

# Mark Scheme (Results) Summer 2008

GCE

## GCE Mathematics (6666/01)

June 2008  
6666 Core Mathematics C4  
Mark Scheme

Question	Scheme	Marks																					
1. (a)	<table border="1" style="margin: auto; border-collapse: collapse;"> <tr> <td style="padding: 2px 10px;">x</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0.4</td> <td style="padding: 2px 10px;">0.8</td> <td style="padding: 2px 10px;">1.2</td> <td style="padding: 2px 10px;">1.6</td> <td style="padding: 2px 10px;">2</td> </tr> <tr> <td style="padding: 2px 10px;">y</td> <td style="padding: 2px 10px;"><math>e^0</math></td> <td style="padding: 2px 10px;"><math>e^{0.08}</math></td> <td style="padding: 2px 10px;"><math>e^{0.32}</math></td> <td style="padding: 2px 10px;"><math>e^{0.72}</math></td> <td style="padding: 2px 10px;"><math>e^{1.28}</math></td> <td style="padding: 2px 10px;"><math>e^2</math></td> </tr> <tr> <td style="padding: 2px 10px;">or y</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">1.08329 ...</td> <td style="padding: 2px 10px;">1.37713...</td> <td style="padding: 2px 10px;">2.05443...</td> <td style="padding: 2px 10px;">3.59664...</td> <td style="padding: 2px 10px;">7.38906...</td> </tr> </table>	x	0	0.4	0.8	1.2	1.6	2	y	$e^0$	$e^{0.08}$	$e^{0.32}$	$e^{0.72}$	$e^{1.28}$	$e^2$	or y	1	1.08329 ...	1.37713...	2.05443...	3.59664...	7.38906...	
x	0	0.4	0.8	1.2	1.6	2																	
y	$e^0$	$e^{0.08}$	$e^{0.32}$	$e^{0.72}$	$e^{1.28}$	$e^2$																	
or y	1	1.08329 ...	1.37713...	2.05443...	3.59664...	7.38906...																	
	<p>Either <math>e^{0.32}</math> and <math>e^{1.28}</math> or awrt 1.38 and 3.60 (or a mixture of e's and decimals)</p>	B1  [1]																					
(b) Way 1	$\text{Area} \approx \frac{1}{2} \times 0.4 \times \left[ e^0 + 2(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}) + e^2 \right]$ $= 0.2 \times 24.61203164... = 4.922406... = \underline{4.922} \text{ (4sf)}$	<p>Outside brackets <math>\frac{1}{2} \times 0.4</math> or 0.2</p> <p><u>For structure of trapezium rule</u> [ ..... ];</p> <p>B1; M1√</p> <p>A1 cao [3]</p>																					
<i>Aliter</i> (b) Way 2	$\text{Area} \approx 0.4 \times \left[ \frac{e^0 + e^{0.08}}{2} + \frac{e^{0.08} + e^{0.32}}{2} + \frac{e^{0.32} + e^{0.72}}{2} + \frac{e^{0.72} + e^{1.28}}{2} + \frac{e^{1.28} + e^2}{2} \right]$ <p>which is equivalent to:</p> $\text{Area} \approx \frac{1}{2} \times 0.4 \times \left[ e^0 + 2(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}) + e^2 \right]$ $= 0.2 \times 24.61203164... = 4.922406... = \underline{4.922} \text{ (4sf)}$	<p>0.4 and a divisor of 2 on all terms inside brackets.</p> <p>One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.</p> <p>B1 M1√</p> <p>A1 cao [3]</p>																					
		4 marks																					

Note an expression like  $\text{Area} \approx \frac{1}{2} \times 0.4 + e^0 + 2(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}) + e^2$  would score B1M1A0

Allow one term missing (slip!) in the ( ) brackets for

The M1 mark for structure is for the material found in the curly brackets ie  
 $\left[ \text{first y ordinate} + 2(\text{intermediate ft y ordinate}) + \text{final y ordinate} \right]$

Question Number	Scheme	Marks
2. (a)	$\left\{ \begin{array}{l} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}$ $\int x e^x dx = x e^x - \int e^x \cdot 1 dx$ $= x e^x - \int e^x dx$ $= x e^x - e^x (+ c)$	<p>Use of 'integration by parts' formula in the <b>correct direction</b>. (See note.) M1</p> <p>Correct expression. (Ignore dx) A1</p> <p>Correct integration with/without + c A1</p> <p style="text-align: right;"><b>[3]</b></p>
	<p>(b)</p> $\left\{ \begin{array}{l} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}$ $\int x^2 e^x dx = x^2 e^x - \int e^x \cdot 2x dx$ $= x^2 e^x - 2 \int x e^x dx$ $= x^2 e^x - 2(x e^x - e^x) + c$ $\left\{ \begin{array}{l} = x^2 e^x - 2x e^x + 2e^x + c \\ = e^x (x^2 - 2x + 2) + c \end{array} \right\}$	<p>Use of 'integration by parts' formula in the <b>correct direction</b>. M1</p> <p>Correct expression. (Ignore dx) A1</p> <p>Correct expression <b>including + c. (seen at any stage! in part (b))</b> A1 ISW</p> <p>You can ignore subsequent working.</p> <p style="text-align: right;"><b>[3]</b></p> <p style="text-align: right;"><b>6 marks</b></p>

Note integration by parts in the **correct direction** means that  $u$  and  $\frac{dv}{dx}$  must be assigned/used as  $u = x$  and  $\frac{dv}{dx} = e^x$  in part (a) for example

+ c is not required in part (a).

Question Number	Scheme	Marks
3. (a)	<p>From question, <math>\frac{dA}{dt} = 0.032</math></p> <p><math>\left\{ A = \pi x^2 \Rightarrow \frac{dA}{dx} = \right\} 2\pi x</math></p> <p><math>\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = (0.032) \frac{1}{2\pi x}; \left\{ = \frac{0.016}{\pi x} \right\}</math></p> <p>When <math>x = 2 \text{ cm}</math>, <math>\frac{dx}{dt} = \frac{0.016}{2\pi}</math></p> <p>Hence, <math>\frac{dx}{dt} = 0.002546479... \text{ (cm s}^{-1}\text{)}</math></p>	<p><math>\frac{dA}{dt} = 0.032</math> seen or implied from working. B1</p> <p><math>2\pi x</math> by itself seen or implied from working B1</p> <p><math>0.032 \div \text{Candidate's } \frac{dA}{dx}</math>; M1;</p> <p>awrt 0.00255 A1 cso</p> <p>[4]</p>
(b)	<p><math>V = \underline{\pi x^2(5x)} = \underline{5\pi x^3}</math></p> <p><math>\frac{dV}{dx} = 15\pi x^2</math></p> <p><math>\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 15\pi x^2 \cdot \left( \frac{0.016}{\pi x} \right); \{ = 0.24x \}</math></p> <p>When <math>x = 2 \text{ cm}</math>, <math>\frac{dV}{dt} = 0.24(2) = \underline{0.48} \text{ (cm}^3 \text{ s}^{-1}\text{)}</math></p>	<p><math>V = \underline{\pi x^2(5x)}</math> or <math>\underline{5\pi x^3}</math> B1</p> <p><math>\frac{dV}{dx} = 15\pi x^2</math> or ft from candidate's V in one variable B1 <math>\sqrt{\quad}</math></p> <p>Candidate's <math>\frac{dV}{dx} \times \frac{dx}{dt}</math>; M1 <math>\sqrt{\quad}</math></p> <p><u>0.48</u> or awrt 0.48 A1 cso</p> <p>[4]</p>
<b>8 marks</b>		

Question Number	Scheme	Marks
4. (a)	<p style="text-align: center;"><math>3x^2 - y^2 + xy = 4</math> ( eqn * )</p> <p style="text-align: center;"><del><math>\frac{dy}{dx}</math></del> <math>\times</math> <math>\left\{ \frac{dy}{dx} \right\}</math> <math>6x - 2y \frac{dy}{dx} + \left( y + x \frac{dy}{dx} \right) = 0</math></p> <p style="text-align: center;"><math>\left\{ \frac{dy}{dx} = \frac{-6x - y}{x - 2y} \right\}</math> or <math>\left\{ \frac{dy}{dx} = \frac{6x + y}{2y - x} \right\}</math></p> <p style="text-align: center;"><math>\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x - y}{x - 2y} = \frac{8}{3}</math></p> <p style="text-align: center;">giving <math>-18x - 3y = 8x - 16y</math></p> <p style="text-align: center;">giving <math>13y = 26x</math></p> <p style="text-align: center;">Hence, <math>y = 2x \Rightarrow \underline{y - 2x = 0}</math></p> <p>(b) At P &amp; Q, <math>y = 2x</math>. Substituting into eqn *</p> <p style="text-align: center;">gives <math>3x^2 - (2x)^2 + x(2x) = 4</math></p> <p style="text-align: center;">Simplifying gives, <math>x^2 = 4 \Rightarrow \underline{x = \pm 2}</math></p> <p style="text-align: center;"><math>y = 2x \Rightarrow y = \pm 4</math></p> <p style="text-align: center;">Hence coordinates are <math>\underline{(2, 4)}</math> and <math>\underline{(-2, -4)}</math></p>	<p style="text-align: center;">Differentiates implicitly to include either <math>\pm ky \frac{dy}{dx}</math> or <math>x \frac{dy}{dx}</math>. (Ignore <math>\left( \frac{dy}{dx} = \right)</math>)</p> <p style="text-align: center;">Correct application <math>\left( \underline{\quad} \right)</math> of product rule</p> <p style="text-align: center;"><math>(3x^2 - y^2) \rightarrow \left( \underline{6x - 2y} \frac{dy}{dx} \right)</math> and <math>(4 \rightarrow \underline{0})</math></p> <p style="text-align: center;"><i>not necessarily required.</i></p> <p style="text-align: center;">Substituting <math>\frac{dy}{dx} = \frac{8}{3}</math> into their equation.</p> <p style="text-align: center;">Attempt to combine either terms in x or terms in y together to give either <i>ax</i> or <i>by</i>.</p> <p style="text-align: center;">simplifying to give <math>\underline{y - 2x = 0}</math> <b>AG</b></p> <p style="text-align: center;">Attempt replacing <math>y</math> by <math>2x</math> in at least one of the <math>y</math> terms in eqn *</p> <p style="text-align: center;">Either <math>x = 2</math> or <math>x = -2</math></p> <p style="text-align: center;">Both <math>\underline{(2, 4)}</math> and <math>\underline{(-2, -4)}</math></p>
		M1 B1 A1  M1 *  dM1 *  A1 cso <b>[6]</b>
		M1 A1  A1 <b>[3]</b>
		<b>9 marks</b>

Question Number	Scheme	Marks
5. (a)	<p><b>** represents a constant (which must be consistent for first accuracy mark)</b></p> $\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)^{-\frac{1}{2}}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}} = \underline{\underline{1}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$ <p style="text-align: right;"><math>(4)^{-\frac{1}{2}}</math> or <math>\frac{1}{2}</math> outside brackets</p> <p>Expands <math>(1+**x)^{-\frac{1}{2}}</math> to give a simplified or an un-simplified <math>1 + (-\frac{1}{2})(**x)</math>;</p> $= \frac{1}{2} \left[ 1 + (-\frac{1}{2})(**x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (**x)^2 + \dots \right]$ <p><b>with ** <math>\neq</math> 1</b></p> $= \frac{1}{2} \left[ 1 + (-\frac{1}{2})(-\frac{3x}{4}) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (-\frac{3x}{4})^2 + \dots \right]$ $= \frac{1}{2} \left[ 1 + \frac{3}{8}x + \frac{27}{128}x^2 + \dots \right]$ $\left\{ = \frac{1}{2} + \frac{3}{16}x + \frac{27}{256}x^2 + \dots \right\}$	<p><b>B1</b></p> <p><b>M1;</b></p> <p><b>A1 <math>\sqrt</math></b></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p><b>Award SC M1 if you see</b>  <math>(-\frac{1}{2})(**x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (**x)^2</math></p> </div> <p><math>\frac{1}{2} [ 1 + \frac{3}{8}x; \dots ]</math></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p><b>SC:</b> <math>K [ 1 + \frac{3}{8}x + \frac{27}{128}x^2 + \dots ]</math></p> </div> <p><math>\frac{1}{2} [ \dots; \frac{27}{128}x^2 ]</math></p> <p><i>Ignore subsequent working</i></p> <p style="text-align: right;"><b>[5]</b></p> <p>Writing <math>(x+8)</math> multiplied by candidate's part (a) expansion. <b>M1</b></p> <p>Multiply out brackets to find a constant term, two <math>x</math> terms and two <math>x^2</math> terms.</p> <p>Anything that cancels to <math>4 + 2x; \frac{33}{32}x^2</math></p> <p style="text-align: right;"><b>A1; A1</b></p> <p style="text-align: right;"><b>[4]</b></p> <p style="text-align: right;"><b>9 marks</b></p>

Question Number	Scheme	Marks
6. (a)	<p>Lines meet where:</p> $\begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 17 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$ <p> <b>i:</b> <math>-9 + 2\lambda = 3 + 3\mu</math> (1)  Any two of <b>j:</b> <math>\lambda = 1 - \mu</math> (2)  <b>k:</b> <math>10 - \lambda = 17 + 5\mu</math> (3) </p> <p>(1) - 2(2) gives: <math>-9 = 1 + 5\mu \Rightarrow \mu = -2</math></p> <p>(2) gives: <math>\lambda = 1 - (-2) = 3</math></p> $\mathbf{r} = \begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + 3 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \quad \text{or} \quad \mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 17 \end{pmatrix} - 2 \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$ <p>Intersect at <math>\mathbf{r} = \begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix}</math> or <math>\mathbf{r} = \underline{-3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}}</math></p> <p>Either check <b>k:</b>  <math>\lambda = 3</math>: LHS = <math>10 - \lambda = 10 - 3 = 7</math>  <math>\mu = -2</math>: RHS = <math>17 + 5\mu = 17 - 10 = 7</math></p> <p>(As LHS = RHS then the lines intersect.)</p>	<p>Need any two of these correct equations seen anywhere in part (a). M1</p> <p>Attempts to solve simultaneous equations to find one of either <math>\lambda</math> or <math>\mu</math> dM1</p> <p>Both <math>\underline{\lambda = 3}</math> &amp; <math>\underline{\mu = -2}</math> A1</p> <p>Substitutes their value of <b>either</b> <math>\lambda</math> or <math>\mu</math> into the line <math>l_1</math> or <math>l_2</math> <b>respectively</b>. This mark can be implied by any two correct components of <math>(-3, 3, 7)</math>. ddM1</p> <p><math>\begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix}</math> or <math>\underline{-3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}}</math> A1</p> <p>or <math>(-3, 3, 7)</math></p> <p>Either check that <math>\lambda = 3, \mu = -2</math> in a third equation or check that <math>\lambda = 3, \mu = -2</math> give the same coordinates on the other line. B1</p> <p>Conclusion not needed. [6]</p>
(b)	<p><math>\mathbf{d}_1 = 2\mathbf{i} + \mathbf{j} - \mathbf{k}</math> , <math>\mathbf{d}_2 = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}</math></p> $\text{As } \mathbf{d}_1 \cdot \mathbf{d}_2 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} = \underline{(2 \times 3) + (1 \times -1) + (-1 \times 5)} = 0$ <p>Then <math>l_1</math> is perpendicular to <math>l_2</math>.</p>	<p>Dot product calculation between the <b>two direction vectors:</b>  <math>\underline{(2 \times 3) + (1 \times -1) + (-1 \times 5)}</math>  or <math>\underline{6 - 1 - 5}</math> M1</p> <p>Result '<math>=0</math>' and appropriate conclusion A1</p> <p>[2]</p>

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6. (c)	<p>Equating <math>\mathbf{i}</math>; <math>-9 + 2\lambda = 5 \Rightarrow \lambda = 7</math></p> $\mathbf{r} = \begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + 7 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}$ <p>(= <math>\overline{OA}</math>. Hence the point A lies on <math>l_1</math>.)</p>	<p>Substitutes candidate's <math>\lambda = 7</math> into the line <math>l_1</math> and finds <math>5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}</math>. The conclusion on this occasion is not needed.</p> <p>B1</p> <p>[1]</p>
(d)	<p>Let <math>\overline{OX} = -3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}</math> be point of intersection</p> $\overline{AX} = \overline{OX} - \overline{OA} = \begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} = \begin{pmatrix} -8 \\ -4 \\ 4 \end{pmatrix}$ $\overline{OB} = \overline{OA} + \overline{AB} = \overline{OA} + 2\overline{AX}$ $\overline{OB} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} + 2 \begin{pmatrix} -8 \\ -4 \\ 4 \end{pmatrix}$ <p>Hence, <math>\overline{OB} = \begin{pmatrix} -11 \\ -1 \\ 11 \end{pmatrix}</math> or <math>\overline{OB} = \underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}</math></p>	<p>Finding the difference between their <math>\overline{OX}</math> (can be implied) and <math>\overline{OA}</math>.</p> $\overline{AX} = \pm \left( \begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} \right)$ <p>M1 <math>\sqrt{\pm}</math></p> $\begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} + 2 \left( \text{their } \overline{AX} \right)$ <p>dM1 <math>\sqrt{\pm}</math></p> $\begin{pmatrix} -11 \\ -1 \\ 11 \end{pmatrix} \text{ or } \underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$ <p>A1</p> <p>[3]</p> <p>12 marks</p>



Question Number	Scheme	Marks
7. (a)	$\frac{2}{4-y^2} \equiv \frac{2}{(2-y)(2+y)} \equiv \frac{A}{(2-y)} + \frac{B}{(2+y)}$ $2 \equiv A(2+y) + B(2-y)$ <p>Let <math>y = -2</math>, <math>2 = B(4) \Rightarrow B = \frac{1}{2}</math></p> <p>Let <math>y = 2</math>, <math>2 = A(4) \Rightarrow A = \frac{1}{2}</math></p> <p>giving <math>\frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}</math></p> <p>(If no <b>working seen</b>, but candidate writes down <b>correct partial fraction</b> then award all three marks. If no working is seen but one of <math>A</math> or <math>B</math> is incorrect then M0A0A0.)</p>	<p>Forming this identity.  <b>NB:</b> A &amp; B are not assigned in this question</p> <p>M1</p> <p>Either one of <math>A = \frac{1}{2}</math> or <math>B = \frac{1}{2}</math></p> <p>A1</p> <p><math>\frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}</math>, aef</p> <p>A1 cao</p> <p>[3]</p>

Question Number	Scheme	Marks
7. (b)	$\int \frac{2}{4-y^2} dy = \int \frac{1}{\cot x} dx$ $\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)} dy = \int \tan x dx$ $\therefore -\frac{1}{2} \ln(2-y) + \frac{1}{2} \ln(2+y) = \ln(\sec x) + (c)$ $y=0, x=\frac{\pi}{3} \Rightarrow -\frac{1}{2} \ln 2 + \frac{1}{2} \ln 2 = \ln\left(\frac{1}{\cos(\frac{\pi}{3})}\right) + c$ $\{0 = \ln 2 + c \Rightarrow \underline{c = -\ln 2}\}$ $-\frac{1}{2} \ln(2-y) + \frac{1}{2} \ln(2+y) = \ln(\sec x) - \ln 2$ $\frac{1}{2} \ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$ $\ln\left(\frac{2+y}{2-y}\right) = 2 \ln\left(\frac{\sec x}{2}\right)$ $\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)^2$ $\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$ <p>Hence, <math>\underline{\underline{\sec^2 x = \frac{8+4y}{2-y}}}</math></p>	<p>Separates variables as shown. Can be implied. Ignore the integral signs, and the '2'.</p> <p>B1</p> <p><math>\ln(\sec x)</math> or <math>-\ln(\cos x)</math> B1 M1; their <math>\int \frac{1}{\cot x} dx = \text{LHS}</math> correct with ft for their A and B and no error with the "2" with or without + c A1 <math>\sqrt{\quad}</math></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">       Use of <math>y=0</math> and <math>x=\frac{\pi}{3}</math> in an integrated equation containing c ;     </div> <p>M1*</p> <p>Using either the quotient (or product) or power laws for logarithms CORRECTLY. M1</p> <p>Using the log laws correctly to obtain a single log term on both sides of the equation. dM1*</p> <p><math>\underline{\underline{\sec^2 x = \frac{8+4y}{2-y}}}</math> A1 aef</p> <p style="text-align: right;"><b>[8]</b></p> <p><b>11 marks</b></p>

Question Number	Scheme	Marks	
<p>8. (a)</p> <p>(b)</p>	<p>At <math>P(4, 2\sqrt{3})</math> either <math>4 = 8\cos t</math> or <math>2\sqrt{3} = 4\sin 2t</math></p> <p><math>\Rightarrow</math> only solution is <math>t = \frac{\pi}{3}</math> where <math>0 \leq t \leq \frac{\pi}{2}</math></p> <p><math>x = 8\cos t, \quad y = 4\sin 2t</math></p> <p><math>\frac{dx}{dt} = -8\sin t, \quad \frac{dy}{dt} = 8\cos 2t</math></p> <p>At <math>P, \quad \frac{dy}{dx} = \frac{8\cos(\frac{2\pi}{3})}{-8\sin(\frac{\pi}{3})}</math></p> $\left\{ = \frac{8(-\frac{1}{2})}{(-8)(\frac{\sqrt{3}}{2})} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58 \right\}$ <p>Hence <math>m(N) = -\sqrt{3}</math> or <math>\frac{-1}{\frac{1}{\sqrt{3}}}</math></p> <p>N: <math>y - 2\sqrt{3} = -\sqrt{3}(x - 4)</math></p> <p>N: <math>y = -\sqrt{3}x + 6\sqrt{3}</math> <b>AG</b></p> <p>or <math>2\sqrt{3} = -\sqrt{3}(4) + c \Rightarrow c = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}</math></p> <p>so N: <math>\boxed{y = -\sqrt{3}x + 6\sqrt{3}}</math></p>	<p><math>4 = 8\cos t</math> or <math>2\sqrt{3} = 4\sin 2t</math></p> <p><math>t = \frac{\pi}{3}</math> or awrt 1.05 (radians) only stated in the range <math>0 \leq t \leq \frac{\pi}{2}</math></p> <p>Attempt to differentiate both <math>x</math> and <math>y</math> wrt <math>t</math> to give <math>\pm p\sin t</math> and <math>\pm q\cos 2t</math> respectively</p> <p>Correct <math>\frac{dx}{dt}</math> and <math>\frac{dy}{dt}</math></p> <p>Divides in correct way round and attempts to substitute their value of <math>t</math> (in degrees or radians) into their <math>\frac{dy}{dx}</math> expression.</p> <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>You may need to check candidate's substitutions for M1*</p> </div> <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>Note the next two method marks are dependent on M1*</p> </div> <p>Uses <math>m(N) = -\frac{1}{\text{their } m(T)}</math>.</p> <p>Uses <math>y - 2\sqrt{3} = (\text{their } m_N)(x - 4)</math> or finds <math>c</math> using <math>x = 4</math> and <math>y = 2\sqrt{3}</math> and uses <math>y = (\text{their } m_N)x + "c"</math>.</p> <p><math>y = -\sqrt{3}x + 6\sqrt{3}</math></p>	<p>M1</p> <p>A1</p> <p>[2]</p> <p>M1</p> <p>A1</p> <p>M1*</p> <p>dM1*</p> <p>dM1*</p> <p>A1 cso AG</p> <p>[6]</p>

Question	Scheme	Marks
8. (c)	$A = \int_0^4 y \, dx = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4 \sin 2t \cdot (-8 \sin t) \, dt$ $A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32 \sin 2t \cdot \sin t \, dt = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2 \sin t \cos t) \cdot \sin t \, dt$ $A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64 \cdot \sin^2 t \cos t \, dt$ $A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 64 \cdot \sin^2 t \cos t \, dt$	<p>attempt at <math>A = \int y \frac{dx}{dt} \, dt</math> correct expression (ignore limits and <math>dt</math>)</p> <p>Seeing <math>\sin 2t = 2 \sin t \cos t</math> anywhere in PART (c).</p> <p>Correct proof. Appreciation of how the negative sign affects the limits. <b>Note that the answer is given in the question.</b></p> <p>M1 A1 M1 A1 AG [4]</p>
(d)	<p>{Using substitution <math>u = \sin t \Rightarrow \frac{du}{dt} = \cos t</math>}</p> <p>{change limits: when <math>t = \frac{\pi}{3}</math>, <math>u = \frac{\sqrt{3}}{2}</math> &amp; when <math>t = \frac{\pi}{2}</math>, <math>u = 1</math>}</p> $A = 64 \left[ \frac{\sin^3 t}{3} \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \quad \text{or} \quad A = 64 \left[ \frac{u^3}{3} \right]_{\frac{\sqrt{3}}{2}}^1$ $A = 64 \left[ \frac{1}{3} - \left( \frac{1}{3} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \right) \right]$ $A = 64 \left( \frac{1}{3} - \frac{1}{8} \sqrt{3} \right) = \frac{64}{3} - 8\sqrt{3}$ <p>(Note that <math>a = \frac{64}{3}</math>, <math>b = -8</math>)</p>	<p><math>k \sin^3 t</math> or <math>ku^3</math> with <math>u = \sin t</math> Correct integration ignoring limits.</p> <p>Substitutes limits of either (<math>t = \frac{\pi}{2}</math> and <math>t = \frac{\pi}{3}</math>) or (<math>u = 1</math> and <math>u = \frac{\sqrt{3}}{2}</math>) and subtracts the correct way round.</p> <p><math>\frac{64}{3} - 8\sqrt{3}</math></p> <p>Aef in the form <math>a + b\sqrt{3}</math>, with awrt 21.3 and anything that cancels to <math>a = \frac{64}{3}</math> and <math>b = -8</math>.</p> <p>M1 A1 dM1 A1 aef isw [4]</p>
		<b>16 marks</b>