

Mark Scheme (Results)

January 2008

GCE

GCE Mathematics (6674/01)

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6674 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Marks
1	Integrating factor = e^{-3x} $\therefore \frac{d}{dx}(ye^{-3x}) = xe^{-3x}$ $\therefore (ye^{-3x}) = \int xe^{-3x} dx = -\frac{x}{3}e^{-3x} + \int \frac{1}{3}e^{-3x} dx$ $= -\frac{x}{3}e^{-3x} - \frac{1}{9}e^{-3x} (+c)$ $\therefore y = -\frac{x}{3} - \frac{1}{9} + ce^{3x}$	B1 M1 M1 A1 A1ft [5]
	Notes: First M for multiplying through by Integrating Factor and evidence of calculus Second M for integrating by parts ‘the right way around’. Be generous – ignore wrong signs and wrong constants. Second M dependent on first. Both As dependent on this M. First A1 for correct expression – constant not required Second A requires constant for follow through. If treated as a second order de with errors then send to review.	

2.	<p>Use $(2x+1)$ as factor to give $f(x) = (2x+1)(x^2 - 6x + 10)$</p> <p>Attempt to solve quadratic to give $x = \frac{6 \pm \sqrt{(36-40)}}{2}$</p> <p>Two complex roots are $= 3 \pm i$</p>	M1 A1 M1 A1 M1 A1 (6) [6]
	<p>Notes:</p> <p>First M if method results in quadratic expression with 3 terms (even with remainder).</p> <p>Second M for use of correct formula on their quadratic.</p> <p>Third M for using i from negative discriminant.</p>	
3. (a) (b)	<p>Consider $\frac{(x+3)(x+9)-(3x-5)(x-1)}{(x-1)}$, obtaining $\frac{-2x^2+20x+22}{(x-1)}$</p> <p>Factorise to obtain $\frac{-2(x-11)(x+1)}{(x-1)}$.</p> <p>Identify $x = 1$ and their two other critical values Obtain one inequality <i>as an answer</i> involving at least one of their critical values To obtain $x < -1, -1 < x < 11$</p>	M1 A1 M1 A1 (4) B1ft M1 A1, A1 (4) [8]
	<p>Notes:</p> <p>Second M attempt to factorise quadratic expression with 3 terms (usual rules).</p> <p>Second A don't require -2 outside but can be part of factors.</p>	

4. (a)	<p>$f(0.7) = -0.195028497$ and $f(0.8) = 0.297206781$</p> <p>Use $\frac{0.8 - \alpha}{\alpha - 0.7} = \frac{f(0.8)}{-f(0.7)}$ to obtain $\alpha = \frac{-0.8f(0.7) + 0.7f(0.8)}{f(0.8) - f(0.7)}$</p> <p>$(=0.739620991) = 0.740$ Answer required to 3 dp or better</p> <p>(b) $f'(x) = 6x + 1 - \frac{1}{2}\sec^2(\frac{x}{2})$</p> <p>Use $x_2 = 0.75 - \frac{f(0.75)}{f'(0.75)}$ ($= 0.741087218$) = 0.741 Answer required to 3 dp or better</p>	B1, B1 M1 A1 (4) M1 A1 M1 A1 (4) [8]
	<p>Notes:</p> <p>(a) Bs for 3dp or better</p> <p>First M for reasonable attempt using fractions and differences.</p> <p>(b) First M attempt to differentiate $f(x)$, term in x is enough.</p> <p>Lose last A if either or both not to 3 dp</p>	

Question Number	Scheme	Marks
5. (a)	Method to obtain partial fractions e.g. $5r + 4 = A(r+1)(r+2) + Br(r+2) + Cr(r+1)$ And equating coefficients, or substituting values for x . $A = 2, B = 1, C = -3$ or $\frac{2}{r+1} + \frac{1}{r+2} - \frac{3}{r+3}$	M1 A1 A1 A1 (4)
(b)	$\sum_{r=1}^n \dots = \frac{2}{1} + \frac{1}{2} - \frac{3}{3} + \frac{2}{2} + \frac{1}{3} - \frac{3}{4} + \frac{2}{3} + \frac{1}{4} - \frac{3}{5} + \dots + \frac{2}{n-1} + \frac{1}{n} - \frac{3}{n+1} + \frac{2}{n} + \frac{1}{n+1} - \frac{3}{n+2}$ $= 2 + \frac{3}{2}, -\frac{2}{n+1} - \frac{3}{n+2} \text{ or equivalent}$ $= \frac{7(n+1)(n+2) - 4(n+2) - 6(n+1)}{2(n+1)(n+2)} = \frac{7n^2 + 11n}{2(n+1)(n+2)} *$	M1 A1, A1 M1 A1 (5)

Notes:

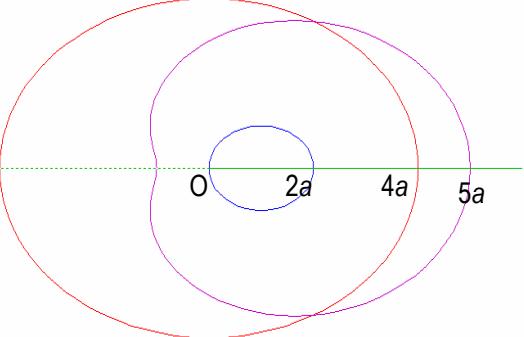
- (a) Require three constants for method.
- (b) Require first 3 and last 2 of their terms for first method

Second method - dependent on first - for attempt to combine to single fraction.

Expansion of $(n+1)(n+2)$ in numerator and correct solution required for final A1

Question Number	Scheme	Marks
6(a)	(i) Multiply top and bottom by conjugate to give $\frac{-2-i}{5}$ (ii) Expand and simplify using $i^2 = -1$ to give $3-4i$	M1 A1
(b)	$z^2 - z = 5 - 5i$, $ z^2 - z = 5\sqrt{2} *$	M1 A1 (4) M1A1 (2)
(c)	$\arg(z^2 - z) = -\frac{\pi}{4}$ or -45° or $7\pi/4$ or 315° or $-0.7853\dots$ or $5.497\dots$	M1 A1 (2)
(d)	<p>one mark for each point</p>	B1, B1 ft (2)
	Notes: (a) $-2-i$ or $2+i$ OK for method. Attempt to expand required. (b) square root required for method (c) 2 for correct answer only, tan required for method. 2dp or better. (d) Position of points not clear but both quadrants correct first B1 only.	[10]

Question Number	Scheme	Marks
7 (a)	<p>Solve auxiliary equation $3m^2 - m - 2 = 0$ to obtain $m = -\frac{2}{3}$ or 1 C.F is $Ae^{-\frac{2}{3}x} + Be^x$</p> <p>Let PI = $\lambda x^2 + \mu x + \nu$. Find $y' = 2\lambda x + \mu$, and $y'' = 2\lambda$ and substitute into d.e. Giving $\lambda = -\frac{1}{2}$, $\mu = \frac{1}{2}$ and $\nu = -\frac{7}{4}$</p> <p>$\therefore y = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{4} + Ae^{-\frac{2}{3}x} + Be^x$</p>	M1 A1 A1ft M1 A1 A1A1 A1ft (8)
(b)	<p>Use boundary conditions: $2 = -\frac{7}{4} + A + B$ $y' = -x + \frac{1}{2} - \frac{2}{3}Ae^{-\frac{2}{3}x} + Be^x$ and $3 = \frac{1}{2} - \frac{2}{3}A + B$</p> <p>Solve to give $A = 3/4$, $B = 3$ ($\therefore y = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{4} + \frac{3}{4}e^{-\frac{2}{3}x} + 3e^x$)</p>	M1A1ft M1 M1 M1 A1 (6) [14]
	<p>Notes:</p> <p>(a) Attempt to solve quadratic expression with 3 terms (usual rules) Both values required for first accuracy. Real values only for follow through Second M 3 term quadratic for PI required Final A1ft for their CF+ their PI dependent upon at least one M</p> <p>(b) Second M for attempt to differentiate their y and third M for substitution</p>	

Question Number	Scheme	Marks
8 (a)	$a(3 + 2 \cos \theta) = 4a$ Solve to obtain $\cos \theta = \frac{1}{2}$ $\theta = \pm \frac{\pi}{3}$ and points are $(4a, \frac{\pi}{3})$ and $(4a, \frac{5\pi}{3})$	M1 M1 A1, A1 (4)
(b)	Use area $= \frac{1}{2} \int r^2 d\theta$ to give $\frac{1}{2} a^2 \int (3 + 2 \cos \theta)^2 d\theta$ Obtain $\int (9 + 12 \cos \theta + 2 \cos 2\theta + 2) d\theta$ Integrate to give $11\theta + 12 \sin \theta + \sin 2\theta$ Use limits $\frac{\pi}{3}$ and π , then double or $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ or theirs Find a third area of circle $= \frac{16\pi a^2}{3}$ Obtain required area $= \frac{38\pi a^2}{3} - \frac{13\sqrt{3}a^2}{2}$	M1 A1 M1 A1 M1 B1 A1, A1 (8)
(c)	 <p style="text-align: center;">correct shape 5a and 4a marked 2a marked and passes through O</p>	B1 B1 B1 (3) [15]
	Notes: (a) First A for $r=4a$ second for both values in radians. Accept 1.0471... and 5.2359....2 dp or better for final A (b) First M for substitution, expansion and attempt to use double angles. Second M for integrating expression of the form $a + b \cos \theta + c \cos 2\theta$ Lose final A only if a^2 missing in last line (c) First B for approximately symmetrical shape about initial line, only 1 loop which is convex strictly within shaded region	