Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	9	/	0	1	Signature	

Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 22 June 2015 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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PEARSON

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1. Solve the equation

$$2\cosh^2 x - 3\sinh x = 1$$

giving your answers in terms of natural logarithms.

(6)

Leave blank

Question 1 continued

Q1

(Total 6 marks)

Leave blank

2. A curve has equation

$$y = \cosh x, \qquad 1 \leqslant x \leqslant \ln 5$$

Find the exact length of this curve. Give your answer in terms of e.

(5)

Leave blank

Question 2 continued

Q2

(Total 5 marks)

Leave blank

3.

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

(a) Find the eigenvalues of **A**.

(5)

(b) Find a normalised eigenvector for each of the eigenvalues of **A**.

(5)

(c) Write down a matrix **P** and a diagonal matrix **D** such that $\mathbf{P}^{\mathrm{T}}\mathbf{AP} = \mathbf{D}$.

(2)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 12 marks)

Leave blank

4. The curve C has equation

$$y = \frac{1}{\sqrt{x^2 + 2x - 3}}, \qquad x > 1$$

(a) Find $\int y \, dx$

(3)

The region R is bounded by the curve C, the x-axis and the lines with equations $x = 2$ and $x = 3$. The region R is rotated through 2π radians about the x-axis.

(b) Find the volume of the solid generated. Give your answer in the form $p\pi \ln q$, where p and q are rational numbers to be found.

(4)

Leave blank

Question 4 continued

Q4

(Total 7 marks)

Leave blank

5. The points A, B and C have position vectors $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ respectively.

(a) Find a vector equation of the straight line AB.

(2)

(b) Find a cartesian form of the equation of the straight line AB.

(2)

The plane Π contains the points A, B and C.

(c) Find a vector equation of Π in the form $\mathbf{r}.\mathbf{n} = p$.

(4)

(d) Find the perpendicular distance from the origin to Π.

(2)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 10 marks)

Leave blank

6. The hyperbola H is given by the equation $x^2 - y^2 = 1$

(a) Write down the equations of the two asymptotes of H.

(1)

(b) Show that an equation of the tangent to H at the point P $(\cosh t, \sinh t)$ is

$$y\sinh t = x\cosh t - 1$$

(3)

The tangent at P meets the asymptotes of H at the points Q and R.

(c) Show that P is the midpoint of QR.

(3)

(d) Show that the area of the triangle OQR, where O is the origin, is independent of t.

(3)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 10 marks)

Leave blank

7. $$I_n = \int \sin^n x \, dx, \quad n \geqslant 0$$

(a) Prove that for $n \geqslant 2$

$$I_n = \frac{1}{n}(-\sin^{n-1} x \cos x + (n-1)I_{n-2})$$

(4)

Given that n is an odd number, $n \geqslant 3$

(b) show that

$$\int_0^{\frac{\pi}{2}} \sin^n x \, dx = \frac{(n-1)(n-3)...6.4.2}{n(n-2)(n-4)...7.5.3}$$

(4)

(c) Hence find $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^2 x \, dx$

(3)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 11 marks)

Leave blank

8. The ellipse E has equation $x^2 + 4y^2 = 4$

(a) (i) Find the coordinates of the foci, F_1 and F_2, of E.

(ii) Write down the equations of the directrices of E.

(4)

(b) Given that the point P lies on the ellipse, show that

$$|PF_1| + |PF_2| = 4$$

(4)

A chord of an ellipse is a line segment joining two points on the ellipse.

The set of midpoints of the parallel chords of E with gradient m, where m is a constant, lie on a straight line l.

(c) Find an equation of l.

(6)

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Question 8 continued

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END