

Mark Scheme (Results) January 2008

GCE

GCE Mathematics (6674/01)

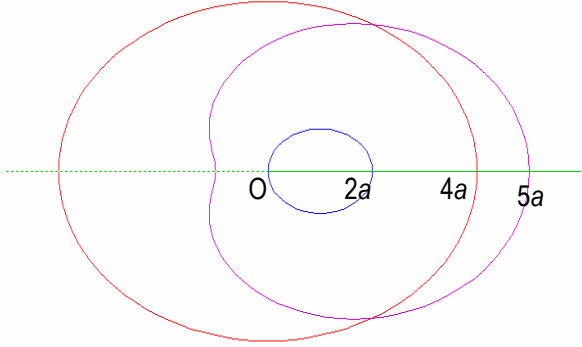
January 2008
6674 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Marks
1	Integrating factor = e^{-3x} $\therefore \frac{d}{dx}(ye^{-3x}) = xe^{-3x}$ $\therefore (ye^{-3x}) = \int xe^{-3x} dx = -\frac{x}{3}e^{-3x} + \int \frac{1}{3}e^{-3x} dx$ $= -\frac{x}{3}e^{-3x} - \frac{1}{9}e^{-3x} (+c)$ $\therefore y = -\frac{x}{3} - \frac{1}{9} + ce^{3x}$	B1 M1 M1 A1 A1ft [5]
	<p>Notes:</p> <p>First M for multiplying through by Integrating Factor and evidence of calculus</p> <p>Second M for integrating by parts ‘the right way around’. Be generous – ignore wrong signs and wrong constants.</p> <p>Second M dependent on first. Both As dependent on this M.</p> <p>First A1 for correct expression – constant not required</p> <p>Second A requires constant for follow through.</p> <p>If treated as a second order de with errors then send to review.</p>	

<p>2.</p>	<p>Use $(2x+1)$ as factor to give $f(x) = (2x+1)(x^2 - 6x + 10)$</p> <p>Attempt to solve quadratic to give $x = \frac{6 \pm \sqrt{(36-40)}}{2}$</p> <p>Two complex roots are $= 3 \pm i$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>(6)</p> <p>[6]</p>
	<p>Notes:</p> <p>First M if method results in quadratic expression with 3 terms (even with remainder).</p> <p>Second M for use of correct formula on their quadratic.</p> <p>Third M for using i from negative discriminant.</p>	
<p>3.</p> <p>(a)</p> <p>(b)</p>	<p>Consider $\frac{(x+3)(x+9) - (3x-5)(x-1)}{(x-1)}$, obtaining $\frac{-2x^2 + 20x + 22}{(x-1)}$</p> <p>Factorise to obtain $\frac{-2(x-11)(x+1)}{(x-1)}$.</p> <p>Identify $x = 1$ and their two other critical values</p> <p>Obtain one inequality <i>as an answer</i> involving at least one of their critical values</p> <p>To obtain $x < -1, 1 < x < 11$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>(4)</p> <p>B1ft</p> <p>M1</p> <p>A1, A1</p> <p>(4)</p> <p>[8]</p>
	<p>Notes:</p> <p>Second M attempt to factorise quadratic expression with 3 terms (usual rules).</p> <p>Second A don't require -2 outside but can be part of factors.</p>	

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<p>5. (a)</p> <p>(b)</p>	<p>Method to obtain partial fractions e.g. $5r + 4 = A(r + 1)(r + 2) + Br(r + 2) + Cr(r + 1)$ And equating coefficients, or substituting values for x.</p> <p>$A = 2, B = 1, C = -3$ or $\frac{2}{r} + \frac{1}{r+1} - \frac{3}{r+2}$</p> $\sum_{r=1}^n \dots = \frac{2}{1} + \frac{1}{2} - \frac{3}{3}$ $+ \frac{2}{2} + \frac{1}{3} - \frac{3}{4}$ $+ \frac{2}{3} + \frac{1}{4} - \frac{3}{5} = 2 + \frac{3}{2}, -\frac{2}{n+1} - \frac{3}{n+2} \text{ or equivalent}$ $+ \dots$ $+ \frac{2}{n-1} + \frac{1}{n} - \frac{3}{n+1}$ $+ \frac{2}{n} + \frac{1}{n+1} - \frac{3}{n+2}$ $= \frac{7(n+1)(n+2) - 4(n+2) - 6(n+1)}{2(n+1)(n+2)} = \frac{7n^2 + 11n}{2(n+1)(n+2)} *$	<p>M1</p> <p>A1 A1 A1 (4)</p> <p>M1 A1, A1</p> <p>M1 A1 (5)</p>
	<p>Notes:</p> <p>(a) Require three constants for method.</p> <p>(b) Require first 3 and last 2 of their terms for first method</p> <p>Second method - dependent on first - for attempt to combine to single fraction.</p> <p>Expansion of $(n+1)(n+2)$ in numerator and correct solution required for final A1</p>	

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7 (a)	<p>Solve auxiliary equation $3m^2 - m - 2 = 0$ to obtain $m = -\frac{2}{3}$ or 1</p> <p>C.F is $Ae^{-\frac{2}{3}x} + Be^x$</p> <p>Let PI = $\lambda x^2 + \mu x + \nu$. Find $y' = 2\lambda x + \mu$, and $y'' = 2\lambda$ and substitute into d.e. Giving $\lambda = -\frac{1}{2}$, $\mu = \frac{1}{2}$ and $\nu = -\frac{7}{4}$</p> <p>$\therefore y = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{4} + Ae^{-\frac{2}{3}x} + Be^x$</p>	<p>M1 A1</p> <p>A1ft</p> <p>M1</p> <p>A1 A1A1</p> <p>A1ft</p> <p>(8)</p>
(b)	<p>Use boundary conditions:</p> <p>$2 = -\frac{7}{4} + A + B$</p> <p>$y' = -x + \frac{1}{2} - \frac{2}{3}Ae^{-\frac{2}{3}x} + Be^x$ and $3 = \frac{1}{2} - \frac{2}{3}A + B$</p> <p>Solve to give $A = 3/4$, $B = 3$ ($\therefore y = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{4} + \frac{3}{4}e^{-\frac{2}{3}x} + 3e^x$)</p>	<p>M1A1ft</p> <p>M1 M1</p> <p>M1 A1</p> <p>(6)</p> <p>[14]</p>
	<p>Notes:</p> <p>(a) Attempt to solve quadratic expression with 3 terms (usual rules)</p> <p>Both values required for first accuracy.</p> <p>Real values only for follow through</p> <p>Second M 3 term quadratic for PI required</p> <p>Final A1ft for their CF+ their PI dependent upon at least one M</p> <p>(b) Second M for attempt to differentiate their y and third M for substitution</p>	

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<p>8 (a)</p> <p>(b)</p> <p>(c)</p>	<p>$a(3 + 2 \cos \theta) = 4a$ Solve to obtain $\cos \theta = \frac{1}{2}$ $\theta = \pm \frac{\pi}{3}$ and points are $(4a, \frac{\pi}{3})$ and $(4a, \frac{5\pi}{3})$</p> <p>Use area = $\frac{1}{2} \int r^2 d\theta$ to give $\frac{1}{2} a^2 \int (3 + 2 \cos \theta)^2 d\theta$ Obtain $\int (9 + 12 \cos \theta + 2 \cos 2\theta + 2) d\theta$ Integrate to give $11\theta + 12 \sin \theta + \sin 2\theta$ Use limits $\frac{\pi}{3}$ and π, then double or $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ or theirs Find a third area of circle = $\frac{16\pi a^2}{3}$ Obtain required area = $\frac{38\pi a^2}{3} - \frac{13\sqrt{3}a^2}{2}$</p>  <p>correct shape 5a and 4a marked 2a marked and passes through O</p>	<p>M1 M1 A1, A1 (4)</p> <p>M1 A1 M1 A1 M1 B1 A1, A1 (8)</p> <p>B1 B1 B1 (3)</p> <p>[15]</p>
	<p>Notes:</p> <p>(a) First A for $r=4a$ second for both values in radians. Accept 1.0471... and 5.2359... 2 dp or better for final A</p> <p>(b) First M for substitution, expansion and attempt to use double angles. Second M for integrating expression of the form $a + b \cos \theta + c \cos 2\theta$ Lose final A only if a^2 missing in last line</p> <p>(c) First B for approximately symmetrical shape about initial line, only 1 loop which is convex strictly within shaded region</p>	