Mark Scheme (Results)

Summer 2013

GCE Decision Mathematics 1 (6689/01)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information, please visit our website at www.edexcel.com.

Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

www.edexcel.com/contactus

Pearson: helping people progress, everywhere

Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

Question Number	Scheme	Marks	
1. (a)	Bipartite (graph)	B1	**(1)**
(b)	e.g. (**see below for alternatives**)		
	First alternating path: B – 4 = L – 3 = H – 2	M1	
	Change status to give B = 4 – L = 3 – H = 2	A1	
	Improved matching: A = 1, B = 4, H = 2, (I unmatched), L = 3 R = 5	A1	
	Second alternating path: I – 1 = A – 3 = L – 5 = R – 6	M1	
	Changing status to give: I = 1 – A = 3 – L = 5 – R = 6	A1	
	Complete matching: A = 3, B = 4, H = 2, I = 1, L = 5, R = 6	A1	**(6)**
		7 marks	

<div align="center">Notes for Question 1</div>

Possible 1st paths	A	B	H	I	L	R	Subsequent 2nd paths
B – 4 – L – 3 – H – 2	1	4	2	-	3	5	I – 1 – A – 3 – L – 5 – R – 6
B – 4 – L – 5 – R – 6	1	4	3	-	5	6	I – 1 – A – 3 – H – 2
I – 1 – A – 3 – H - 2	3	-	2	1	4	5	B – 4 – L – 5 – R – 6

a1B1: CAO, but be charitable on spelling, award if phonetically close.

b1M1: An alternating path (e.g. letter – number – letter – ...) from either B to 2 or 6 or from I to 2 – or vice   versa

b1A1: CAO – a correct path including change status **either** stated (only accept 'change (of) status' **or** 'c.s.') **or** shown (**all** symbols e.g. (...–... = ...) **interchanged** (... = ... – ...)). Chosen path clear.

b2A1: CAO must follow from the correct stated path. Accept on a **clear** diagram (with five arcs **only**).

b2M1: A second alternating path from the remaining (unused) I or B to the remaining (unused) 6 or 2 - or vice versa.

b3A1: CAO including change status (stated **or** shown), chosen path clear

b4A1: CAO must follow from **two correct** stated paths (so **both** previous M marks must have been awarded).

Accept on a **clear** diagram (with six arcs only).

Question Number	Scheme	Marks
2.(a)	Bin 1: **0.6 0.2** 0.4 0.5 0.1 Bin 3: **1.6** Bin 2: **1.5** 0.3 Bin 4: 0.7 0.9	M1 A1 A1 **(3)**
(b)	0.6 1.5 1.6 0.2 0.4 0.5 0.7 0.1 0.9 0.3 pivot 0.5 0.6 1.5 1.6 0.7 0.9 0.5 0.2 0.4 0.1 0.3 pivots 1.6 0.1 1.6 0.6 1.5 0.7 0.9 0.5 0.2 0.4 0.3 0.1 pivots 0.7 0.4 1.6 1.5 0.9 0.7 0.6 0.5 0.4 0.2 0.3 0.1 pivots 0.9 0.3 (0.6) 1.6 1.5 0.9 0.7 0.6 0.5 0.4 0.3 0.2 0.1 sort complete	M1 A1 A1ft A1cso **(4)**
(c)	Bin 1: **1.6** 0.4 Bin 2: **1.5** 0.5 Bin 3: **0.9 0.7** 0.3 0.1 Bin 4: 0.6 0.2	M1 A1 A1 **(3)**
(d)	e.g. $6.8/2 = 3.4$ so yes a minimum of 4 bins is needed	B1 **(1)** **11 marks**

Notes for Question 2

a1M1: First four items placed correctly and at least six values put in bins. (Condone cumulative totals here only.)

a1A1: Bin 1 correct

a2A1: CSO All correct

b1M1: Quick sort – pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0) and first pass gives >p, p, <p. So after the first pass the list should read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing 1 pivot per iteration M1 only**

b1A1: First pass correct, next two pivots chosen consistently for second pass.

b2A1ft: second and third passes correct (ft from their first pass and choice of pivots) – need not be choosing pivots for the fourth pass for this mark.

b3A1: CSO (correct solution only – all previous marks in this part **must** have been awarded) including 'sort complete' – this could be shown by the final list being re-written or 'sorted' statement or each item being used as a pivot.

c1M1: **Must be using 'sorted' list** in descending order. First four items placed correctly and at least six values put in bins. (Condone cumulative totals here only.)

c1A1: First seven items placed correctly (so Bin 1 and 2 correct, Bin 3 containing 0.9 and 0.7 and Bin 4 containing 0.6)

c2A1: CSO

SC for part (c) If '**sorted**' list is wrong from part (b) (i.e. one error e.g. a missing number, an extra number or one number incorrectly placed) then award M1 only in (c) for their first seven items correctly placed.

d1B1: A conclusion based on their answer to part (c) together with either a correct lower bound calculation or based on the total > 6 or full bins (three of the bins are full in part (c)).

Part (b) **Using middle left as pivot**

0.6	1.5	1.6	0.2	0.4	0.5	0.7	0.1	0.9	0.3	pivot 0.4	M1
0.6	1.5	1.6	0.5	0.7	0.9	0.4	0.2	0.1	0.3	pivots 1.6 0.1	A1
1.6	0.6	1.5	0.5	0.7	0.9	0.4	0.2	0.3	0.1	pivots 0.5 0.2	
1.6	0.6	1.5	0.7	0.9	0.5	0.4	0.3	0.2	0.1	pivots 1.5 (0.3)	A1ft
1.6	1.5	0.6	0.7	0.9	0.5	0.4	0.3	0.2	0.1	pivot 0.7	
1.6	1.5	0.9	0.7	0.6	0.5	0.4	0.3	0.2	0.1	sort complete	A1cso

Misreads

- If they have misread a number **at the start of part (a), so genuinely miscopied** and got say 1.0 instead of 0.1 then mark the whole question as a misread – removing the last two A or B marks earned. This gives a maximum total of 9.
- If they have used the correct numbers in part (a) and they then use incorrect numbers in part (b) (say 1.0 instead of 0.1) from the beginning of the sort or misread their own numbers **during part (b)** then count it as an **error in part (b)** but mark part (c) as a misread – giving a maximum of 8 or maybe 7 marks depending on how many marks they lose in (b).
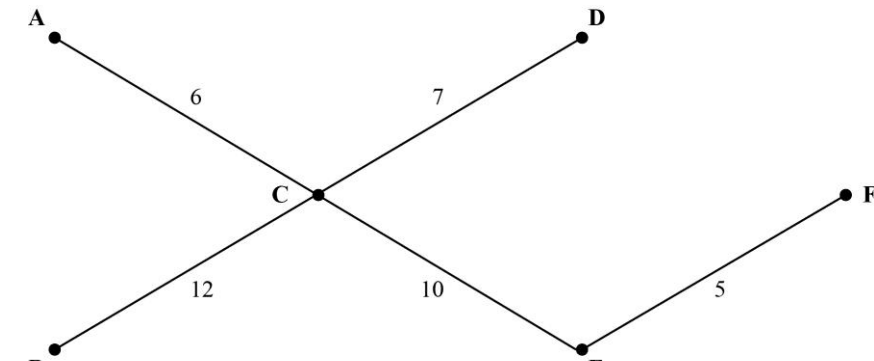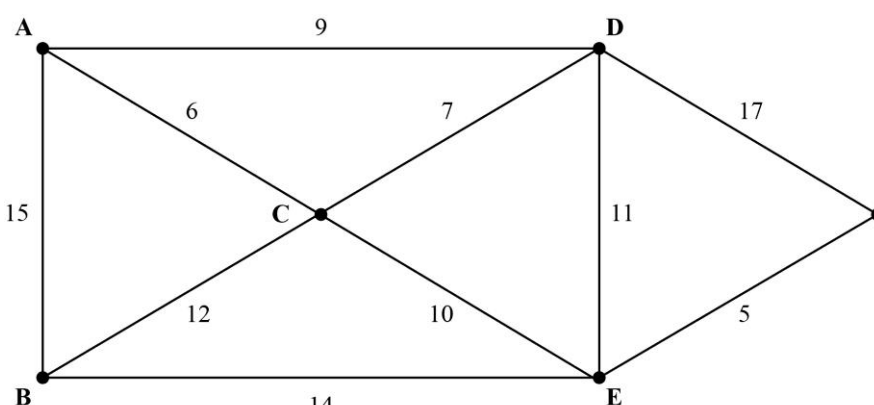
Sorting list into ascending order in (b)

- If the candidate sorts the list into ascending order and reverses the list **in part (b)** then they can score full marks.
- If the list is not reversed in part (b) then mark as a misread (so remove the last two A marks if earned in part (b)). If the list is reversed at the start of part (c) but not in part (b) then still treat this as a misread. If the list is still in ascending order in part (c) award no marks for first fit increasing. If the candidate says that the list needs reversing in part (b) but doesn't actually show the reversed list in part (b) then remove the final A mark.

Ascending (middle left)

0.6	1.5	1.6	0.2	0.4	0.5	0.7	0.1	0.9	0.3	(0.4)	M1
0.2	0.1	0.3	0.4	0.6	1.5	1.6	0.5	0.7	0.9	(0.1, 0.6)	A1
0.1	0.2	0.3	0.4	0.6	1.5	0.5	0.7	0.9	1.6	(0.2, 0.5)	
0.1	0.2	0.3	0.4	0.5	0.6	1.5	0.7	0.9	1.6	((0.3), 1.5)	A1ft
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.9	1.5	1.6	(0.7)	
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.9	1.5	1.6		A1cso+complete

Ascending (middle right)

0.6	1.5	1.6	0.2	0.4	0.5	0.7	0.1	0.9	0.3	(0.5)	M1
0.2	0.4	0.1	0.3	0.5	0.6	1.5	1.6	0.7	0.9	(0.1, 1.6)	A1
0.1	0.2	0.4	0.3	0.5	0.6	1.5	0.7	0.9	1.6	(0.4, 0.7)	
0.1	0.2	0.3	0.4	0.5	0.6	0.7	1.5	0.9	1.6	(0.3, (0.6), 0.9)	A1ft
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.9	1.5	1.6		A1cso+complete

Question Number	Scheme	Marks
3.(a)	AC, CD, CE; EF; BC	M1; A1; A1 **(3)**
(b)		B1 **(1)**
(c)		B1 B1 **(2)**
(d)	EF, AC, CD, reject AD, CE, reject DE, CB	M1 A1 A1 **(3)**
(e)	Time = 40 (days)	B1 **(1)** **10 marks**

Notes for Question 3

Accept the **weight** of each arc to represent the arcs (as each value is unique).

a1M1: Prim's – first three arcs correctly chosen **or** first four nodes correctly chosen {A, C, D, E, ...}. Any rejections seen during selection **M0**. Order of nodes may be seen at the top of the matrix {1, −, 2, 3, 4, −}

a1A1: First four arcs correctly chosen **or** all six nodes correctly chosen {A, C, D, E, F, B}. Order of nodes may be seen at the top of the matrix {1, 6, 2, 3, 4, 5}

a2A1: CSO (must be considering arcs for this final mark).

Misread: Starting at a node other than A scores **M1 only** – **must** have the first three arcs (or four nodes or numbers) correct.

Starting at	Minimum arcs required for M1	Nodes	Order
A	AC CD CE	ACDE(FB)	1(6)234(5)
B	BC AC CD	BCAD(EF)	3124(56)
C	AC CD CE	CADE(FB)	2(6)134(5)
D	CD AC CE	DCAE(FB)	3(6)214(5)
E	EF CE AC	EFCA(DB)	4(6)3(5)12
F	EF CE AC	FECA(DB)	4(6)3(5)21

b1B1: CAO (weights not required)

c1B1: Any four arcs added correctly (weights not required)

c2B1: CAO (including weights) – bod if arcs 'appear' to be crossed out (they may be using the network diagram to answer part (d)).

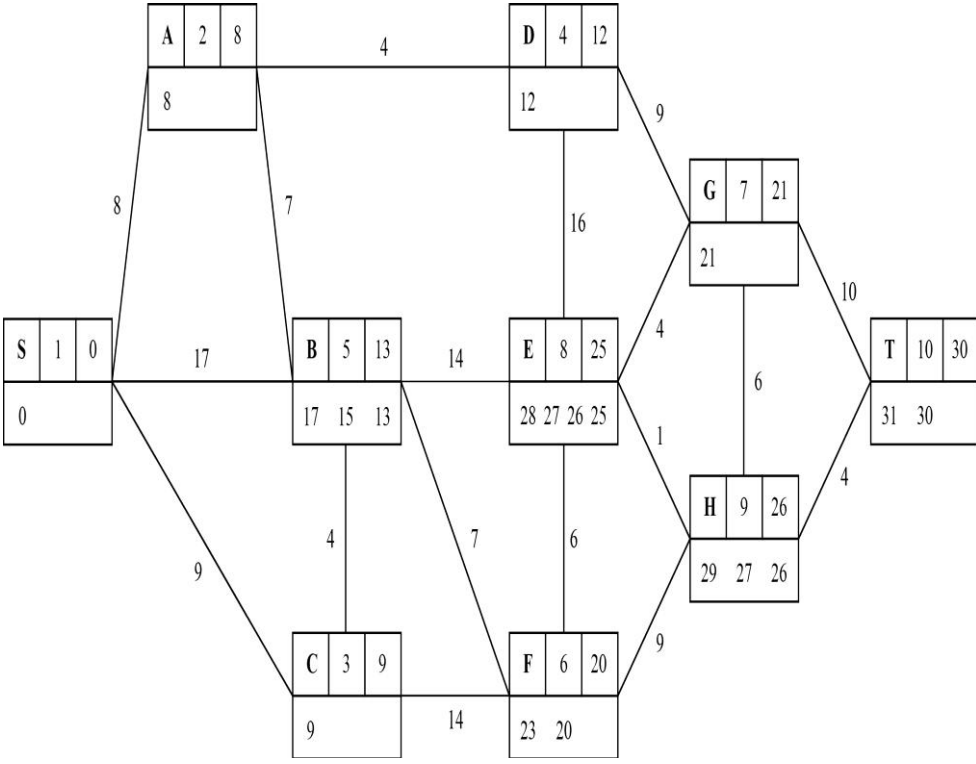d1M1: Kruskal's – first three arcs correctly chosen and **at least one rejection seen at some point**.

d1A1: All five arcs selected correctly EF, AC, CD, CE, CB.

d2A1: CAO All selections and rejections correct (in correct order and at the correct time).

- Listing all the arcs in order and then listing those arcs in the tree in the correct order is fine for **full marks** (this implies that rejections are correct and at the correct time)
- Listing all the arcs in order and just drawing the MST is **M0**

SC for part (d): If the network diagram is incorrect in part (c) **and it is clear that the candidate has used part (c) (instead of the original table) to answer part (d)** then award **M1 only** for the first three arcs correctly chosen and at least one rejection seen at some point provided their network is connected and contains at least nine arcs.

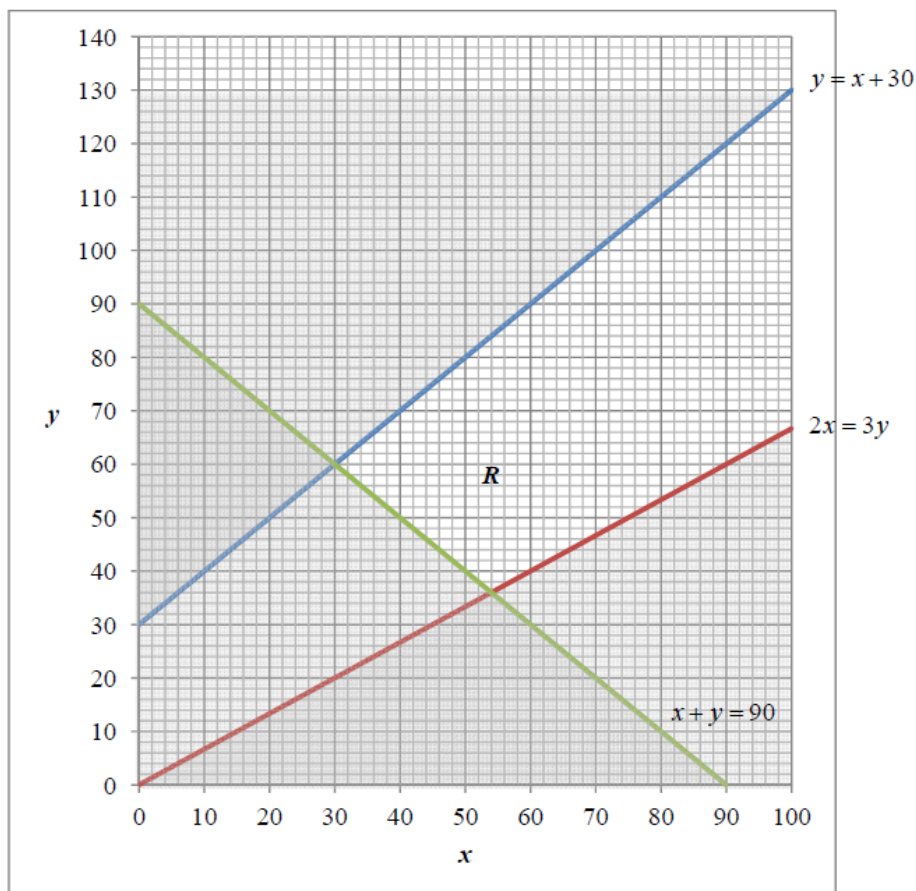e1B1: CAO (ignore lack/incorrect units)

Question Number	Scheme	Marks
4.(a)		M1

Dijkstra network diagram (node boxes shown as: Vertex | Order of labelling | Final label; working values below):

- A 2 8 — working: 8
- D 4 12 — working: 12
- G 7 21 — working: 21
- S 1 0 — working: 0
- B 5 13 — working: 17 15 13
- E 8 25 — working: 28 27 26 25
- T 10 30 — working: 31 30
- H 9 26 — working: 29 27 26
- C 3 9 — working: 9
- F 6 20 — working: 23 20

Edge weights: A–D 4, A–S 8, A–B 7, D–E 16, D–G 9, G–E 4, G–T 10, G–H 6, S–B 17, S–C 9, B–E 14, B–C 4, E–H 1, E–F 6, H–T 4, C–F 14, F–H 9

A1 (A,B,C,D)

A1 (E,F,G)
A1ft(H and T)

Shortest path S to T: SADGEHT
Length of shortest path S to T: 30 (miles)

A1
A1ft   **(6)**

(b) Shortest path S to T via F: SCBFEHT
Length is 31 (miles)

B1
B1   **(2)**

8 marks

Notes for Question 4

a1M1: A larger value replaced by a smaller value at least once in the working values at either B or E or F or H or T.

a1A1: **All** values in A, B, C and D correct. The working values at B must be in the correct order.

a2A1: **All** values in E, F and G correct and the working values in the correct order. Penalise order of labelling only once per question (F, G and E labelled in that order and F must be labelled after A, B, C and D).

a3A1ft: **All** values in H and T ft correct and the working values in the correct order. Penalise order of labelling only once per question (H and T labelled in that order and H labelled after all other nodes).

a4A1: Route CAO.

a5A1ft: ft on their final value (if answer is not 30 ft their final value at T).

b1B1: Route CAO

b2B1: Length CAO (condone lack of (or incorrect) units throughout).

Question Number	Scheme	Marks
5. (a)	AB + DE = 44 + 30 = 74* AD + BE = 42 + 35 = 77 AE + BD = 39 + 38 = 77 Repeat arcs AC, BC and DE	M1 A3.2.1.0 A1 **(5)**
(b)	E.g. ABCADCBEDFGDEGHECA (18 nodes) Length: 344 + 74 = 418	B1 B1ft **(2)**
(c)	One of AB (44), AD (42) or BD (38) will still have to be repeated. **BD(38) is the shortest** So start at E and **finish at A**, route length now is 344 + 38 = **382**	M1 A1 DA1 **(3)**
		10 marks

Notes for Question 5

a1M1: Three distinct pairings of **their** four odd nodes

a1A1: Any one row correct including pairing **and** total

a2A1: Any two rows correct including pairing **and** total

a3A1: All three rows correct including pairing **and** total

a4A1: CAO correct **arcs** identified AC, BC and DE. Accept ACB or AB via C (check to see if via C appears in working) but **do not** accept AB for this mark

b1B1: Any correct route (checks: eighteen nodes (or seventeen arcs), the route starts and ends at A, pairings AC, BC and DE appear twice in the route and that every letter (A to H inclusive) appears at least once).

b2B1ft: correct answer of 418 **or** 344 + their least out of a choice of at least **two** totals given in part (a)

c1M1: **Either** identifies the need to repeat one pairing which does not include E (could list potential repeats) **or** identifies the need to repeat BD (or 38).

c1A1: Identifies the need to repeat one pairing which does not include E **and** this is BD (38) **because it is the least**. To score the first two marks the candidate must make it clear that they need to repeat **BD because it has the least weight of those pairings that do not include E**.

c2DA1: correct finishing point (A) and length (382). This mark is dependent on them identifying BD (38) as the repeat.

Question Number	Scheme	Marks
6. (a)	He must buy at least 90 boats in total ($x + y \geq 90$)	B1 **(1)**
(b)	E.g. The number of 2-seater boats(x) must be less than or equal to 1.5 times the number of 4-seater boats (y). (check: y = 2, x = 3, 2, 1, ...) ($2x \leq 3y$) E.g. The number of 4-seater boats (y) must be greater than or equal to 2/3 the number of 2-seater boats (x). (check: x = 3, y = 2, 3, 4, ...)	B1 B1 **(2)**
(c)	The correct 3 lines added; x + y = 90; 3y = 2x; y = x + 30 Region, R labelled	B1; B1; B1 B1 **(4)**
(d)	(minimise C =) 100x + 300y	B1 **(1)**
(e)	Method clear – either at least 2 vertices tested or objective line drawn (54, 36), so 54 2-seater and 36 4-seater At a cost of £16 200	M1 A1 B1 B1 **(4)**
		12 marks

Notes for Question 6

a1B1: CAO (must have 'boats', 'least', '90', must be talking about boats **not** cost)

b1B1: For a statement in context with either the ratio of coefficients correct (the 2 with the 2-seater and the 3 with the 4-seater) **or** inequality correct with correct numbers present but not in the correct ratio.

b2B1: Clear accurate correct statement in context.

c1B1: $x + y = 90$ correctly drawn. Must pass within one small square of the points of intersection with the axes

c2B1: $3y = 2x$ correctly drawn. Must pass within one small square of the origin and (90, 60).

c3B1: $y = x + 30$ correctly drawn. Must pass within one small square of (0, 30) and (60, 90).

c4B1: Region, R, correctly labelled – not just implied by shading – must have scored all three previous marks in this part.

d1B1: CAO (isw if $100x + 300y$ 'simplified' to $k(100x + 300y)$ but if $100x + 300y$ not stated then B0)

e1M1: Line must be correct to within one small square if extended from axis to axis **OR** attempting to find two vertices of their R (or the correct R) by either reading off their graph or using simultaneous equations **and** testing using **their** objective function.

e1A1: Correct objective line (same condition that the line must be correct to within one small square if extended from axis to axis) **OR** testing (30, 60) correctly (giving 21 000) **and** testing (54, 36) correctly (giving 16 200).

e1B1: Correct point identified. (Condone in terms of x and y rather than in terms of boats.)

e2B1: CAO – condone lack of/incorrect units on the cost.

Examples for part (b) scoring B1 B1 (**useful check:** when $y = 2$, $x = 3, 2, 1, \ldots$ **or** when $x = 3$, $y = 2, 3, 4, \ldots$)

- Twice the number of 2-seater boats must be at most three times the number of 4-seater boats
- Three times the number of 4-seater must be at least twice the number of 2-seater boats
- **For every three 2-seater boats there must be at least two 4-seater boats** (or multiple of this ratio)
- **For every two 4-seater boats there must be at most three 2-seater boats** (or multiple of this ratio)
- At most 60% of the total boats are 2-seater
- At least 40% of the total boats are 4-seater

Examples of B1 B0 – **in each case either the inequality is the correct way round OR the 2 is with 2-seater boats and the 3 is with the 4-seater boats** (accept multiples of 2 and 3) (useful numbers: when $y = 2$, $x = 3$, 4, 5,… when $x = 3$, $y = 2, 1, \ldots$, when $y = 3$, $x = 2, 1, \ldots$, when $x = 2$, $y = 3, 4, 5, \ldots$)

- Twice the number of 2-seater boats must be at least three times the number of 4-seater boats
- Three times the number of 4-seater must be at most twice the number of 2-seater boats
- Three times the number of 2-seater must be at least twice the number of 4-seater boats
- **For every three 2-seater boats there must be at most two 4-seater boats (or multiple of this ratio)**
- **For every two 4-seater boats there must be at least three 2-seater boats (or multiple of this ratio)**
- **For every two 2-seater boats there must be at least three 4-seater boats (or multiple of this ratio)**
- **For every three 4-seater boats there must be at most two 2-seater boats (or multiple of this ratio)**
- At least 60% of the total boats are 2-seater
- At most 40% of the total boats are 4-seater
- At least 60% of the total boats are 4-seater
- At most 40% of the total boats are 2-seater

Question Number	Scheme	Marks
7.(a)		M1 A1
		M1 A1 **(4)**
(b)	Float on M = 42 − 26 − 8 = 8	M1 A1 **(2)**
(c)(i)	2 day delay on P – no effect on the project completion date (float on P is 4)	B1
(c)(ii)	2 day delay on Q – project finishes 2 days late (Q is a critical activity)	B1 **(2)**
(d)	(172/53 = 3.245, so) a minimum of 4 workers needed	B1 **(1)**

Question Number	Scheme	Marks
(e)		M1 A1 (any 6 more)
		M1 A1 (all 11)
		(4)
(f)	E.g. Activities H, I, J, K and L together with $22 < \text{time} < 26$ stated. So 5 workers needed	M1 A1 **(2)**
(g)	The cascade gives a higher lower bound, so (f) is better.	M1 A1 **(2)**
		17 marks

Notes:
a1M1: All top boxes complete, values generally increasing left to right, condone one 'rogue' (if values do not increase from left to right then if one value is ignored and then the values do increase from left to right then this is considered to be only one rogue value)
a1A1: CAO.
a2M1: All bottom boxes complete, values generally decreasing right to left, condone one 'rogue'.
a2A1: CAO
b1M1: Correct calculation seen – all three numbers correct (ft), float $\geqslant$ 0.
b1A1: Float correct (no ft on this mark)
c1B1: CAO
c2B1: CAO
d1B1: 4 with (or without) working scores this mark
e1M1: At least six activities added including six floats. Scheduling diagram scores M0.
e1A1: Six activities including their floats dealt with correctly.
e2M1: All remaining eleven activities including all eleven floats.
e2A1: CAO.

Examples for part (f):

Example 1: Activities H, I, J, K and L with $22 < time < 26$ so 5 workers needed.
Example 2: At $10 < time < 14$, F, D, E and H must be happening. Activity G must be happening $7 < time < 18$ but its duration is 5 so it must also occur at some point in the interval $10 < time < 14$ so 5 workers needed.

f1M1: Example 1: A statement with the correct number of workers (5) **and** the correct activities (H, I, J, K and L) with some mention of time, **or**
Example 2: A statement with the correct number of workers (5), the correct activities (F,D,E and H) with some mention of time **and** an indication that G **must** be happening with the other four activities at some point - give bod but e.g. 'at time 11 F, D, E, G and H must be happening' is **M0**). **Scheduling** the activities only scores **M0**.

f1A1: A correct, complete full statement with details of both time **and** activities. Candidates only need to give a time within the intervals stated.

Please note strict inequalities for the time. Allow e.g. **on** 'day 23' as equivalent to $22 < time < 23$.

g1M1: **Must have attempted both parts (d) and (f). Their** higher lower bound chosen + attempt at a reason.

Allow for the M mark a reason which argues that e.g. the cascade chart gives a better lower bound (e.g. it takes into account exactly when activities must be taking place) **or** e.g. the calculation gives a better lower bound (e.g. as it takes into account the sum of all the activities) but without specifically answering the question of which of the two bounds is better. Give bod on an attempt at a reason.

g1A1: CAO plus a correct reason given. Acceptable reasons e.g. the cascade gives a larger value **or** the bound for the cascade shows that the project cannot be done with fewer workers, etc.