac{1}{2}}=(4+x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}=(a)\times\left(1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}\right)$		For 1st M1
(c)	$\left(x^2+5x+4\right)^{\frac{1}{2}}=(4+x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}$ | $\lvert 5x+x^2\rvert<4$ so $5x+x^2=4$	M1	Attempt factors or a correct equ
	| $x=\frac{-5\pm\sqrt{41}}{2}$	A1	Correct fact. Or cvs
	$-4<x<4$ and $-1<x<1$ | $5x+x^2=-4 \Rightarrow (x+4)(x+1)$, $x=-1$ or -4	M1 A1	Solves 2nd eqn or 1 correct interval. A1 All correct
	So $-1<x<1$ | So $-1<x<\frac{\sqrt{41}-5}{2}$ or ...	M1	M1 for a suitable combined region
	So series is convergent for $-\frac{1}{2}\leqslant x\leqslant\frac{1}{2}$	A1 (6)	A1 cso
(d)	$\int ... = \left[2x+\frac{5x^2}{8}-\frac{9x^3}{3\times64}+\frac{45x^4}{4\times512}\right]_{-\frac{1}{2}}^{\frac{1}{2}}$	M1	Some correct integration. Ignore limits
	$= 2\left[1-\frac{3\times1}{8\times64}\right]$	M1	Attempt both limits S+ for props of odd and even functions
	$= \frac{509}{256}$ (o.e)	A1 (3)	
		[15]	

Question	Scheme	Marks	Notes
5. (a)	$f(x)=\frac{1}{3}x+\frac{16}{3}x^{-1} \Rightarrow f'(x)=\frac{1}{3}-\frac{16}{3}x^{-2}$ or quotient rule	M1	Some correct diff
	$y'=0 \Rightarrow x^2=16$	M1	$y'=0 \rightarrow x^2=\ldots$
	$\boldsymbol{a}=-4$ and $\boldsymbol{b}=-\frac{8}{3}$ (o.e.)	A1A1 (4)	$(-4, \frac{-8}{3})$ is OK
(b)	$g(x)\leqslant -\frac{8}{3}$ (Accept $y\leqslant -\frac{8}{3}$ or $(-\infty, -\frac{8}{3}]$ o.e.)	B1ft (1)	ft their value of b
(c)	 Values are not required. Graphs should not go beyond – 4. Penalise once only S+ for values	B1 B1 B1 (3)	Cor' posit' of g(x) [not crossing axes] Cor' posit' of $g^{-1}(x)$ [not crossing axes] Intersection at roughly correct pt And labels

5 (d)	y = g(x) gives $x^2-3xy+16=0$	M1	3TQ in x (needn't = 0)
	$x=\dfrac{3y\pm\sqrt{9y^2-64}}{2}$	M1 A1	Attempt formula or complete square. Leading to x = ... A1 for $\pm$ or + or -
	Consideration of + and – and take + $g^{-1}(x)=\dfrac{3x+\sqrt{9x^2-64}}{2}$	A1	Must have chosen + S+ for good reason
	Domain [**x**] $\leqslant -\frac{8}{3}$ (o.e.) [<u>Not</u> $g^{-1}(x)\leqslant -\frac{8}{3}$]	B1ft (5)	ft their (b) or b
(e)	Simpler to do g(x) = x leading to $x^2+16=3x^2$	M1	Write down a correct eqn (ft their g^{-1}) and attempt to simplify to quad or quartic
	$x^2=8$	M1	Solving 2T quad or 3T quartic
	$x=-2\sqrt{2}$ or $-\sqrt{8}$	A1 (3) **[16]**	S+ for reason for -

Question	Scheme	Marks	Notes
6 (a)	$\begin{pmatrix}2\\-5\\4\end{pmatrix}\bullet\begin{pmatrix}1\\2\\2\end{pmatrix}=2-10+8=0$ so, the lines are perpendicular	M1 A1 (2)	Attempt correct scalar product Correct calc and conclusion
(b)	$10-5\lambda=2+2\mu$ and $-3+4\lambda=3+2\mu$ (or $1+2\lambda=-1+\mu$)	M1	Form suitable eqns
	$\lambda=\frac{14}{9},\mu=\frac{1}{9}$	A1	
	In 3rd equation: LHS $1+\frac{28}{9}=\frac{37}{9}$ and RHS = $-1+\frac{1}{9}=-\frac{8}{9}$ so skew	M1 (3)	Check and comment
Solve i,j	Should get: $\lambda=\frac{4}{9}$ and $\mu=\frac{26}{9}$ Check: LHS = $-\frac{11}{9}$ RHS = $\frac{79}{9}$		
Solve i,k	Get no solution e.g. – 5 = 5		
(c)	Let R be a point on L_1 then $\overrightarrow{AR}=\begin{pmatrix}2+2\lambda\\8-5\lambda\\-6+4\lambda\end{pmatrix}$ and $\overrightarrow{AR}$ is $\perp$ to $\begin{pmatrix}2\\-5\\4\end{pmatrix}$	M1 M1	Forming vector *AR* Attempt suitable scalar product. Must see some products
	$4+4\lambda-40+25\lambda-24+16\lambda=0$ so $\lambda=\frac{4}{3}$	M1A1	Solve for λ
	$\overrightarrow{OX}=\frac{11}{3}\mathbf{i}+\frac{10}{3}\mathbf{j}+\frac{7}{3}\mathbf{k}$	A1 (5)	Allow coord form
(d)	$\overrightarrow{AX}=\begin{pmatrix}\frac{14}{3}\\\frac{4}{3}\\-\frac{2}{3}\end{pmatrix}$, so $\lvert\overrightarrow{AX}\rvert=\frac{2}{3}\sqrt{7^2+2^2+1^2}=\underline{2\sqrt{6}}$ (Allow $\sqrt{24}$ or $\sqrt{\frac{216}{9}}$)	M1, A1 (2)	Attempt vector *AX*
(e)	Let M be midpoint of AB $\overrightarrow{AM}=\overrightarrow{AX}\bullet$ (unit vector in direction of L_2)	M1	Suitable strategy
	$\lvert\overrightarrow{AM}\rvert=\frac{1}{3}\left(\frac{14}{3}\times1+\frac{4}{3}\times2-\frac{2}{3}\times2\right)[=2]$	M1	Correct calcs
	So $\lvert\overrightarrow{AB}\rvert=4$ or $\lvert\overrightarrow{AM}\rvert=2$	A1	o.e.
	$\overrightarrow{OB}=\overrightarrow{OA}+4\times$(unit vector in direction of L_2)	M1	Strategy
	$\left[\overrightarrow{OB}=\begin{pmatrix}-1\\2\\3\end{pmatrix}+\frac{4}{3}\begin{pmatrix}1\\2\\2\end{pmatrix}\right]=\begin{pmatrix}\frac{1}{3}\\\frac{14}{3}\\\frac{17}{3}\end{pmatrix}$	A1 (5)	
(f)	$\overrightarrow{XA}\bullet\overrightarrow{XB}=\begin{pmatrix}-\frac{14}{3}\\-\frac{4}{3}\\\frac{2}{3}\end{pmatrix}\bullet\begin{pmatrix}-\frac{10}{3}\\\frac{4}{3}\\\frac{10}{3}\end{pmatrix}=\frac{1}{9}(140-16+20)=\frac{144}{9}=2\sqrt{6}.2\sqrt{6}\cos\theta$	M1	Attempt a correct scalar product o.e. and set = to $\lvert AX\rvert\lvert BX\rvert\cos\theta$
	$\cos\theta=\frac{2}{3}$	A1cao (2) **[19]**	

Question	Scheme	Marks	Notes
7 (a)	$x=\sec\theta \quad \Rightarrow \dfrac{dx}{d\theta}=\sec\theta\tan\theta$	M1	
	$I=\int\dfrac{\sec\theta\tan\theta}{\tan^3\theta}\,d\theta$ or $\int\sec\theta\cot^2\theta\,d\theta$, $=\int\text{cosec}\theta\cot\theta\,d\theta$	A1, M1	Correct θ integral Prep to integrate
	$=\left[-\text{cosec}\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$, $=-\left(\dfrac{2}{\sqrt{3}}-\sqrt{2}\right)=\dfrac{\sqrt{6}-2}{\sqrt{3}}$ **(*)**	A1, A1cso	For $-\text{cosec}\theta$ o.e. Changing limits and cso
		(5)	
(b)	$J=\left[-\text{cosec}\theta\cot\theta\right]-\int\text{cosec}^3\theta\,d\theta$	M1	Suitable 1st step Condone sign slips
	$J=\left[-\text{cosec}\theta\cot\theta\right]-\int\text{cosec}\theta[1+\cot^2\theta]\,d\theta$	M1	Use of $1+\cot^2$
	$J=\left[-\text{cosec}\theta\cot\theta\right]-\int\text{cosec}\theta\,d\theta-J$ [Allow $\int\text{cosec}\theta\cot^2\theta\,d\theta$]	A1	Correctly dealing with 2nd int
	Use of $\int\text{cosec}\theta\,d\theta=-\ln\left\|\text{cosec}\theta+\cot\theta\right\|$	B1	Int of cosecθ Must come from their working
	$2J=\ldots$ so , $J=\dfrac{1}{2}\left[\ln\left\|\text{cosec}\theta+\cot\theta\right\|-\text{cosec}\theta\cot\theta\right]$ **(*)**	M1A1	Identify J and cso
		(6)	
(c)	$V=\pi\int_{\sqrt{2}}^{2}\dfrac{1}{\left(x^2-1\right)^3}\,dx$ [= πK]	B1	Correct integral + limits (Ignore π)
	$x=\sec\theta\Rightarrow K=\int\sec\theta\cot^5\theta\,d\theta$ (o.e.)	M1	Changes to int in θ
	$K=\int\text{cosec}\theta\cot^4\theta\,d\theta=\left[-\text{cosec}\theta\cot^3\theta\right]-\int3\cot^2\theta\,\text{cosec}^3\theta\,d\theta$	M1	Attempt int by parts
	$\int3\cot^2\theta\,\text{cosec}^3\theta\,d\theta=\int3\cot^2\theta\,\text{cosec}\theta\,d\theta+\int3\cot^4\theta\,\text{cosec}\theta\,d\theta$	M1	Split 2nd int Award after 2nd M1
	$K=\left[-\text{cosec}\theta\cot^3\theta\right]-3J-3K$	M1	K = f(θ) $\underline{+}J$ - nK
	$\left[-\text{cosec}\theta\cot^3\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=-\left[\left(\dfrac{2}{\sqrt{3}}\times\dfrac{1}{3\sqrt{3}}\right)-\left(\sqrt{2}\right)\right]=\sqrt{2}-\dfrac{2}{9}$	A1	Only award A marks once <u>all</u> the integration is completed
	$\left[J\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=\frac{1}{2}\left[\ln\left(\dfrac{\sqrt{3}}{\sqrt{2}+1}\right)-\dfrac{2}{3}+\sqrt{2}\right]$	A1	
	$K=\frac{1}{4}\left[\sqrt{2}-\frac{2}{9}-\frac{3}{2}\left\{\ln\left(\frac{\sqrt{3}}{\sqrt{2}+1}\right)-\frac{2}{3}+\sqrt{2}\right\}\right]$ so $V=\pi\left[\dfrac{3}{8}\ln\left(\dfrac{\sqrt{2}+1}{\sqrt{3}}\right)+\dfrac{7}{36}-\dfrac{\sqrt{2}}{8}\right]$	A1cso (8)	Including π
		[19]	

Questions	Mark	Awarding of S and T marks
2, 3	S1	For a fully correct solution that is succinct or includes an S+ point
4-7	S2	For a fully correct solution that is succinct or includes an S+ point
4-7	S1	For a fully correct solution that is succinct but has an S– point
4-7	S1	For a fully correct solution that is slightly laboured but includes an S+ point
4-7	S1	For a score of n -1 but solution is otherwise succinct or contains an S+ point
		Maximum of 6 S marks
ALL	T1	For at least half marks on every question

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