January 2007
6679 Mechanics M3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) Maximum speed when accel. = 0 (o.e.)	B1 (1)
	(b) $\frac{1}{12}(30 - x) = v\frac{dv}{dx}$ (acceln = … + attempt to integrate)	M1 ↓
	Use of $v\frac{dv}{dx}$: $\frac{v^2}{2} = \frac{1}{12}\left(30x - \frac{x^2}{2}\right)$ $(+c)$	M1 A1 ↓
	Substituting $x = 30$, $v = 10$ and finding c (= 12.5), or limits	M1
	$v^2 = 25 + 5x - \frac{1}{12}x^2$ (o.e.)	A1 (5)
	(a) Allow "acceln > 0 for $x < 30$, acceln < 0 for $x > 30$" Also "accelerating for $x < 30$, decelerating for $x > 30$" But "acceln < 0 for $x > 30$" only is B0	
	(b) 1st M1 will be generous for wrong form of acceln (e.g. dv/dx)! 3rd M1 If use limits, they must use them in correct way with correct values Final A1. Have to accept any expression, but it must be for v^2 explicitly (not $1/2v^2$), and if in separate terms, one can expect like terms to be collected. Hence answer in form as above, or e.g. $\frac{1}{12}(300 + 60x - x^2)$; also $100 - \frac{1}{12}(30 - x)^2$	

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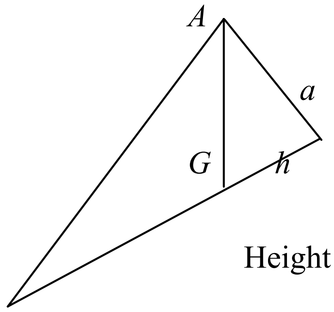

Height of cone = $\dfrac{a}{\tan\alpha} = 3a$

Hence $h = \frac{3}{4}a$

$\tan\theta = \dfrac{a}{\frac{3}{4}a} = \dfrac{4}{3} \Rightarrow \theta = 53.1°$

1st M1 (generous) allow any trig ratio to get height of cone (e.g. using sin)

3rd M1 For correct trig ratio on a suitable triangle to get θ or complement (even if they call the angle by another name – hence if they are aware or not that they are getting the required angle)
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M1 A1
↓
M1

↓
M1 A1
(5)
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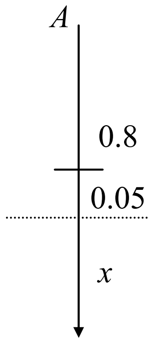
3	(a) E.P.E. $= \frac{1}{2}\frac{3.6mg}{a}x^2 = \frac{1}{2}\frac{3.6mg}{a}\left(\frac{a}{3}\right)^2$ $= \underline{0.2\ mga}$	M1 A1 A1 (3)
	(b) Friction = $\mu mg \Rightarrow$ work done by friction = $\mu mg\left(\frac{4a}{3}\right)$ Work-energy: $\frac{1}{2}m.2ga = \mu mgd + 0.2\ mga$ (3 relevant terms) Solving to find μ: $\underline{\mu = 0.6}$ (b) 1st M1: allow for attempt to find work done by frictional force (i.e. not just finding friction). 2nd M1: "relevant" terms, i.e. energy or work terms! A1 f.t. on their work done by friction	M1 A1 M1 A1√ ↓ M1 A1 (6)

4.	(a) Energy: $\frac{1}{2}m.3ag - \frac{1}{2}mv^2 = mga(1+\cos\theta)$	M1 A1
	$\underline{v^2 = ag(1-2\cos\theta)}$ **(o.e.)**	A1 (3)
	(b) $T + mg\cos\theta = m\frac{v^2}{a}$	M1 A1
	Hence $\underline{T = (1-3\cos\theta)mg}$ (*)	A1 cso (3)
	(c) Using $T = 0$ to find $\cos\theta$	M1
	Hence height above A = $\underline{\frac{4}{3}a}$ Accept 1.33a (but must have 3+ s.f.)	A1 (2)
	(d) $v^2 = \frac{1}{3}ag$ **(o.e.)** f.t. using $\cos\theta = \frac{1}{3}$ in v^2	B1√
	consider vert motion: $(v\sin\theta)^2 = 2gh$ (with v resolved)	M1 A1 ↓
	$\sin^2\theta = \frac{8}{9}$ (or θ = 70.53, sin θ = 0.943) and solve for h (as ka)	M1
	$h = \underline{\frac{4}{27}a}$ or 0.148a (awrt)	A1
	OR consider energy: $\frac{1}{2}m(v\cos\theta)^2 + mgh = \frac{1}{2}mv^2$ (3 non-zero terms)	M1 A1 ↓
	Sub for v, θ and solve for h	M1
	$h = \underline{\frac{4}{27}a}$ or 0.148a (awrt)	A1

Question Number	Scheme	Marks
5.	(a) $\updownarrow\ T\cos\theta = mg$	B1
	$\leftrightarrow\ T + T\sin\theta = mr\omega^2$ (3 terms)	M1 A1
	$r = h\tan\theta$	B1
	$\dfrac{mg}{\cos\theta}(1+\sin\theta) = \dfrac{m\omega^2 h\sin\theta}{\cos\theta}$ (eliminate r)	↓ M1
	$\underline{\omega^2 = \dfrac{g}{h}\left(\dfrac{1+\sin\theta}{\sin\theta}\right)}$ (*) (solve for ω^2)	↓ M1 A1 (7)
	(b) $\omega^2 = \dfrac{g}{h}\left(\dfrac{1}{\sin\theta}+1\right) > \dfrac{2g}{h}$ $(\sin\theta < 1)$ $\Rightarrow$ $\omega > \sqrt{\dfrac{2g}{h}}$ (*)	M1 A1 (2)
	(c) $\dfrac{3g}{h} = \dfrac{g}{h}\left(\dfrac{1+\sin\theta}{\sin\theta}\right)$ $\Rightarrow$ $\sin\theta = \frac{1}{2}$	M1 A1
	$T\cos\theta = mg$ $\Rightarrow$ $\underline{T = \dfrac{2\sqrt{3}}{3}mg}$ or $\underline{1.15mg}$ (awrt)	↓ M1 A1 (4)
	(a) Allow first B1 M1 A1 if assume different tensions (so next M1 is effectively for eliminating r **and** T. (b) M1 requires a *valid* attempt to derive an *in*equality for ω. (Hence putting sin θ = 1 immediately into expression of ω^2 [assuming this is the critical value] is M0.)	

6.	(a) Moments: $\pi \int_1^2 xy^2 \mathrm{d}x = V\bar{x}$ or $\int_1^2 xy^2 \mathrm{d}x = \bar{x}\int_1^2 y^2 \mathrm{d}x$	M1
	$\int_1^2 y^2\,\mathrm{d}x = \int_1^2 \frac{1}{4x^4}\mathrm{d}x = \left[-\frac{1}{12x^3}\right]_1^2 \quad (= \frac{7}{96})$ (either)	M1 A1
	$\int_1^2 xy^2\,\mathrm{d}x = \int_1^2 \frac{1}{4x^3}\mathrm{d}x = \left[-\frac{1}{8x^2}\right]_1^2 \quad (= \frac{3}{32})$ (both)	A1 ↓
	Solving to find $\bar{x}$ $(= \frac{9}{7})$ $\Rightarrow$ required dist $= \frac{9}{7} - 1 = \frac{2}{7}$ m (*)	M1 A1 cso (6)
	(b) Mass: H: $(\rho)\frac{2}{3}\pi\left(\frac{1}{2}\right)^3$ $\left[= \frac{1}{12}(\rho)\pi\right]$, S: $(\rho)\frac{7\pi}{96}$, T: $H + S$ $\left[= \frac{5}{32}(\rho)\pi\right]$	B1, M1
	Dist of CM from base: H: $\frac{19}{16}$ m, S: $\frac{5}{7}$ m, T: $\bar{x}$	B1 B1
	Moments: $\left[= \frac{1}{12}(\rho)\pi\right]\left(\frac{19}{16}\right) + (\rho)\frac{7\pi}{96}\left(\frac{5}{7}\right) = \left[\frac{5}{32}(\rho)\pi\right]\bar{x}$	M1 A1
	$\bar{x} = \frac{29}{30}$ m or 0.967 m (awrt)	A1 (7)
	Allow distances to be found from different base line if necessary	

7.	(a)  $T = \frac{\lambda}{0.8}(0.05) = 0.25g$	M1
	$\lambda = \frac{(0.8)(0.25g)}{0.05} = 39.2$ (*)	A1 (2)
	(b) $T = \frac{39.2}{0.8}(x + 0.05)$	M1
	$mg - T = ma$ (3 term equn)	M1
	$0.25g - \frac{39.2}{0.8}(x + 0.05) = 0.25\ \ddot{x}$ (or equivalent)	A1
	$\ddot{x} = -196x$	A1
	SHM with period $\frac{2\pi}{\omega} = \frac{2\pi}{14} = \frac{\pi}{7}$ s (*)	↓ M1 A1 cso (6)
	(c) $v = 14\sqrt{\{(0.1)^2 - (0.05)^2\}}$	M1 A1√
	$= 1.21(24\ldots) \approx$ <u>1.21 m s^{-1}</u> (3 s.f.) Accept $7\sqrt{3}/10$	A1 (3)
	(d) Time T under gravity $= \frac{1.21..}{g}$ ($= 0.1237s$)	B1√
	Complete method for time T' from B to slack.	
	[↑ e.g. $\frac{\pi}{28} + t$, where $0.05 = 0.1\sin 14t$ OR T', where $-0.05 = 0.1\cos 14T'$]	M1 A1
	$T'' = 0.1496$s	A1
	Total time $= T + T' =$ <u>0.273 s</u>	A1 (5)
	(b) 1st M1 must have extn as $x + k$ with $k \neq 0$ (but allow M1 if e.g. $x + 0.15$), or must justify later For last four marks, *must* be using $\ddot{x}$ (not a) (c) Using $x = 0$ is M0 (d) M1 – must be using distance for when string goes slack. Using $x = -0.1$ (i.e. assumed end of the oscillation) is M0	