

# Mark Scheme (Results) January 2009

GCE

GCE Mathematics (6679/01)

**January 2009  
6679 Mechanics M3  
Mark Scheme**

| Question Number | Scheme   | Marks  |
|-----------------|--|--|
| <b>1</b>        | <p>N2L</p> $3a = -\left(9 + \frac{15}{(t+1)^2}\right)$ $3v = -9t + \frac{15}{t+1} (+A)$ $v = 0, t = 4 \Rightarrow 0 = -36 + 3 + A \Rightarrow A = 33$ $v = -3t + \frac{5}{t+1} + 11$ $t = 0 \Rightarrow v = 16$  | <p>B1</p> <p>M1 A1ft</p> <p>M1 A1</p> <p>M1 A1 (7)<br/>[7]</p>                         |
| <b>2</b>        | <div style="text-align: center;"> </div> <p>(a)</p> <p>(←) <math>T \sin \theta = \frac{4}{3}mg</math></p> <p>(↑) <math>T \cos \theta = mg</math></p> $T^2 = \left(\frac{4}{3}mg\right)^2 + (mg)^2$ <p>Leading to <math>T = \frac{5}{3}mg</math></p> <p>(b)</p> <p>HL <math>T = \frac{\lambda x}{a} \Rightarrow \frac{5}{3}mg = \frac{3mge}{a}</math>      ft their <math>T</math></p> $e = \frac{5}{9}a$ $E = \frac{\lambda x^2}{2a} = \frac{3mg}{2a} \times \left(\frac{5}{9}a\right)^2 = \frac{25}{54}mga$ | <p>M1 A1</p> <p>A1</p> <p>M1</p> <p>A1 (5)</p> <p>M1 A1ft</p> <p>M1 A1 (4)<br/>[9]</p> |

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| 3               | $\omega = \frac{80 \times 2\pi}{60} \text{ rad s}^{-1} \left( = \frac{8\pi}{3} \approx 8.377... \right)$ <p style="text-align: center;">Accept <math>v = \frac{16\pi}{75} \approx 0.67 \text{ ms}^{-1}</math> as equivalent</p> $(\uparrow) R = mg$ <p>For least value of <math>\mu</math> <math>(\leftarrow) \mu mg = mr\omega^2</math></p> $\mu = \frac{0.08}{9.8} \times \left( \frac{8\pi}{3} \right)^2 \approx 0.57 \quad \text{accept } 0.573$   | <p>B1</p> <p>B1</p> <p>M1 A1=A1</p> <p>M1 A1 (7)</p> <p>[7]</p>                                    |
| 4               | <p>(a)</p> $a = 8$ $T = \frac{25}{2} = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{4\pi}{25} (\approx 0.502 \dots)$ $v^2 = \omega^2 (a^2 - x^2) \Rightarrow v^2 = \left( \frac{4\pi}{25} \right)^2 (8^2 - 3^2) \quad \text{ft their } a, \omega$ $v = \frac{4\pi}{25} \sqrt{55} \approx 3.7 \text{ (m h}^{-1}\text{)} \quad \text{awrt } 3.7$ <p>(b)</p> $x = a \cos \omega t \Rightarrow 3 = 8 \cos \left( \frac{4\pi}{25} t \right) \quad \text{ft their } a, \omega$ $t \approx 2.3602 \dots$ <p>time is 12 22</p> | <p>B1</p> <p>M1 A1</p> <p>M1 A1ft</p> <p>M1 A1 (7)</p> <p>M1 A1ft</p> <p>M1 A1 (4)</p> <p>[11]</p> |

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|-----------------|--|--|
| 5               | <p>(a) Let <math>x</math> be the distance from the initial position of <math>B</math> to <math>C</math><br/> GPE lost = EPE gained<br/> <math display="block">mgx \sin 30^\circ = \frac{6mgx^2}{2a}</math> Leading to <math>x = \frac{a}{6}</math><br/> <math display="block">AC = \frac{7a}{6}</math></p> <p>(b) The greatest speed is attained when the acceleration of <math>B</math> is zero, that is where the forces on <math>B</math> are equal.<br/> <math display="block">(\curvearrowright) \quad T = mg \sin 30^\circ = \frac{6mge}{a}</math> <math display="block">e = \frac{a}{12}</math> CE <math display="block">\frac{1}{2}mv^2 + \frac{6mg}{2a} \left(\frac{a}{12}\right)^2 = mg \frac{a}{12} \sin 30^\circ</math> Leading to <math display="block">v = \sqrt{\left(\frac{ga}{24}\right)} = \frac{\sqrt{6ga}}{12}</math></p> <p><i>Alternative approaches to (b) are considered on the next page.</i></p> | <p>M1 A1=A1</p> <p>M1</p> <p>A1 (5)</p> <p>M1</p> <p>A1</p> <p>M1 A1=A1</p> <p>M1 A1 (7)</p> <p>[12]</p> |

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|-----------------|--|--|
| 5               | <p><i>Alternative approach to (b) using calculus with energy.</i></p> <p>Let distance moved by <math>B</math> be <math>x</math></p> <p>CE <math>\frac{1}{2}mv^2 + \frac{6mg}{2a}x^2 = mgx \sin 30^\circ</math></p> $v^2 = gx - \frac{6g}{a}x^2$ <p>For maximum <math>v</math> <math>\frac{d}{dx}(v^2) = 2v \frac{dv}{dx} = g - \frac{12g}{a}x = 0</math></p> $x = \frac{a}{12}$ $v^2 = g\left(\frac{a}{12}\right) - \frac{6g}{a}\left(\frac{a}{12}\right)^2 = \frac{ga}{24}$ $v = \sqrt{\left(\frac{ga}{24}\right)}$ | <p>M1 A1=A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (7)</p> |
|                 | <p><i>Alternative approach to (b) using calculus with Newton's second law.</i></p> <p>As before, the centre of the oscillation is when extension is <math>\frac{a}{12}</math></p> <p>N2L <math>mg \sin 30^\circ - T = m\ddot{x}</math></p> $\frac{1}{2}mg - \frac{6mg\left(\frac{a}{12} + x\right)}{a} = m\ddot{x}$ $\ddot{x} = -\frac{6g}{a}x \Rightarrow \omega^2 = \frac{6g}{a}$ $v_{\max} = \omega a = \sqrt{\left(\frac{6g}{a}\right)} \times \frac{a}{12} = \sqrt{\left(\frac{ga}{24}\right)}$                 | <p>M1 A1</p> <p>M1 A1</p> <p>A1</p> <p>M1 A1 (7)</p> |

| Question Number | Scheme   | Marks   |
|-----------------|--|---|
| 6 (a)           | $\int y^2 dx = \int (4-x^2)^2 dx = \int (16-8x^2+x^4) dx$ $= 16x - \frac{8x^3}{3} + \frac{x^5}{5}$ $\left[ 16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_0^2 = \frac{256}{15}$<br>$\int xy^2 dx = \int x(4-x^2)^2 dx = \int (16x-8x^3+x^5) dx$ $= 8x^2 - 2x^4 + \frac{x^6}{6}$ $\left[ 8x^2 - 2x^4 + \frac{x^6}{6} \right]_0^2 = \frac{32}{3}$<br>$\bar{x} = \frac{\int xy^2 dx}{\int y^2 dx} = \frac{32}{3} \times \frac{15}{216} = \frac{5}{8} *$ | <br>M1 A1<br>M1 A1<br><br>M1 A1<br>M1A1<br><br>M1 A1 (10) |
| (b)             | $A \times \bar{x} = (\pi r^2 l) \times \frac{l}{2}$ $\frac{256}{15} \pi \times \frac{5}{8} = \pi \times 16l \times \frac{l}{2}$<br>Leading to $l = \frac{2\sqrt{3}}{3}$ accept exact equivalents or awrt 1.15  | <br>M1<br>A1 ft<br><br>M1 A1 (4)                          |

[14]

| Question Number         | Scheme   | Marks  |
|-------------------------|--|--|
| <p>7 (a)</p> <p>(b)</p> | <p>Let speed at C be <math>u</math></p> <p>CE <math>\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mga(1 - \cos\theta)</math></p> $u^2 = \frac{9ga}{4} - 2ga \cos\theta$ $mg \cos\theta (+R) = \frac{mu^2}{a}$ $mg \cos\theta = \frac{9mg}{4} - 2mg \cos\theta \quad \text{eliminating } u$ <p>Leading to <math>\cos\theta = \frac{3}{4} *</math></p> <p>At C <math>u^2 = \frac{9ga}{4} - 2ga \times \frac{3}{4} = \frac{3}{4}ga</math></p> <p>(<math>\rightarrow</math>) <math>u_x = u \cos\theta = \sqrt{\left(\frac{3ga}{4}\right)} \times \frac{3}{4} = \sqrt{\left(\frac{27ga}{64}\right)} = 2.033\sqrt{a}</math></p> <p>(<math>\downarrow</math>) <math>u_y = u \sin\theta = \sqrt{\left(\frac{3ga}{4}\right)} \times \frac{\sqrt{7}}{4} = \sqrt{\left(\frac{21ga}{64}\right)} = 1.792\sqrt{a}</math></p> $v_y^2 = u_y^2 + 2gh \Rightarrow v_y^2 = \frac{21}{64}ga + 2g \times \frac{7}{4}a = \frac{245}{64}ga$ $\tan\psi = \frac{v_y}{u_x} = \sqrt{\left(\frac{245}{27}\right)} \approx 3.012 \dots$ <p><math>\psi \approx 72^\circ</math>                      awrt <math>72^\circ</math></p> <p>Or <math>1.3^\circ</math> (1.2502<math>^\circ</math>)                      awrt <math>1.3^\circ</math></p> | <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1 (7)</p> <p>B1</p> <p>M1 A1ft</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (8)</p> <p>[15]</p> |
|                         | <p><i>Alternative for the last five marks</i></p> <p>Let speed at P be <math>v</math>.</p> <p>CE <math>\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mg \times 2a</math>                      or equivalent</p> $v^2 = \frac{17mga}{4}$ $\cos\psi = \frac{u_x}{v} = \sqrt{\left(\frac{27}{64} \times \frac{4}{17}\right)} = \sqrt{\left(\frac{27}{272}\right)} \approx 0.315$ <p><math>\psi \approx 72^\circ</math>                      awrt <math>72^\circ</math></p> <p><i>Note: The time of flight from C to P is <math>\frac{\sqrt{235} - \sqrt{21}}{8} \sqrt{\left(\frac{a}{g}\right)} \approx 1.38373 \sqrt{\left(\frac{a}{g}\right)}</math></i></p>  | <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p>   |