Mark Scheme (Results)

Summer 2013

GCE Further Pure Mathematics 2 (6668/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- ⌈ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2+bx+c)=(x+p)(x+q), \text{ where } |pq|=|c|, \text{ leading to x =}$$

$$(ax^2+bx+c)=(mx+p)(nx+q), \text{ where } |pq|=|c| \text{ and } |mn|=|a|, \text{ leading to x =}$$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0, \quad$ leading to x =...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1.		
(a)	$\dfrac{2}{(2r+1)(2r+3)}=\dfrac{A}{2r+1}+\dfrac{B}{2r+3}=,\ \ \dfrac{1}{2r+1}-\dfrac{1}{2r+3}$	M1,A1 (2)
(b)	$\dfrac{1}{3}-\dfrac{1}{5}+\dfrac{1}{5}-\dfrac{1}{7}+....\dfrac{1}{2n+1}-\dfrac{1}{2n+3}$	
	$=\dfrac{1}{3}-\dfrac{1}{2n+3}=\dfrac{2n+3-3}{3(2n+3)}$	M1
	$\sum_1^n \dfrac{3}{(2r+1)(2r+3)}=\dfrac{3}{2}\times\dfrac{2n}{3(2n+3)}=\dfrac{n}{2n+3}$	M1depA1 (3)
		[5]

Notes for Question 1

(a)

M1 for any valid attempt to obtain the PFs

A1 for $\dfrac{1}{2r+1}-\dfrac{1}{2r+3}$

NB With no working shown award M1A1 if the correct PFs are written down, but M0A0 if either one is incorrect

(b)

M1 for using **their** PFs to split each of the terms of the sum or of $\sum\dfrac{2}{(2r+1)(2r+3)}$ into 2 PFs.

At least 2 terms at the start and 1 at the end needed to show the diagonal cancellation resulting in two remaining terms.

M1dep for simplifying to a single fraction and multiplying it by the appropriate constant

A1cao for $\sum=\dfrac{n}{2n+3}$

NB: If *r* is used instead of *n* (including for the answer), only M marks are available.

Question Number	Scheme	Marks
2 **(a)**	$z = 5\sqrt{3} - 5\text{i} = r(\cos\theta + \text{i}\sin\theta)$	
	$r = \sqrt{(5^2 \times 3 + 5^2)} = 10$	B1 (1)
(b)	$\arg z = \arctan\left(-\dfrac{5}{5\sqrt{3}}\right) = -\dfrac{\pi}{6}$ $\left(\text{or } -\dfrac{\pi}{6} \pm 2n\pi\right)$	M1A1 (2)
(c)	$\left\lvert\dfrac{w}{z}\right\rvert = \dfrac{2}{10} = \dfrac{1}{5}$ or 0.2	B1 (1)
(d)	$\arg\left(\dfrac{w}{z}\right) = \dfrac{\pi}{4} - \left(-\dfrac{\pi}{6}\right),\ = \dfrac{5\pi}{12}$ $\left(\text{or } \dfrac{5\pi}{12} \pm 2n\pi\right)$	M1,A1 (2)
		[6]

Notes for Question 2
(a) B1 for $\lvert z\rvert = 10$ no working needed (b) M1 for $\arg z = \arctan\left(\pm\frac{5}{5\sqrt{3}}\right)$, $\tan(\arg z) = \pm\frac{5}{5\sqrt{3}}$, $\arg z = \arctan\left(\pm\frac{5\sqrt{3}}{5}\right)$ or $\tan(\arg z) = \pm\frac{5\sqrt{3}}{5}$ OR use their $\lvert z\rvert$ with sin or cos used correctly A1 for $= -\frac{\pi}{6}$ $\left(\text{or } -\frac{\pi}{6} \pm 2n\pi\right)$ (must be 4th quadrant) (c) B1 for $\left\lvert\frac{w}{z}\right\rvert = \frac{2}{10}$ or $\frac{1}{5}$ or 0.2 (d) M1 for $\arg\left(\frac{w}{z}\right) = \frac{\pi}{4} - \arg z$ using **their** $\arg z$ A1 for $\frac{5\pi}{12}$ $\left(\text{or } \frac{5\pi}{12} \pm 2n\pi\right)$ *Alternative for* (d): Find $\frac{w}{z} = \frac{\left(\sqrt{6}-\sqrt{2}\right)+\left(\sqrt{6}+\sqrt{2}\right)\mathrm{i}}{20}$ $\tan\left(\arg\frac{w}{z}\right) = \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}$ M1 from their $\frac{w}{z}$ $\arg\left(\frac{w}{z}\right) = \frac{5\pi}{12}$ A1 cao Work for (c) and (d) may be seen together – give B and A marks only if modulus and argument are clearly identified ie $\frac{1}{5}\left(\cos\frac{5\pi}{12} + \mathrm{i}\sin\frac{5\pi}{12}\right)$ alone scores B0M1A0

Question Number	Scheme	Marks
3	$(x=0)\quad \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\sin 0-4\times\frac{1}{2}=-2$	B1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+4\frac{\mathrm{d}y}{\mathrm{d}x}-\cos x\,(=0)$	M1
	$(x=0)\quad \frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\cos 0-4\times\frac{1}{8}=\frac{1}{2}$	A1
	$(y=)\,y_0+x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0+\frac{x^2}{2!}\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0+\frac{x^3}{3!}\left(\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)_0+....$	M1 (2! or 2 and 3! or 6)
	$(y=)\frac{1}{2}+x\times\frac{1}{8}+\frac{x^2}{2}\times(-2)+\frac{x^3}{6}\times\frac{1}{2}$	
	$y=\frac{1}{2}+\frac{x}{8}-x^2+\frac{x^3}{12}$	A1 cao **[5]**
	Alt:	
	$y=\frac{1}{2}+\frac{x}{8}+ax^2+bx^3+...$	B1
	$y''=2a+6bx+...$	M1 Diff twice
	$2a+6bx+...=\sin x-\left(\frac{1}{2}+\frac{x}{8}+ax^2+bx^3...\right)$	A1 Correct differentiation and equation used
	$2a+2=0\quad a=-1$	M1
	$6b+\frac{1}{2}=1\quad b=\frac{1}{12}$	
	$y=\frac{1}{2}+\frac{x}{8}-x^2+\frac{x^3}{12}$	A1cao

Notes for Question 3
B1 for $\left(\dfrac{d^2y}{dx^2}\right)_0 = -2$ wherever seen M1 for attempting the differentiation of the given equation. To obtain $\dfrac{d^3y}{dx^3} \pm k\dfrac{dy}{dx} \pm \cos x\left(=0\right)$ oe A1 for substituting $x=0$ to obtain $\left(\dfrac{d^3y}{dx^3}\right)_0 = \dfrac{1}{2}$ M1 for using the expansion $\left[y = f\left(x\right)\right] = f\left(0\right) + xf'\left(0\right) + \dfrac{x^2}{2(!)}f''\left(0\right) + \dfrac{x^3}{3!}f'''\left(0\right)$ with their values for $\dfrac{d^3y}{dx^3}$ and $\dfrac{d^2y}{dx^2}$. Factorial can be omitted in the x^2 term but must be shown explicitly in the x^3 term or implied by further working eg using 6. A1cao for $y = \dfrac{1}{2} + \dfrac{x}{8} - x^2 + \dfrac{x^3}{12}$ (Ignore any higher powers included) Exact decimals allowed. **Must include *y* =….** *Alternative*: B1 for $y = \dfrac{1}{2} + \dfrac{x}{8} + ax^2 + bx^3 + ...$ M1 for differentiating this twice to get $y'' = 2a + 6bx + ...$ (may not be completely correct) A1 for correct differentiation and using the given equation and the expansion of sin *x* to get $2a + 6bx + ... = \left(x - \dfrac{x^3}{3} + ...\right) - 4\left(\dfrac{1}{2} + \dfrac{x}{8} + ...\right)$ M1 for equating coefficients to obtain a value for *a* or *b* A1 cao for $y = \dfrac{1}{2} + \dfrac{x}{8} - x^2 + \dfrac{x^3}{12}$ (Ignore any higher powers included)

Question Number	Scheme	Marks
4 **(a)**	Assume true for $n=k$: $z^k = r^k(\cos k\theta + \mathrm{i}\sin k\theta)$	
	$n=k+1$: $z^{k+1} = (z^k \times z =) r^k(\cos k\theta + \mathrm{i}\sin k\theta) \times r(\cos\theta + \mathrm{i}\sin\theta)$	M1
	$= r^{k+1}(\cos k\theta\cos\theta - \sin k\theta\sin\theta + \mathrm{i}(\sin k\theta\cos\theta + \cos k\theta\sin\theta))$	M1
	$= r^{k+1}(\cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta)$	M1depA1cso
	$\therefore$ <u>if true for $n=k$,</u> also <u>true for $n=k+1$</u>	
	$k=1$ <u>$z^1 = r^1(\cos\theta + \mathrm{i}\sin\theta)$</u>; <u>True for $n=1$</u> $\therefore$ <u>true for all n</u>	A1cso (5)
	Alternative: See notes for use of $r\mathrm{e}^{\mathrm{i}\theta}$ form	
(b)	$w = 3\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$	
	$w^5 = 3^5\left(\cos\frac{15\pi}{4} + \mathrm{i}\sin\frac{15\pi}{4}\right)$	M1
	$w^5 = 243\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\mathrm{i}\right)$ $\left[= \frac{243\sqrt{2}}{2} - \frac{243\sqrt{2}}{2}\mathrm{i} \text{ or}\right]$ oe	A1 (2)
		[7]

Notes for Question 4

(a)

NB: Allow each mark if *n*, *n* + 1 used instead of *k*, *k* + 1

M1 for using the result for $n = k$ to write $z^{k+1}\left(= z^k \times z\right) = r^k\left(\cos k\theta + \mathrm{i}\sin k\theta\right) \times r\left(\cos\theta + \mathrm{i}\sin\theta\right)$

M1 for multiplying out and collecting real and imaginary parts, using $\mathrm{i}^2 = -1$
OR using sum of arguments and product of moduli to get $r^{k+1}\left(\cos\left(k\theta+\theta\right) + \mathrm{i}\sin\left(k\theta+\theta\right)\right)$

M1dep for using the addition formulae to obtain single cos and sin terms
OR factorise the argument $r^{k+1}\left(\cos\theta\left(k+1\right) + \mathrm{i}\sin\theta\left(k+1\right)\right)$
Dependent on the second M mark.

A1cso for $r^{k+1}\left(\cos\left(k+1\right)\theta + \mathrm{i}\sin\left(k+1\right)\theta\right)$ Only give this mark if all previous steps are fully correct.

A1cso All 5 underlined statements must be seen

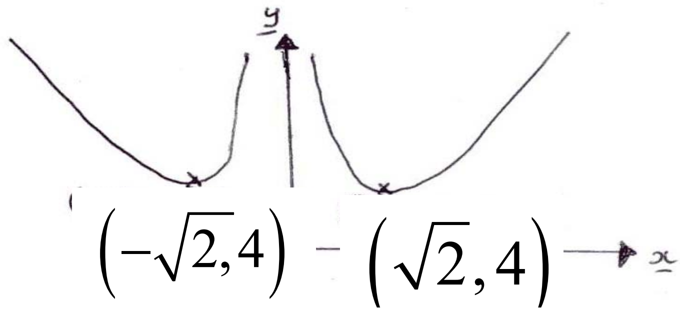
Alternative: Using Euler's form

$z = r\left(\cos\theta + i\sin\theta\right) = r\,\mathrm{e}^{\mathrm{i}\theta}$	M1 May not be seen explicitly
$z^{k+1} = z^k \times z = \left(r\,\mathrm{e}^{\mathrm{i}\theta}\right)^k \times r\,\mathrm{e}^{\mathrm{i}\theta} = r^k\,\mathrm{e}^{\mathrm{i}k\theta} \times r\,\mathrm{e}^{\mathrm{i}\theta}$	M1
$= r^{k+1}\,\mathrm{e}^{\mathrm{i}(k+1)\theta}$	M1dep on 2nd M mark
$= r^{k+1}\left(\cos\left(k+1\right)\theta + \mathrm{i}\sin\left(k+1\right)\theta\right)$	A1cso
$k = 1 \quad z^1 = r^1\left(\cos\theta + \mathrm{i}\sin\theta\right)$	
True for $n = 1$ $\therefore$ true for all *n* etc	A1 cso All 5 underlined statements must be seen

(b)

M1 for attempting to apply de Moivre to *w* or attempting to expand w^5 and collecting real and imaginary parts, but no need to simplify these.

A1cao for $243\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\mathrm{i}\right)$ $\left[= \frac{243\sqrt{2}}{2} - \frac{243\sqrt{2}}{2}\mathrm{i}\right]$ (oe eg 3^5 instead of 243)

Question Number	Scheme	Marks
5		
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}+2\frac{y}{x}=4x$	M1
	I F: $\mathrm{e}^{\int \frac{2}{x}\mathrm{d}x}=\mathrm{e}^{2\ln x}=\left(x^2\right)$	M1
	$x^2\frac{\mathrm{d}y}{\mathrm{d}x}+2xy=4x^3$	M1dep
	$yx^2=\int 4x^3\mathrm{d}x=x^4\left(+c\right)$	M1dep
	$y=x^2+\frac{c}{x^2}$	A1cso (5)
(b)	$x=1,\ y=5\Rightarrow c=4$	M1
	$y=x^2+\frac{4}{x^2}$	A1ft (2)
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x}=2x-\frac{8}{x^3}$	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=0\quad x^4=4\ ,\ x=\pm\sqrt{2}\quad$ or $\pm\sqrt[4]{4}$	M1,A1
	$y=2+\frac{4}{2}=4$	A1cao
	Alt: Complete square on $y=...$ or use the original differential equation	M1
	$x=\pm\sqrt{2},\quad y=4$	A1,A1
		B1 shape B1 turning points shown *somewhere* (5)
		[12]

Notes for Question 5

(a)

M1 for dividing the given equation by x May be implied by subsequent work.

M1 for IF= $e^{\int \frac{2}{x}dx} = e^{2\ln x} = (x^2)$ $\int \frac{2}{x}dx$ must be seen together with an attempt at integrating this. $\ln x$ must be seen in the integrated function.

M1dep for multiplying the equation $\frac{dy}{dx} + 2\frac{y}{x} = 4x$ by their IF dep on 2nd M mark

M1dep for attempting the integration of the resulting equation - constant not needed. Dep on 2nd and 3rd M marks

A1cso for $y = x^2 + \frac{c}{x^2}$ oe eg $yx^2 = x^4 + c$

Alternative: for first three marks: Multiply given equation by x to get straight to the third line. All 3 M marks should be given.

(b)

M1 for using $x = 1,\ y = 5$ in **their** expression for y to obtain a value for c

A1ft for $y = x^2 + \frac{4}{x^2}$ follow through their result from (a)

(c)

M1 for differentiating **their** result from (b), equating to 0 and solving for x

A1 for $x = \pm\sqrt{2}$ (no follow through) or $\pm\sqrt[4]{4}$ No extra real values allowed but ignore any imaginary roots shown.

A1cao for using the particular solution to obtain $y = 4$. No extra values allowed.

Alternatives for these 3 marks:

M1 for making $\frac{dy}{dx} = 0$ in the given differential equation to get $y = 2x^2$ and using this with their particular solution to obtain an equation in one variable

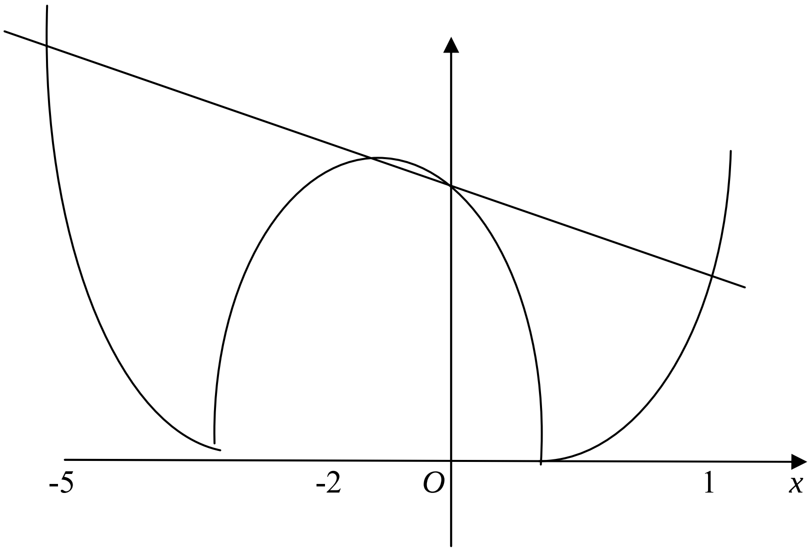
OR complete the square on **their** particular solution to get $y = \left(x + \frac{2}{x}\right)^2 - 4$

A1 for $x = \pm\sqrt{2}$ (no follow through)

A1cao for $y = 4$ No extra values allowed

B1 for the correct shape - must have two minimum points and two branches, both asymptotic to the y-axis

B1 for a fully correct sketch with the coordinates of the minimum points shown somewhere on or beside the sketch. Decimals accepted here.

Question Number	Scheme	Marks
6 **(a)**	$2x^2+6x-5=5-2x$	M1
	$2x^2+8x-10=0$	
	$x^2+4x-5=0$	
	$(x+5)(x-1)=0$ or by formula	M1
	$x=-5,\ x=1$	A1
	$-2x^2-6x+5=5-2x$	M1
	$2x^2+4x=0$	A1
	$x=0\ \ x=-2$	A1 (6)
(b)		B1 line B1 quad curve B1ft (on x-coords from (a)) (3)
(c)	$x<-5, \quad -2<x<0, \quad x>1$	B1,B1,B1 (3)
	Special case: Deduct the last B mark earned1 if $\leqslant$ or $\geqslant$ used	**[12]**

Notes for Question 6

(a)**NB: Marks for (a) can only be awarded for work shown in (a):**

M1 for $2x^2 + 6x - 5 = 5 - 2x$

M1 for obtaining a 3 term quadratic and attempting to solve by factorising, formula or completing the square

A1 for $x = -5,\ x = 1$

M1 for considering the part of the quadratic that needs to be reflected ie for $-2x^2 - 6x + 5 = 5 - 2x$ oe

A1 for a correct 2 term quadratic, terms in any order $2x^2 + 4x = 0$ oe

A1 for $x = 0\ \ x = -2$

NB: The question demands that algebra is used, so solutions which do not show how the roots have been obtained will score very few if any marks, depending on what is written on the page.

Alternative: Squaring both sides:

M1 Square both sides and simplify to a quartic expression
M1 Take out the common factor x
A1 x, a correct linear factor and a correct quadratic factor
M1 x and 3 linear factors
A1 any two of the required values
A1 all 4 values correct

(b)

B1 for a line drawn, with negative gradient, crossing the positive y-axis

B1 for the quadratic curve, with part reflected and the correct shape. It should cross the y-axis at the same point as the line and be pointed where it meets the x-axis (ie not U-shaped like a turning point)

B1ft for showing the x coordinates of the points where the line crosses the curve. They can be shown on the x-axis as in the MS (accept O for 0) or written alongside the points as long as it is clear the numbers are the x coordinates
The line should cross the curve at all the crossing points found *and no others* for this mark to be given.

(c)**NB: No follow through for these marks**

B1 for any one of $x < -5$, $-2 < x < 0$, $x > 1$ correct

B1 for a second one of these correct

B1 for the third one correct

Special case: if $\leqslant$ or $\geqslant$ is used, deduct the last B mark earned.

Question Number	Scheme	Marks
7		
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=v+x\frac{\mathrm{d}v}{\mathrm{d}x}$	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}v}{\mathrm{d}x}+\frac{\mathrm{d}v}{\mathrm{d}x}+x\frac{\mathrm{d}^2v}{\mathrm{d}x^2}$	M1A1
	$4x^2\left(2\frac{\mathrm{d}v}{\mathrm{d}x}+x\frac{\mathrm{d}^2v}{\mathrm{d}x^2}\right)-8x\left(v+x\frac{\mathrm{d}v}{\mathrm{d}x}\right)+\left(8+4x^2\right)\times xv=x^4$	M1
	$4x^3\frac{\mathrm{d}^2v}{\mathrm{d}x^2}+4x^3v=x^4$	M1
	$4\frac{\mathrm{d}^2v}{\mathrm{d}x^2}+4v=x$ *	A1 (6)
	See end for an alternative for (a)	
(b)	$4\lambda^2+4=0$	
	$\lambda^2=-1$ oe	M1A1
	$(v=)C\cos x+D\sin x$ (or $(v=)Ae^{\mathrm{i}x}+Be^{-\mathrm{i}x}$)	A1
	P.I: Try $v=kx\ (+l)$	
	$\frac{\mathrm{d}v}{\mathrm{d}x}=k \quad \frac{\mathrm{d}^2v}{\mathrm{d}x^2}=0$	M1
	$4\times 0+4(kx\ (+l))=x$	M1dep
	$k=\frac{1}{4} \quad (l=0)$	
	$v=C\cos x+D\sin x+\frac{1}{4}x$ (or $v=Ae^{\mathrm{i}x}+Be^{-\mathrm{i}x}+\frac{1}{4}x$)	A1 (6)
(c)	$y=x\left(C\cos x+D\sin x+\frac{1}{4}x\right)$ (or $y=x\left(Ae^{\mathrm{i}x}+Be^{-\mathrm{i}x}+\frac{1}{4}x\right)$)	B1ft (1)

Question 7 continued		
	Alternative for (a):	
	$v=\dfrac{y}{x}$	
	$\dfrac{\mathrm{d}v}{\mathrm{d}x}=\dfrac{\mathrm{d}y}{\mathrm{d}x}\times\dfrac{1}{x}-y\times\dfrac{1}{x^2}$	M1
	$\dfrac{\mathrm{d}^2v}{\mathrm{d}x^2}=\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\times\dfrac{1}{x}-\dfrac{\mathrm{d}y}{\mathrm{d}x}\times\dfrac{1}{x^2}-\dfrac{\mathrm{d}y}{\mathrm{d}x}\times\dfrac{1}{x^2}+2y\times\dfrac{1}{x^3}$	M1A1
	$x^3\dfrac{\mathrm{d}^2v}{\mathrm{d}x^2}=x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-2x\dfrac{\mathrm{d}y}{\mathrm{d}x}+2y$	M1
	$4x^3\dfrac{\mathrm{d}^2v}{\mathrm{d}x^2}+4x^3v=4x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-8x\dfrac{\mathrm{d}y}{\mathrm{d}x}+8y+4x^2y=x^4$	M1
	$4\dfrac{\mathrm{d}^2v}{\mathrm{d}x^2}+4v=x$ *	A1

Notes for Question 7

(a)

M1 for attempting to differentiate $y = xv$ to get $\frac{dy}{dx}$ - product rule must be used

M1 for differentiating **their** $\frac{dy}{dx}$ to obtain an expression for $\frac{d^2y}{dx^2}$ - product rule must be used

A1 for $\frac{d^2y}{dx^2} = \frac{dv}{dx} + \frac{dv}{dx} + x\frac{d^2v}{dx^2}$

M1 for substituting **their** $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ and $y = xv$ in the original equation to obtain a differential equation in v and x

M1 for collecting the terms to have at most a 4 term equation - 4 terms only if a previous error causes $\frac{dv}{dx}$ to be included, otherwise 3 terms

A1cao and cso for $4\frac{d^2v}{dx^2} + 4v = x$ *

Alternative: (see end of mark scheme)

M1 for writing $v = \frac{y}{x}$ and attempting to differentiate by quotient or product rule to get $\frac{dv}{dx}$

M1 for differentiating **their** $\frac{dv}{dx}$ to obtain an expression for $\frac{d^2v}{dx^2}$ - product or quotient rule must be used

A1 for $\frac{d^2v}{dx^2} = \frac{d^2y}{dx^2} \times \frac{1}{x} - \frac{dy}{dx} \times \frac{1}{x^2} - \frac{dy}{dx} \times \frac{1}{x^2} + 2y \times \frac{1}{x^3}$

M1 for multiplying **their** $\frac{d^2v}{dx^2}$ by x^3

M1 for multiplying by 4 **and** adding $4x^2y$ to each side and equating to x^4 (as rhs is now identical to the original equation.

A1cao and cso for $4\frac{d^2v}{dx^2} + 4v = x$ *

(b)

M1 for forming the auxiliary equation and attempting to solve

A1 for $\lambda^2 = -1$ oe

A1 for the complementary function in either form. Award for a correct CF even if $\lambda = \mathrm{i}$ only is shown.

Notes for Question 7 continued
M1 for trying one of $v = kx,\ k \neq 1$ or $v = kx + l$ and $v = mx^2 + kx + l$ as a PI and obtaining $\frac{\mathrm{d}v}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2v}{\mathrm{d}x^2}$ M1dep for substituting their differentials in the equation $4\frac{\mathrm{d}^2v}{\mathrm{d}x^2} + 4v = x$. Award M0 if the original equation is used. Dep on 2nd M mark of (b) A1cao for obtaining the correct result (either form) (c) B1ft for reversing the substitution to get $y = x\left(C\cos x + D\sin x + \frac{1}{4}x\right)$ $\left(\text{or } y = x\left(Ae^{ix} + Be^{-ix} + \frac{1}{4}x\right)\right)$ follow through their answer to (b)

Question Number	Scheme	Marks
8 (a)	$(y=)\, r\sin\theta = a\sin 2\theta \sin\theta$	M1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}=\right) a\left(2\cos 2\theta\sin\theta + \sin 2\theta\cos\theta\right)$	M1depA1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}=\right) 2a\sin\theta\left(\cos 2\theta + \cos^2\theta\right)$	M1
	At P $\frac{\mathrm{d}y}{\mathrm{d}\theta}=0 \Rightarrow \sin\theta = 0$ (n/a) or $2\cos^2\theta - 1 + \cos^2\theta = 0$	M1 $\sin\theta = 0$ not needed
	$3\cos^2\theta = 1$	
	$\cos\theta = \frac{1}{\sqrt{3}}$ *	A1cso
		(6)
(b)	$r = a\sin 2\theta = 2a\sin\theta\cos\theta$	
	$r = 2a\sqrt{\left(1-\frac{1}{3}\right)}\sqrt{\frac{1}{3}} = 2a\frac{\sqrt{2}}{3}$	M1A1 (2)
(c)	Area $= \int_0^\phi \frac{1}{2}r^2\,\mathrm{d}\theta = \frac{1}{2}a^2\int_0^\phi \sin^2 2\theta\,\mathrm{d}\theta$	M1
	$= \frac{1}{2}a^2\int_0^\phi \frac{1}{2}(1-\cos 4\theta)\,\mathrm{d}\theta$	M1
	$= \frac{1}{4}a^2\left[\theta - \frac{1}{4}\sin 4\theta\right]_0^\phi$	M1A1
	$= \frac{1}{4}a^2\left[\phi - \frac{1}{4}\left(4\sin\phi\cos\phi\left(2\cos^2\phi - 1\right)\right)\right]$	M1dep on 2nd M mark
	$= \frac{1}{4}a^2\left[\arccos\left(\frac{1}{\sqrt{3}}\right) - \left(\sqrt{\frac{2}{3}}\times\sqrt{\frac{1}{3}}\times\left(\frac{2}{3}-1\right)\right)\right]$	M1 dep (all Ms)
	$\frac{1}{36}a^2\left[9\arccos\left(\frac{1}{\sqrt{3}}\right) + \sqrt{2}\right]$ *	A1 (7)
		[15]

Notes for Question 8

(a)

M1 for obtaining the y coordinate $y = r\sin\theta = a\sin 2\theta \sin\theta$

M1dep for attempting the differentiation to obtain $\frac{dy}{d\theta}$ Product rule and/or chain rule must be used; sin to become $\pm\cos$ (cos to become $\pm\sin$). The 2 may be omitted. Dependent on the first M mark.

A1 for correct differentiation eg $\frac{dy}{d\theta} = a\left(2\cos 2\theta \sin\theta + \sin 2\theta \cos\theta\right)$ oe

M1 for using $\sin 2\theta = 2\sin\theta\cos\theta$ anywhere in their solution to (a)

M1 for setting $\frac{dy}{d\theta} = 0$ and getting a quadratic factor with no $\sin^2\theta$ included.

Alternative: Obtain a quadratic in $\sin\theta$ or $\tan\theta$ and complete to $\cos\theta$ = later.

A1cso for $\cos\theta = \frac{1}{\sqrt{3}}$ or $\cos\phi = \frac{1}{\sqrt{3}}$ *

Question 8 (a) Variations you may see:

y = rsinθ = asin2θsinθ

y = asin2θsinθ	y = 2asin²θcosθ	y = 2a(cosθ – cos³θ)
dy/dθ = a(2cos2θsinθ + sin2θcosθ) = a(2cos2θsinθ + 2sinθcos²θ) = 2asinθ(cos2θ + cos²θ) = 2asinθ(3cos²θ -1) or = 2asinθ(2cos²θ – sin²θ) or = 2asinθ(2 – 3sin²θ)	dy/dθ = 2a(2sinθcos²θ – sin³θ) = 2asinθ(2cos²θ – sin²θ)	dy/dθ = 2a(-sinθ +3sinθcos²θ) = 2asinθ(3cos²θ -1)

At P: dy/dθ = 0 => sin θ = 0 or:		
$2\cos^2\theta - \sin^2\theta = 0$	$3\cos^2\theta - 1 = 0$	$2 - 3\sin^2\theta = 0$
$\tan^2\theta = 2$	$\cos^2\theta = 1/3$	$\sin^2\theta = 2/3$
$\tan\theta = \pm\sqrt{2} \Rightarrow \cos\theta = \pm\frac{1}{\sqrt{3}}$	$\cos\theta = \pm\frac{1}{\sqrt{3}}$	$\text{Sin }\theta = \pm\frac{\sqrt{2}}{\sqrt{3}} = \pm\frac{\sqrt{6}}{3} \Rightarrow \cos\theta = \pm\frac{1}{\sqrt{3}}$

(b)

M1 for using $\sin 2\theta = 2\sin\theta\cos\theta$, $\cos^2\theta + \sin^2\theta = 1$ and $\cos\phi = \frac{1}{\sqrt{3}}$ in $r = a\sin 2\theta$ to obtain a numerical multiple of a for R. Need not be simplified.

A1cao for $R = 2a\frac{\sqrt{2}}{3}$

Can be done on a calculator. Completely correct answer with no working scores 2/2; incorrect answer with no working scores 0/2

Notes for Question 8 continued
(c) M1 for using the area formula $\int_0^{\phi} \frac{1}{2} r^2 \mathrm{d}\theta = \frac{1}{2} a^2 \int_0^{\phi} \sin^2 2\theta \,\mathrm{d}\theta$ Limits not needed M1 for preparing $\int \sin^2 2\theta \,\mathrm{d}\theta$ for integration by using $\cos 2x = 1 - 2\sin^2 x$ M1 for attempting the integration: $\cos 4\theta$ to become $\pm \sin 4\theta$ - the $\frac{1}{4}$ may be missing but inclusion of 4 implies differentiation - and the constant to become $k\theta$. Limits not needed. A1 for $= \frac{1}{4} a^2 \left[\theta - \frac{1}{4} \sin 4\theta \right]$ Limits not needed M1dep for changing **their** integrated function to an expression in $\sin\theta$ and $\cos\theta$ and substituting limits 0 and ϕ. Dependent on the second M mark of (c) M1dep for a numerical multiple of a^2 for the area. Dependent on all previous M marks of (c) A1cso for $\frac{1}{36} a^2 \left[9 \arccos\left(\frac{1}{\sqrt{3}} \right) + \sqrt{2} \right]$ * This is a given answer, so check carefully that it can be obtained from the previous step in their working. **Also**: The final 3 marks can only be awarded if the working is **shown** ie $\sin 4\theta$ cannot be obtained by calculator.

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